Ph1a Homework Solution 2

Fall 2009

Each homework problem is worth 5 points. Please disregard the point values listed on the problem itself. Use these instead.

2.1 QP9 (5 points)

2.1.a (1 point)

To find the velocity we just use a definition

$$\vec{V} \equiv \frac{(\vec{r}_f - \vec{r}_0)}{(t_f - t_0)}$$

where

$$\vec{r}_f = [0,\, 2\,\text{cm},\, 0];\ \vec{r}_0 = [1\,\text{cm},\, 0,\, 0];\ t_f = 5\,\text{sec};\ t_0 = 0$$

so that

$$\vec{V} = \frac{1}{5} \cdot [-1,\, 2,\, 0]\,\frac{\text{cm}}{\text{sec}}.$$

The speed being the magnitude of the velocity is thus

$$|\vec{V}| = \sqrt{\frac{1}{5}}\,\frac{\text{cm}}{\text{sec}} \approx 0.447\frac{\text{cm}}{\text{sec}}.$$

2.1.b (2 points)

The vector from **B** to **C** was found in the last part to be

$$\vec{\ell}_1 = [-1,\, 2,\, 0]\,\text{cm}$$

and similarly the vector from **B** to **D** is given by

$$\vec{\ell}_2 = [-1,\, 0,\, 3]\,\text{cm}$$

The area $\mathcal{A}$ of the side with vertices **B**, **C** and **D** is given by

$$\mathcal{A} = \frac{1}{2}|\vec{\ell}_1 \times \vec{\ell}_2|$$

$$\vec{\ell}_1 \times \vec{\ell}_2 = [-1,\, 2,\, 0] \times [-1,\, 0,\, 3]\,\text{cm}^2 = [6,\, 3,\, 2]\,\text{cm}^2$$

$$|\vec{\ell}_1 \times \vec{\ell}_2| = [\sqrt{36 + 9 + 4}\,]\,\text{cm}^2 = 7\,\text{cm}^2$$

$$\rightarrow \mathcal{A} = \frac{7}{2}\,\text{cm}^2$$

2.1.c (2 points)

The fly must travel in the direction that is perpendicular to the side in part (b). However the vector that is found in the cross product in part (b) is perpendicular to this face. So we can use this to first calculate a unit vector in the direction that the fly travels. (Note, there are two valid answers.)

$$\hat{n} = \pm\,\frac{1}{7}\,[6,\, 3,\, 2]$$

Since the fly's speed is 3 (cm/sec), it follows that its velocity is

$$\vec{V} = \pm\,\frac{3}{7}\,[6,\, 3,\, 2]\,\frac{\text{cm}}{\text{sec}}$$

and since it travels for 7 seconds, its has a displacement vector given by

$$\vec{s} \ = \ \pm [18,\ 9,\ 6]\,\mathrm{cm}.$$

Since the bug took off from the position described by $r_0 = [0,\ 2\,\mathrm{cm},\ 0]$, its final location must be

$$\vec{r}_f \ = \ [18,\ 11,\ 6]\,\mathrm{cm} \text{ or } [-18,\ -7,\ -6]\,\mathrm{cm}.$$

2.2 QP43 (5 points)

2.2.a (1 point)

$$l = |\vec{b}| \cos\theta = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{3 \cdot 1 + 1 \cdot 3}{\sqrt{3^2 + 1^2}} = \frac{6}{\sqrt{10}}$$

2.2.b (1.5 points)

$$\begin{aligned} \cos\theta &= \frac{\vec{b} \cdot \vec{a}}{|\vec{b}||\vec{a}|} = \frac{3 \cdot 1 + 1 \cdot 3}{\sqrt{1^2 + 3^2}\sqrt{3^2 + 1^2}} = 0.6 \\ \theta &= \cos^{-1}\theta = 0.9 \text{ rad} = 53 \text{ degrees} \end{aligned}$$

2.2.c (1.5 points)

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta = (\sqrt{3^2 + 1^2})^2 \sqrt{1 - 0.6^2} = 8.0$$

Following the right-hand rule, the direction is out of the page.

2.2.d (1 point)

$$A = \frac{1}{2}|\vec{a} \times \vec{b}| = 4.0$$

2.3 Frautschi 4.17 (5 points)

2.3.a (2 points)

Let the initial velocity be v_o and the angel be θ. The range will be

$$R = \frac{2v_0 \sin\theta v_o \cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

The optimal happens at $\theta = 45^o$, where the ranage takes the vaule $R = \frac{v_0^2}{g}$. Given $R = 500m$ and $g = 9.8m/s^2$, we get $v_0 = 70m/s$.

2.3.b (3 points)

Let R_2 be the case with the horse, and the only difference now is the horizontal initial velocity, while the vertical initial velocity is the same for with and without the horse. The horizontal initial velocity gains an extra of V_H, so it is $V_H + V_0 \cos\theta$.

$$\frac{R_2}{R} = \frac{V_H + V_0 \cos\theta}{V_0 \cos\theta}$$

2.4 Frautschi 4.20 (5 points)

2.4.a (3 points)

From problem 17 of Chapter 4, we know that the total horizontal distance

$$x_f = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

Now we need a relation between v_0 and the hight h. If the maximum hight h is reached at $t_{1/2} = t_f/2$, then

$$h = \frac{1}{2} g (t_f/2)^2$$

$$t_f/2 = \frac{v_0 \sin\theta}{g}$$

Eliminating t_f from the above two equations, we get

$$v_0^2 = \frac{2gh}{\sin\theta}$$

which, substitute into the top equation of x_f, gives

$$x_f = 4h \cot\theta$$

Or,

$$\theta = \arctan \frac{4h}{x_f}$$

2.4.b (2 points)

Use $x_f = 0.2m$ and $0.4m$ and the equation above, we get the values as follows.

	h	a	b	θ_a	θ_b	$\theta_a - \theta_b$
3 balls	0.35	0.2	0.4	82^0	74^0	8^0
7 balls	3.25	0.2	0.4	89^0	88^0	1^0
13 balls	13	0.2	0.4	89.9^0	89.4^0	0.5^0