Capacity Region of Multi-Resolution Streaming in Peer-to-Peer Networks

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***Abstract*—We consider multi-resolution streaming in fully-connected peer-to-peer networks, where transmission rates are constrained by arbitrarily specified upload capacities of the source and peers. We fully characterize the capacity region of rate vectors achievable with arbitrary coding, where an achievable rate vector describes a vector of throughputs of the different resolutions that can be supported by the network. We then prove that all rate vectors in the capacity region can be achieved using pure routing strategies. This shows that coding has no capacity advantage over routing in this scenario.**

I. Introduction

We consider multi-resolution streaming in a heterogeneous peer-to-peer setting, where peers have different upload capacities and demand an information stream at different resolutions. The information stream is layered, such as in Scalable Video Coding [1], which generates a base video layer and a number of enhancement layers that depend on the base layer and all lower layers.

We assume a fully-connected overlay network in which transmission rates are constrained by the upload capacity of the source and each peer, a model introduced in [2] to capture the most important constraints in peer-to-peer networks. A problem instance is defined by specifying the number of layers demanded by each peer and the upload capacity constraints of the source and each peer. Our goal is to find the capacity region of achievable rate vectors, where an achievable rate vector describes a vector of throughputs of the different resolutions that can be supported by the network.

Solutions can be classified as follows. Inter-session coding solutions are the most general, allowing coding across information from different sessions (i.e. layers). Intra-session coding solutions restrict coding to occur only within each session. Routing solutions allow only replication and forwarding of information at each node. Intra-session coding corresponds to independent multicast network coding for each layer, for which the capacity region is given by a linear program. In contrast, characterizing inter-session coding capacity, which corresponds to the information theoretic capacity, is open for general networks.

Related work by Chiu et al. [3] studies the special case of a single resolution. That case corresponds to a single multicast, and [3] shows that network coding is not needed to achieve capacity. In [4], Ponec et al. consider the multi-resolution case restricted to intra-session coding, showing that intra-session coding does not improve the capacity region over routing. A different objective of minimizing average finish times for file download was studied in [5], [6].

In this paper we provide a complete characterization of the capacity region of feasible rate vectors achievable with arbitrary (inter or intra-session) coding, and show that the entire capacity region can be achieved with routing.

II. Problem Definition

A peer to peer network is modeled as a complete directed graph with a single source node p_0 and $k \geq 1$ peer nodes $\{p_0, p_1, \ldots, p_k\}$. The upload capacities of nodes $p_0, \ldots, p_k$ are $C_0, \ldots, C_k$ respectively.

Information originates at the source node and is distributed to the peers, which help the distribution process by uploading information to other peers. Coding may occur at the source and peers.

Let n be the number of different resolutions in a layered data stream. We denote by $x_1, \ldots, x_n$ the data streams corresponding to the different layers, such that the jth resolution corresponds to $\{x_1, x_2, \ldots, x_j\}$. The rate of x_j is denoted by L_j. For simplicity, we assume that the upload capacities $C_0, \ldots, C_k$ and the data rates $L_1, \ldots, L_n$ are integers, which can be approached arbitrarily closely by scaling the unit appropriately.

We are given nested sets $X_1, \ldots, X_n$ specifying the demands:

$$X_j = \{p_i | p_i \text{ } demands \text{ } x_j\}.$$

We also define $X_{n+1} = \{p_0\}$, so we have

$$\{p_0\} = X_{n+1} \subseteq X_n \subseteq \ldots \subseteq X_1 = \{p_0, p_1, \ldots, p_k\}.$$

For all $S_1, S_2 \subseteq X_1$, $S_1 \to S_2$ is defined as the set of all links coming from S_1 and going to S_2. We also write

$p_i \to p_j$ instead of $\{p_i\} \to \{p_j\}$ for brevity. The constraints on a successful transmission scheme are as follows:

i) Each outgoing link of the source p_0 is a function of $x_1, \ldots, x_n$:

$$\forall p_i \in X_1,\ H(p_0 \to p_i | x_1, \ldots, x_n) = 0.$$

ii) Each outgoing link of $p_i \in X_1 \setminus \{p_0\}$ is a function of incoming links:

$$\forall p_i, p_j \in X_1 \setminus \{p_0\},\ H(p_i \to p_j | p_0 \to p_i, \ldots, p_n \to p_i) = 0.$$

We assume that $H(p_i \to p_i) = 0$ without loss of optimality.

iii) Each peer $p_i \in X_1$ can transmit at rate at most C_i:

$$\forall p_i \in X_1\ C_i \geq \sum_{p_j \in X_1} H(p_i \to p_j)$$

iv) Each peer $p_i \in X_j \setminus \{p_0\}$ is able to decode x_j from its received information:

$$I(X_1 \to p_i; x_j) = H(x_j) = L_j.$$

III. Approach

In this section we provide some intuition for our approach. A first observation is that total upload capacity should be greater than the total rate of data which has to be delivered:

$$\sum_{i=0} C_i \geq \sum_{i=1}^{n} |X_i| L_i. \tag{1}$$

This condition is necessary but not sufficient. The following sequence of lemmas leads to a sufficient condition for a rate vector to be achievable. Owing to space constraints, the proofs of lemmas in this and the next section can be found in the extended version of this paper [7].

Lemma 1. *Let k and C_0 be positive integers and $C_1, C_2, \ldots, C_k$ be nonnegative integers such that*

$$C_0 + \sum_{i=1}^{k} C_i = k$$

Then there exists a directed tree rooted at v_0 with vertices $v_1, \ldots, v_k$ such that

$$outdeg(v_0) = C_0,$$

$$\forall i \in \{1, \ldots, k\}\ outdeg(v_i) = C_i,\ indeg(v_i) = 1.$$

Lemma 2. *Data x with rate L can be transmitted to peers $p_1, \ldots, p_k$ by using source capacity C_0 and peer upload capacities $C_1, \ldots, C_k$ if*

$$C_0 \geq L \text{ and}$$

$$\sum_{i=0}^{k} C_i \geq kL.$$

Lemma 3. *Given the sets of peers $X_1, X_2, \ldots, X_n$ and upload capacities $C_0, C_1, \ldots, C_k$, the rate vector $(L_1, L_2, \ldots, L_n)$ is achievable if for every $j \in \{1, \ldots, n\}$*

$$\sum_{p_i \in X_j} C_i \geq \sum_{i=1}^{j-1} L_i + \sum_{i=j}^{n} |X_i| L_i \tag{2}$$

and

$$C_0 \geq \sum_{i=1}^{n} L_i. \tag{3}$$

Intuitively, if one of the inequalities in Lemma 3, say the jth one, does not hold, this means that the nodes in set X_j cannot handle the transmission of data layers x_j through x_n. Hence, some peers from the set $X_1 \setminus X_j$ need to help in transmitting those data layers, necessitating some additional capacity for transmitting this data to peers in $X_1 \setminus X_j$ which do not themselves demand it. This requires additional capacity beyond that given in (1).

To characterize this explicitly, it is useful to define the margin of the jth inequality:

$$N_j = \sum_{i=j}^{n} |X_i| L_i + \sum_{i=1}^{j-1} L_i - \sum_{p_i \in X_j} C_i.$$

For completeness we also define the $(n+1)$-th margin N_{n+1} as zero. The capacity region derived in the next section is stated in terms of these margins. In fact, not all of them, but a special subset of them, will be used. This subset is defined as follows:

Definition 1. *For a finite sequence $\{a_n\} = a_1, \ldots, a_s$, the dominant subsequence of $\{a_n\}$ is the subsequence $\{a_{i_n}\} = a_{i_1}, \ldots, a_{i_h}$ defined by*

i) $i_h = s$

ii) i_j is the greatest index such that $i_j < i_{j+1}$ and $a_{i_j} > a_{i_{j+1}}$.

IV. Converse Bound for Capacity Region

In this section, we present a converse bound on the capacity region, which is shown to be tight in the following section.

Theorem 1. *Given the sets of peers $X_1, X_2, \ldots, X_n$ and upload capacities $C_0, C_1, \ldots, C_k$, if the rate vector $(L_1, L_2, \ldots, L_n)$ is achievable by any coding scheme, then*

$$\sum_{p_i \in X_1} C_i \geq \sum_{i=1}^{n} |X_i| L_i + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1}$$

where $N_{d_1}, \ldots, N_{d_{h+1}}$ is the dominant subsequence of $N_1, \ldots, N_{n+1}$.

Proof: For a resolution x_j and a peer $p_i \in X_j$ we have, from property iv in Section II,

$$H(x_j) = I(X_1 \to p_i; x_j) = I(X_1 \setminus X_j \to p_i, X_j \to p_i; x_j).$$

If we view $X_1 \setminus X_j$ as a supernode, outgoing links should be functions of incoming links, since peers in the set $X_1 \setminus X_j$ do not create additional data besides incoming data (property ii). Hence, links in set $X_1 \setminus X_j \to p_i$ are completely dependent on links in set $X_j \to X_1 \setminus X_j$. Then, we may write:

$$H(x_j) = I((X_1 \setminus X_j) \to p_i, X_j \to p_i; x_j) \leq$$
$$I(X_j \to (X_1 \setminus X_j), X_j \to p_i; x_j) \leq H(x_j)$$
$$\Rightarrow \quad H(X_j) = I(X_j \to (X_1 \setminus X_j); x_j)$$
$$+ I(X_j \to p_i; x_j | X_j \to (X_1 \setminus X_j)).$$

Summing this for all peers in X_j yields

$$|X_j| H(X_j) = |X_j| I(X_j \to (X_1 \setminus X_j); x_j) +$$
$$\sum_{p_i \in X_j} I(X_j \to p_i; x_j | X_j \to (X_1 \setminus X_j)).$$

By rearranging this, we can obtain

$$I(X_j \to (X_1 \setminus X_j); x_j) = \frac{1}{|X_j| - 1} [|X_j| H(x_j) -$$
$$I(X_j \to (X_1 \setminus X_j); x_j) -$$
$$\sum_{p_i \in X_j} I(X_j \to p_i; x_j | X_j \to (X_1 \setminus X_j))]. \quad (4)$$

The left hand side of this equation can be replaced by parameters which are independent from the transmission scheme by using the following lemma:

Lemma 4. $\sum_{j=1}^{n} I(X_j \to (X_1 \setminus X_j); x_j) \leq \sum_{p_i \in X_j} C_i - \sum_{j=1}^{n} |X_j| H(x_j)$.

By putting the right hand side of Equation (4) in place of the term $I(X_j \to (X_1 \setminus X_j); x_j)$ at Lemma 4, we can obtain:

$$\sum_{p_i \in X_j} C_i \geq \sum_{j=1}^{n} |X_j| H(x_j) + \sum_{j=1}^{n} \frac{1}{|X_j| - 1} [|X_j| H(x_j)$$
$$- I(X_j \to (X_1 \setminus X_j); x_j)$$
$$- \sum_{p_i \in X_j} I(X_j \to p_i; x_j | X_j \to (X_1 \setminus X_j))]. \quad (5)$$

Now define A_j such that

$$\begin{aligned} A_{n+1} &= 0 \\ A_j - A_{j+1} &= |X_j| H(x_j) - I(X_j \to (X_1 \setminus X_j); x_j) \\ &= (|X_j| - 1) I(X_j \to (X_1 \setminus X_j); x_j) \geq 0. \end{aligned}$$

Putting this into Inequality (5), we obtain:

$$\sum_{p_i \in X_j} C_i \geq \sum_{j=1}^{n} |X_j| H(x_j) + \sum_{j=1}^{n} \frac{A_j - A_{j+1}}{|X_j| - 1}. \quad (6)$$

Note that the A_j values are determined by the transmission scheme. To obtain a bound which is independent from transmission scheme, we use the following lemma:

Lemma 5. $A_j \geq N_j$.

Let us examine the last sum in (6):

$$\sum_{j=1}^{n} \frac{A_j - A_{j+1}}{|X_j| - 1} = \sum_{j=1}^{d_1 - 1} \frac{A_j - A_{j+1}}{|X_j| - 1} + \sum_{j=d_1}^{d_2 - 1} \frac{A_j - A_{j+1}}{|X_j| - 1} + \ldots +$$
$$\sum_{j=d_h}^{n} \frac{A_j - A_{j+1}}{|X_j| - 1}$$

Since $A_j - A_{j+1} \geq 0$ and X_is are nested,

$$\begin{aligned} &\geq 0 + \sum_{j=d_1}^{d_2 - 1} \frac{A_j - A_{j+1}}{|X_{d_1}| - 1} + \ldots + \sum_{j=d_h}^{n} \frac{A_j - A_{j+1}}{|X_{d_h}| - 1} \\ &= \frac{A_{d_1}}{|X_{d_1}| - 1} + A_{d_2} \left(\frac{1}{|X_{d_2}| - 1} - \frac{1}{|X_{d_1}| - 1} \right) + \ldots + \\ &\quad A_{d_h} \left(\frac{1}{|X_{d_h}| - 1} - \frac{1}{|X_{d_{h-1}}| - 1} \right) \\ &\geq \frac{N_{d_1}}{|X_{d_1}| - 1} + N_{d_2} \left(\frac{1}{|X_{d_2}| - 1} - \frac{1}{|X_{d_1}| - 1} \right) + \ldots + \\ &\quad N_{d_h} \left(\frac{1}{|X_{d_h}| - 1} - \frac{1}{|X_{d_{h-1}}| - 1} \right) \\ &= \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1}. \end{aligned}$$

The last inequality is due to Lemma 5 . By putting this result in (6), we obtain:

$$\sum_{p_i \in X_1} C_i \geq \sum_{i=1}^{n} |X_i| L_i + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1}$$

■

V. A Routing Scheme that Achieves the Capacity Region

In this section, we give a transmission scheme using multicast routing trees that achieves the bound in Theorem 1.

Theorem 2. *Given the sets of peers $X_1, X_2, \ldots, X_n$ and upload capacities $C_0, C_1, \ldots, C_k$, the rate vector $(L_1, L_2, \ldots, L_n)$ is achievable by routing if*

$$C_0 \geq \sum_{i=1}^{n} L_i, \quad (7)$$

$$\sum_{p_i \in X_1} C_i \geq \sum_{i=1}^{n} |X_i| L_i + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1} \quad (8)$$

where $N_{d_1}, \ldots, N_{d_{h+1}}$ is the dominant subsequence of $N_1, \ldots, N_{n+1}$.

Proof: The proof is by induction on the number of inequalities from Lemma 3 which are not satisfied. For this purpose let us define the set

$$I = \{i | N_i > 0\}.$$

We will show that we can reduce the size of I by at least one, by using some amount of capacity, such that the residual system also satisfies (7) and (8).

The base case is the one where I is the empty set, i.e. all N_j values are less than or equal to zero and it is examined in Lemma 3. Let m and M denote the minimum and maximum elements of I, respectively. The following lemma is essential for our method:

Lemma 6. $\forall p_i \in X_1 \setminus X_m,\ \exists C_{iM}$ *with* $0 \le C_{iM} \le C_i$ *such that*

$$\sum_{p_i \in X_j \setminus X_m} C_{iM} \le N - N_j$$

$\forall j \in \{1, 2, \ldots, m-1\}$ *and*

$$\sum_{p_i \in X_1 \setminus X_m} C_{iM} = \frac{|X_M| N}{|X_M| - 1}$$

where $N = min(N_m, N_M)$.

Now, for each $p_i \in X_1 \setminus X_m$ let us choose C_{iM} as in Lemma 6. Then we have

$$\sum_{p_i \in X_1 \setminus X_m} C_{iM} = \frac{|X_M| N}{|X_M| - 1} \tag{9}$$

where $N = \min(N_m, N_M)$. Let us also define

$$C_{0M} = \frac{N}{|X_M| - 1}.$$

Now take a rate-$\frac{N}{|X_M|-1}$ portion of x_M, called s. Then, using equality (9), the rate of s is given by

$$\frac{N}{|X_M| - 1} = \sum_{p_i \in X_1 \setminus X_m} \frac{C_{iM}}{|X_M|}.$$

Hence we can divide s into $|X_1 \setminus X_m|$ portions s_i corresponding to peers $p_i \in X_1 \setminus X_m$, where the rate of portion s_i is given by $\frac{C_{iM}}{|X_M|}$. To each peer $p_i \in X_1 \setminus X_m$, send s_i from the source. This consumes C_{0M} amount of capacity of the source. Then, from each $p_i \in X_1 \setminus X_m$, send s_i to the peers in X_M. This consumes C_{iM} amount of capacity of each $p_i \in X_1 \setminus X_m$. In this way, transmission of portion s is completed. After the procedure, we have residual capacities

$$C'_i = \begin{cases} C_i - C_{iM} & if\ p_i \in X_1 \setminus X_m \\ C_i & if\ p_i \in X_m \setminus \{p_0\} \\ C_0 - \frac{N}{|X_M|-1} & if\ p_i = p_0. \end{cases}$$

and residual data rates

$$L'_i = \begin{cases} L_i - \frac{N}{|X_M|-1} & if\ i = M \\ L_i & otherwise. \end{cases}$$

The N_i values are updated accordingly. Denoting the updated value of N_j by N'_j, we calculate it differently for three cases:

i) If $j < m$:

$$\begin{aligned} N'_j &= \sum_{i=j}^{n} |X_i| L'_i - C'_0 + \sum_{i=1}^{j-1} L'_i - \sum_{p_i \in X_j \setminus \{p_0\}} C'_i \\ &= \sum_{i=j}^{n} |X_i| L_i - \frac{|X_M| N}{|X_M| - 1} - C_0 + \frac{N}{|X_M| - 1} + \sum_{i=1}^{j-1} L_i \\ &\quad - \sum_{p_i \in X_j \setminus \{p_0\}} C_i + \sum_{p_i \in X_j \setminus X_m} C_{iM} \\ &= (\sum_{i=j}^{n} |X_i| L_i + \sum_{i=1}^{j-1} L_i - \sum_{p_i \in X_j} C_i) \\ &\quad -(\frac{|X_M| N}{|X_M| - 1} - \frac{N}{|X_M| - 1}) + \sum_{p_i \in X_j \setminus X_m} C_{iM} \\ &= N_j - N + \sum_{p_i \in X_j \setminus X_m} C_{iM}. \end{aligned}$$

By the choice of C_{iM} values, using Lemma 6, we have

$$N'_j = N_j - N + \sum_{p_i \in X_j \setminus X_m} C_{iM} \le 0 \tag{10}$$

ii) If $m \le j \le M$:

$$\begin{aligned} N'_j &= \sum_{i=j}^{n} |X_i| L'_i - C'_0 + \sum_{i=1}^{j-1} L'_i - \sum_{p_i \in X_j \setminus \{p_0\}} C'_i \\ &= \sum_{i=j}^{n} |X_i| L_i - \frac{|X_M| N}{|X_M| - 1} - C_0 + \frac{N}{|X_M| - 1} \\ &\quad + \sum_{i=1}^{j-1} L_i - \sum_{p_i \in X_j \setminus \{p_0\}} C_i \end{aligned}$$

$$\Rightarrow \quad N'_j = N_j - N. \tag{11}$$

iii) If $j > M$:

$$\begin{aligned} N'_j &= \sum_{i=j}^{n} |X_i| L'_i - C'_0 + \sum_{i=1}^{j-1} L'_i - \sum_{p_i \in X_j \setminus \{p_0\}} C'_i \\ &= \sum_{i=j}^{n} |X_i| L_i - C_0 + \frac{N}{|X_M| - 1} + \sum_{i=1}^{j-1} L_i \\ &\quad - \frac{N}{|X_M| - 1} - \sum_{p_i \in X_j \setminus \{p_0\}} C_i \end{aligned}$$

$$\Rightarrow \quad N'_j = N_j < 0. \tag{12}$$

Now let us examine the dominant subsequence $N'_{d'_1}, \ldots, N'_{d'_{h'+1}}$ of $N'_1, \ldots, N'_{n+1}$. Since N'_{n+1} is zero by definition, $N'_{d'_{h'+1}}$ is also zero. Therefore for all $i \in \{1, \ldots, h'\}$

$$N'_{d'_i} > N'_{d'_{h'+1}} = 0 \Rightarrow m \le d'_i \le M.$$

But the order of N'_i values for $m \leq i \leq M$ is the same as the order of N_i values for $m \leq i \leq M$ since $N'_j = N_j - N$. Also we know that for all $i \in \{1, \ldots, h\}$

$$m \leq d_i \leq M.$$

Noting that $N_{d_h} = N_M$, we have two cases:

$$(N'_{d'_1}, \ldots, N'_{d'_{h'+1}}) = (N_{d_1} - N, \ldots, N_{d_h} - N, 0)$$

if $N = N_m < N_M$, and

$$(N'_{d'_1}, \ldots, N'_{d'_{h'+1}}) = (N_{d_1} - N, \ldots, N_{d_h - 1} - N, 0)$$

if $N = N_M \leq N_m$.
These two cases can be considered as one since even if N is equal to N_M, we can consider as $N'_{d'_{h'}} = 0 = N_{d_h} - N$ so that it does not affect inequality (8). Hence we can write:

$$(N'_{d'_1}, \ldots, N'_{d'_{h'}}, N'_{d'_{h'+1}}) = (N_{d_1} - N, \ldots, N_{d_h} - N, 0).$$

Now let us calculate the left hand side of the inequality (8) with updated values:

$$\sum_{p_i \in X_1} C'_i = C_0 - \frac{N}{|X_M| - 1} + \sum_{p_i \in X_1 \setminus \{p_0\}} C_i - \sum_{p_i \in X_1 \setminus X_m} C_{iM}$$
$$= \sum_{p_i \in X_1} C_i - \frac{N}{|X_M| - 1} - \frac{|X_M| N}{|X_M| - 1}$$
$$\geq \sum_{i=1}^{n} |X_i| L_i + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1} - \frac{(|X_M| + 1)N}{|X_M| - 1}$$
$$= \sum_{i=1}^{n} |X_i| L'_i + \frac{|X_M| N}{|X_M| - 1} + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1} - \frac{(|X_M| + 1)N}{|X_M| - 1}$$
$$= \sum_{i=1}^{n} |X_i| L'_i + \sum_{i=1}^{h-1} \frac{(N_{d_i} - N) - (N_{d_{i+1}} - N)}{|X_{d_i}| - 1} + \frac{N_{d_h} - N}{|X_{d_h}| - 1}$$
$$= \sum_{i=1}^{n} |X_i| L'_i + \sum_{i=1}^{h-1} \frac{N'_{d'_i} - N'_{d'_i + 1}}{|X_{d_i}| - 1} + \frac{N'_{d'_h} - 0}{|X_{d_h}| - 1}$$
$$= \sum_{i=1}^{n} |X_i| L'_i + \sum_{i=1}^{h} \frac{N'_{d'_i} - N'_{d'_i + 1}}{|X_{d_i}| - 1}.$$

This shows that inequality (8) is preserved after the procedure. Inequality (7) is also preserved since

$$C'_0 = C_0 - \frac{N}{|X_M| - 1} \geq \sum_{i=1}^{n} L_i - \frac{N}{|X_M| - 1} = \sum_{i=1}^{n} L'_i.$$

Now let us look at the updated version I' of I. From (10), (11) and (12) we know that $I' \subseteq I$. If $N = N_m$, then $N'_m = N_m - N = 0 \Rightarrow m \notin I' \Rightarrow |I'| \leq |I| - 1$. Similarly if $N = N_M$, then $M \notin I' \Rightarrow |I'| \leq |I| - 1$. Finally we can say that after reducing the rate of x_M by $\frac{N}{|X_M|-1}$, inequalities (7) and (8) are still correct and number of inequalities from Lemma 3 is reduced by at least one. Hence, by the induction hypothesis, we can complete transmission of the remaining data. This completes the proof. ■

Combining Theorem 1 and Theorem 2 with the addition of trivial condition $C_0 \geq \sum_{i=1}^{n} L_i$, we obtain the exact capacity region:

Corollary 1. *Given the sets of peers $X_1, X_2, \ldots, X_n$ and upload capacities $C_0, C_1, \ldots, C_k$, the rate vector $(L_1, L_2, \ldots, L_n)$ is achievable if and only if the following inequalities hold*

$$C_0 \geq \sum_{i=1}^{n} L_i$$

$$\sum_{p_i \in X_1} C_i \geq \sum_{i=1}^{n} |X_i| L_i + \sum_{i=1}^{h} \frac{N_{d_i} - N_{d_{i+1}}}{|X_{d_i}| - 1}$$

where $N_{d_1}, \ldots, N_{d_{h+1}}$ is the dominant subsequence of $N_1, \ldots, N_{n+1}$. Furthermore, the capacity region is achievable using routing.

VI. Conclusion

We have characterized the capacity region of achievable rates for multi-resolution streaming in peer-to-peer networks with upload capacity constraints, and shown that this region can be achieved by routing. This represents a new class of non-multicast network problems for which we have a capacity characterization. Although coding is not needed to achieve capacity in this scenario, it can nevertheless be useful in scenarios with losses or without centralized control.

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