# Ph1a Homework Solution 1

Fall 2009

*Each homework problem is worth 5 points. Please disregard the point values listed on the problem itself. Use these instead.*

## 1.1 QP1 (5 points)

### 1.1.a (0.5 points)

$h = \frac{1}{2}gt^2$, so $t = \sqrt{\frac{2h}{g}}$. For $h = 2$ m, we get $t = 0.64$ sec.

### 1.1.b (1 point)

This situation is symmetric that in part a). The ball will bounce back to the original height $h = 2$ m since the collision is elastic, and so the time it takes to go up is the same $t = 0.64$ sec. You can double check this by calculating the velocity right before impact in part a, reversing its direction, and finding the time to stop on the way back up.

### 1.1.c (1.5 points)

To find the time the ball collides with the floor, we express the height of the elevator floor, $z_{fl}(t)$, and the height of the ball, $z_b(t)$, and solve for when they are equal. We measure $z$ as the height above the level of the 2nd floor.

$$z_{fl}(t) = v_e t \quad \text{where} \quad v_e = 0.5\,\text{m/s}$$

For the motion of the ball, it is just falling under gravity. It is important to note that the ball initially is moving upwards with velocity $v_e$ since both it and the person holding it are travelling upwards at the same speed as the elevator.

$$z_b(t) = z_0 + v_e t - \frac{1}{2}gt^2 \quad \text{where} \quad z_0 = h = 2\,\text{m}$$

$$v_e t = h + v_e t - \frac{1}{2}gt^2$$

Solving this gives the same solution as parts a & b above, $t = 0.64$ sec. Another way to see this is to change reference frames to the one moving upwards at constant velocity with the elevator. Since this is an inertial (ie non-accelerated) frame, the laws of physics are unchanged and the situation is equivalent to a & b.

### 1.1.d (1 point)

The elevator is initially moving upwards with the same $v_e = 0.5$ m/s as in part c. Now, as the ball is dropped, the elevator accelerates with $a = kt^2$ where $k = 4\,\text{m/s}^4$. As in part c, the height of the ball measured above the 4th floor is the same:

$$z_b(t) = h + v_e t - \frac{1}{2}gt^2$$

We must now solve for $z_{fl}(t)$ from the given acceleration.

$$a = \frac{dv_{fl}}{dt} = kt^2$$

$$\int [dv_{fl} = kt^2\, dt]$$

$$v_{fl} = \frac{1}{3}kt^3 + C$$

We get $C = v_e$ by using the initial condition that $v_{fl}(0) = v_e$. Integrating again gives $z_{fl}(t)$.

$$\frac{dz_{fl}}{dt} = v_{fl}(t) = \frac{1}{3}kt^3 + v_e$$

$$\int [dz_{fl} = (\frac{1}{3}kt^3 + v_e)dt]$$

$$z_{fl} = \frac{1}{12}kt^4 + v_e t + C'$$

We get $C' = 0$ assuming we measure $z = 0$ starting at the 4th floor. Now we solve for the time when $z_{fl}(t) = z_b(t)$.

$$h + v_e t - \frac{1}{2}gt^2 = \frac{1}{12}kt^4 + v_e t$$

$$\frac{1}{12}kt^4 + \frac{1}{2}gt^2 - h = 0$$

Solving for the quadratic gives $t^2 = 0.397$ or $-15.10$ sec. Taking the first one, we get $t = 0.63$ sec.

### 1.1.e (1 point)

Consider the ball going up backwards in time compared to the ball going down forwards it time. Each situation then starts with initial velocity zero. However, the ball going down has air resistance and gravity pointing in opposite directions, while the backwards ball going up has them pointing in the same direction. Thus the backwards ball going up experiences at every point a greater acceleration than the ball going down. Since both balls travel the same distance, the backwards ball going up travels the distance in less time, since it has greater acceleration and same initial condition. Therefore, the ball takes longer to fall down.

*Alternative answer*:
The idea is to write the equation of motion for the falling ball in the rest frame of the earth. This can be solved exactly except for the double time integration of the acceleration due to air resistance, but this can be replaced with some quantity "C" whose sign is known. One obtains:

$$H = \frac{1}{2}gT_d^2 - C$$

where H is the height from which the ball drops, $T_d$ is the time it takes to fall and C is a positive unknown. To solve for the upward equation of motion, *solve in a frame moving with velocity V in the upward direction* where V is the initial velocity of the ball. In this frame the ball starts at rest and falls. The time it takes to fall a distance H is the same as the time it takes for the ball to reach its maximum height and solving for this gives:

$$H = \frac{1}{2}gT_u^2 + D$$

where $T_u$ is the time it takes to reach this height and D is another positive unknown resulting from integrating the air resistance. The quantity H is a distance and is therefore coodinate-invariant, equate the two equations and you see clearly that $T_d > T_u$

## 1.2 QP17 (5 points)

### 1.2.a (1.5 points)

Measure the positions of the Officer, $x_O(t)$, and Indiana, $x_I(t)$, from the turnout.

$$x_O(t) = \frac{1}{2}at^2 \quad \text{and} \quad x_I(t) = v_0 t$$

Solving for $x_O(t) = x_I(t)$ for the time to catch Indiana, we get:

$$\frac{1}{2}at^2 = v_0 t$$
$$t = \frac{2v_0}{a} = 20\,\text{s}$$

### 1.2.b (0.5 points)

At the time found above, we want to know the Officer's speed.

$$v_O(t) = at = 160\,\text{mph}$$

### 1.2.c (0.5 points)

See figure

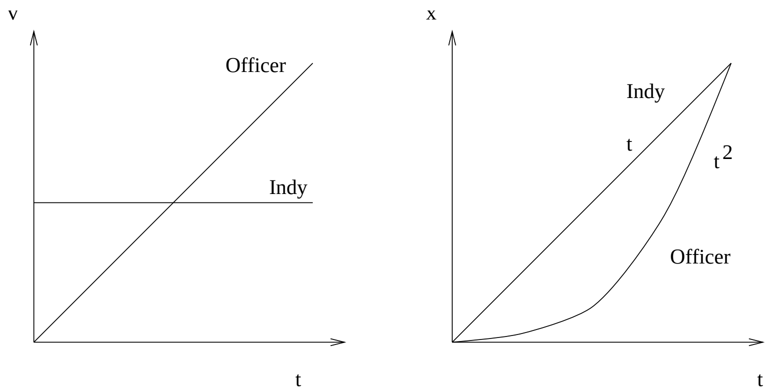


Figure 1: QP17, Part c

### 1.2.d (1.5 points)

We set our origin for $x$ and $t$ at the Officer, $L = 100$ yards behind Indiana, and when Indiana first hits the brakes. Since both cars brake at the same rate, we have:

$$x_O(t) = v_O t - \frac{1}{2}dt^2$$
$$x_I(t) = L + v_I t - \frac{1}{2}dt^2$$

where $v_0$ and $v_I$ are the initial speeds of the cars before the brakes begin to work. We know $v_O = 160$ mph from (b), and $v_I = 80$ mph.

$$x_O(t) = x_I(t)$$
$$v_O t - \frac{1}{2}dt^2 = L + v_I t - \frac{1}{2}dt^2$$

$$t_{crash} = \frac{L}{v_O - v_I} = 2.56\,\mathrm{s}$$

### 1.2.e (1 point)

From the equations for their respective velocities, we get the relative speed.

$$v_O(t) = v_O - dt$$
$$v_I(t) = v_I - dt$$
$$v_{rel}(t) = v_O(t) - v_I(t) = v_O - v_I$$
$$v_{rel}(t_{crash}) = 80\,\mathrm{mph}$$

As a side note, both parts (d) and (e) assume that the amount of deceleration provided by the brakes, $d$, is not enough to prevent the collision. Considering the critical value of $d$ for which this no longer holds, and the corresponding changes to (d) and (e), were not required.

## 1.3 Frautschi 3.10 (5 points)

### 1.3.a (5 point)

We will derive the time analytically first then substitute with given statistics.

$$dN/dt = kN$$
$$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t kdt$$
$$\log N/N_0 = kt$$
$$t = \frac{1}{k} \log \frac{N}{N_0}$$

where $N_0 = 2$ and N is the current world population.

Knowing that the birth rate is 20.3 per year per 1000 population and the death rate is 9.3 per year per 1000 population, we get

$$k = 20.3 - 9.6 = 10.7$$

per year per 1000 population or $k \approx 10^{-2}$ per year per person.

The world population now is $\approx 6.7$ billions. Use the estimation of k we get previously,

$$t = 10^2 \times \log \frac{6.7 \times 10^9}{2} = 2000$$

years.

The estimation of $k$ uses current birth rate, which is very high comparing with historic data, and current death rate, which is very low. If we take into account of wars and food shortage, and estimate $k \approx 10^{-4}$, then we get $200,000$ years.

## 1.4 Frautschi 3.13 (5 points)

### 1.4.a (3 point)

Assume that the decay obeys the following:

$$\frac{dN}{dt} = -kN$$

then we get the function of $N$ as

$$N = N_0 e^{-kt} \tag{1}$$

where $N_0$ is the number of radioactive carbons at $t = 0$. We need to find out the constant $k$ first, using the condition that the half-life time is 5568 years. Let $t_{half}$ be the half life time. We take logarithm on both side of equation (**??**)

$$\log N/N_0 = -kt_{half} = \log \frac{1}{2}$$

This is because $N/N_0 = 1/2$. Then we get

$$k = \frac{\log 2}{t_{half}}$$

.

Let the time when the scroll was made be $t_1$, and the time "‘now"’ be $t_2$. Notice that $t_2 > t_1$.

$$N_1 = N_0 e^{-kt_1}$$
$$N_2 = N_0 e^{-kt_2}$$

Divide the first equation by the second, we get

$$\begin{aligned}
\frac{N_1}{N_2} &= e^{-k(t_1-t_2)} \\
\log\frac{N_1}{N_2} &= -k(t_1-t_2) \\
t_2 - t_1 &= \frac{1}{k}\log\frac{N_1}{N_2} \\
t_2 - t_1 &= \frac{1}{k}\log\frac{dN_1/dt}{dN_2/dt}
\end{aligned}$$

The reason why the ratio of the numbers and the decay rates are the same can be seen from the equation

$$dN/dt = -kN$$

.

Put $k$ into the equation, we get

$$t_2 - t_1 = \frac{t_{half}}{\log 2}\log\frac{dN_1/dt}{dN_2/dt} \tag{2}$$

For part (a), we use $dN_1/dt = 15.3$ and $dN_2/dt = 12.0$, then we get

$$\frac{5568}{\log 2}\log\frac{15.3}{12.0} \approx 1952$$

years. So the scroll was made about 1952 years ago.

### 1.4.b (1 point)

Use eq. (**??**) and $dN_1/dt = 15.3$ and $dN_2/dt = 9.7$,

$$\frac{5568}{\log 2}\log\frac{15.3}{9.7} \approx 3661$$

years ago.

### 1.4.c (1 point)

Use eq. (**??**) and $dN_1/dt = 15.3$ and $dN_2/dt = 2.8$,

$$\frac{5568}{\log 2}\log\frac{15.3}{2.8} \approx 13642$$

years ago.