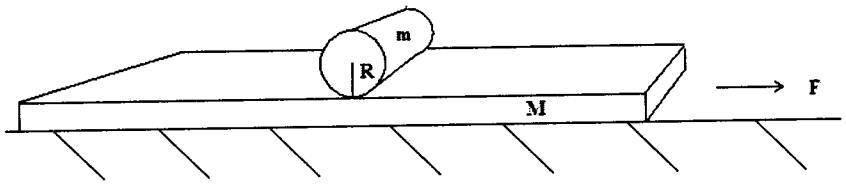
FP23

Roll Out the Barrel



As depicted in the diagram above, a large flat board, sitting on the ground, of mass M is pulled with a force F, while a cylinder of mass m, radius R and moment of inertia $I = (1/2)mR^2$ rolls without slipping on the board. Assume there is friction between the board and the cylinder to ensure rolling without slipping, but for simplicity, assume there is no friction between the board and the ground. In addition, assume that the board is sufficiently long that the cylinder remains on the board for the problem. Finally, for calculations in this problem, define a to be the acceleration of the board with respect to the ground, a_1 to be the acceleration of the centre of mass of the cylinder *with respect to the accelerating board*, and a_2 to be the acceleration of the cylinder *with respect to the ground.*

(2 points) (a) Draw free body diagrams for the cylinder and the board.

(3 points) (b) Write down Newton's equations (listing all relevant forces) for the linear motion of the board and cylinder. In addition, write down an expression describing the rotational motion of the cylinder. Finally, find a constraint associated with rolling without slipping. For each of the equations, list which frame of reference it is in.

(1 point) (c) Does the cylinder roll clockwise or counter-clockwise?

(2 points) (d) Solve the equations you found for a, in terms of F, M, and m.

(2 points) (e) Solve the equations for a_1 and a_2, again in terms of F, M, and m.