Dynamic Algorithms for Multicast with Intra-session Network Coding

Tracey Ho and Harish Viswanathan
Lucent Technologies, Bell Labs
Murray Hill, NJ
{traceyho, harishv}@lucent.com

Abstract

The problem of multiple multicast sessions with intra-session network coding in ergodically time-varying networks is considered. The capacity region of input rates that can be stably supported is established, and dynamic algorithms for multicast routing, network coding, rate control, power allocation, and scheduling that achieve stability for rates within the capacity region are presented. This work builds on the back-pressure approach introduced by Tassiulas et al., extending it to network coding and correlated sources. In the proposed algorithms, decisions on routing, network coding, and scheduling between different sessions at a node are made locally at each node based on virtual queues for different sinks. For correlated sources, the sinks locally determine and control transmission rates across the sources. The proposed approach yields a completely distributed algorithm for wired networks. In the wireless case, scheduling and power control among different transmitters are centralized while routing, network coding, and scheduling between different sessions at a given node are distributed.

Index Terms

Back pressure, network coding, multicast, correlated sources, multi-hop, scheduling

I. Introduction

Network coding has recently been shown to improve performance compared to that of routing for multicasting information over wired and wireless networks [3], [15], [27]. Most of the work in network coding to date assumes a flow model for transmission in which sources generate, at fixed rates, data

Tracey Ho is currently with the California Institute of Technology, Pasadena, CA.

that is then transmitted over a network with fixed link capacities. However, in real networks, traffic is usually bursty because either the sources generate traffic in bursts or the network nodes employ queuing and scheduling across multiple sessions. In such scenarios, optimal multicasting of information involves not only routing and network coding but also link scheduling and scheduling of different flows on the active links. Furthermore, optimal network coding in this context may depend on the current state of the network - link data rates and buffer occupancy.

Routing, scheduling, rate, and power control in networks with bursty traffic has recently received significant attention in the context of wireless networks [4], [14], [18], [19], [20], [21], [25], [28], [29]. Much of the recent work in this area builds on the ideas of [5], [24] that describe algorithms for routing and scheduling flows using queue sizes, or differences in queue size between the queues at the source and the destination of a link, as the metric to select between different flows. Such an approach is usually said to be *back-pressure* based since heavily loaded nodes downstream push back and slow down the flow coming down from nodes upstream. Such a back-pressure approach is generally optimal in the sense that it allows transmission at the maximum possible arrival rates into the network for which the queues at the various network nodes are still stable. Furthermore, the routing and scheduling algorithms are distributed in the wired network case in the sense that decisions are made locally at each node based on feedback from only the immediate destination nodes of the transmission links at the node.

While the back-pressure approach has mostly been applied in the context of unicast transmissions, it has also been extended to the case of multicast transmissions [4], [22]. However, in the multicast case without network coding the algorithms are significantly more complex, even for wired networks.

We extend the above back-pressure based dynamic routing and scheduling algorithms to include network coding and correlated sources. Unlike in the case of a fixed set of flows, for time-varying queuing networks employing the back-pressure approach, network coding also needs to be dynamic and dependent on the state of the network. Random network coding [9], introduced for the flow model, extends naturally to a time-varying network with bursty traffic and provides a distributed implementation. In this paper we consider dynamic multi-session multicast with network coding in wired and ergodically varying wireless networks with ergodic sources. We consider the general case in which each multicast session consists of a set of sources and sinks such that data from all the sources is intended for all the sinks. We establish the capacity region of input rates that can be stabilized with intra-session network coding, and present dynamic algorithms for routing, network coding, scheduling and rate control that achieve stability for rates within this capacity region.

By using network coding within multicast sessions, we not only enlarge the capacity region, but also obtain simpler, distributed back-pressure algorithms for multicasting from independent and correlated sources. The multicast algorithm described in [22] involves enumeration of all multicast trees used, while that in [4] involves maintaining a virtual queue for every subset of sinks for every session. In our approach, each node has just one virtual queue for each sink of each session (for independent sources) or for each source-sink pair of each session (for correlated sources). Routing, network coding and scheduling decisions are made locally by comparing, for each link, the difference in length of corresponding virtual queues, summed over each session's queues. For correlated sources, the sinks locally determine and control transmission rates across the sources. This gives a completely distributed algorithm for wired networks; in the wireless case, scheduling and power control among interfering transmitters is done centrally.

We also show how the algorithm can be extended to wireless multicast while exploiting the wireless multicast advantage [26], where a single transmission can simultaneously be received by multiple nodes, and the combining advantage [2], where nodes can soft-combine the same information signal transmitted by different nodes possibly at different times.

This paper is organized as follows. We present the system model, discuss some network coding considerations and define the notion of stability for multicast in Section II. The characterization of the capacity region and the capacity achieving back-pressure algorithm are presented in Sections III and IV for independent sources on wired and the wireless networks, respectively. In Section V, we treat the general case where each multicast session consists of multiple correlated sources. We conclude with a summary and discussion of future work in Section VI. Portions of this work have appeared in [12].

II. Preliminaries

A. Network model

We consider a network composed of a set $\mathcal{N}$ of $N = |\mathcal{N}|$ nodes with communication links between them that are fixed or time-varying according to some specified ergodic processes, and transmission of a set of multicast sessions $\mathcal{C}$ through the network. Each session $c \in \mathcal{C}$ is associated with a set $\mathcal{S}_c \subset \mathcal{N}$ of sources, and an exogenous process of data arrivals at each of these sources which must be transmitted over the network to each of a set $\mathcal{T}_c \subset \mathcal{N}$ of sinks. Transmissions are assumed to occur in slotted time, with time slots of length T. Decisions on routing, scheduling, etc. are made at most once a slot. For simplicity, we assume fixed length packets and link transmission rates that are restricted to integer multiples of the packet-length/time-slot quotient. That is, an integer number of packets can be transmitted in each slot.

We consider both wired and wireless networks. In our model, for the wired network case the network connectivity and the link rates are explicitly specified. For wireless networks, the network connectivity and link transmission rates depend on the transmitted signal and interference powers and the channel propagation conditions. For example, the transmission rate per unit bandwidth μ_{ij} from node i to node j, with other nodes $n \in \mathcal{N}$ transmitting independent information simultaneously, may be given by the Shannon formula [8]

$$\mu_{ij}(\underline{P}, \underline{S}) = \log\left(1 + \frac{P_i S_{ij}}{N_0 + \sum_{n \in \mathcal{N}} P_n S_{nj}}\right)$$

where P_l is the power transmitted by node l, S_{lj} is the channel gain from node l to node j and N_0 is additive white Gaussian noise power over the signaling bandwidth. Alternate expressions for the transmission rate that take into account practical non-idealities can also be considered within our framework. We also consider more complex scenarios where multiple transmitters transmit the same information or receivers combine information received at different times. The above transmission rate formula can naturally be extended to such *compound* transmission scenarios. This is described in more detail in Section IV.A. We assume that the channel conditions are fixed over the duration of a slot, and known at the beginning of the slot. For simplicity of exposition we assume that the channel and arrival processes are independent and identically distributed across slots; a straightforward generalization to ergodic processes is possible using a similar approach as that in [21].

B. Network coding considerations

Network coding within a multicast session allows traffic for different sinks of a session to share network capacity [3]. How to determine or achieve the optimal capacity for multiple multicast sessions with coding across sessions is an open question. We consider the simpler problem of achieving optimal capacity for the case where coding is done only across packets of the same session, and give policies that asymptotically achieve optimality in this case.

We use the approach of distributed random linear network coding [7], [9], [11], in which network nodes form output data by taking random linear combinations of input data. The contents of each packet, as a linear combination of the input packets, are specified by a *coefficient vector* in the packet header, updated by applying to the coefficient vectors the same linear transformations as to the data. The coefficient vector is thus a function of the random code coefficients specifying the linear combinations at intermediate nodes. A sink is able to decode when it receives a full set of packets with linearly independent coefficient vectors.

The networks considered in [9], [11] are static, and thus for a fixed set of linear combinations employed at the network nodes, a fixed transfer matrix exists between the input processes at the sources and the output observed at the sink nodes. However, this is not the case in the dynamic network model considered here, since different packets from the source may be routed through different nodes and combined with other packets generated at different times. Thus a fixed transfer matrix does not exist. Nevertheless, one can define a matrix whose rows are formed by the coefficient vectors of all the packets received at the sink. Let the determinant of such a matrix be denoted by $f(\underline{\xi})$ where $\underline{\xi}$ are the random code coefficients. If a sink receives as many packets with linearly independent coefficient vectors as the number of source packets, then the determinant $f(\underline{\xi})$ of the corresponding matrix is nonzero. If, for some choice of code coefficients $\underline{\xi}$, the matrices corresponding to each sink's set of received packets each have nonzero determinant, then the product of these determinant polynomials as a function of the code coefficients $\underline{\xi}$ is not identically zero. Our back pressure algorithms ensure that this is indeed the case for rates within the capacity region. By the Schwartz-Zippel theorem, choosing the code coefficients uniformly at random from a finite field of size q yields a zero value with probability inversely proportional to q. Thus the randomized network coding approach is also applicable to the dynamic network setting considered in this paper. A formal statement of this result is presented in Theorem 3 in the next section.

For simplicity, we analyze the case where no restrictions are placed on coding among packets from the same multicast session. This asymptotically achieves optimal capacity, but in the worst-case decoding may not be possible until the end of the entire session. In practice, to reduce the code description overhead and decoding complexity, we can delay the transmission of some packets so that coding occurs primarily among packets formed around the same time. Packets formed within some time interval are grouped into a batch and labeled with a batch index, where the number of packets in a batch is relatively large compared to variations in source rates and link capacities. Packets in each queue are transmitted in increasing order of batch index. Packets arriving at a node from different paths may have experienced different delays and thus may be from widely separated batches. At each time slot, of the packets scheduled to be transmitted in combination, only those of the two earliest batches are combined and transmitted. This, in effect, inserts additional delay for some streams so as to roughly equalize the delay of incoming streams that are to be coded together. The queues whose transmissions are delayed experience a resulting increase

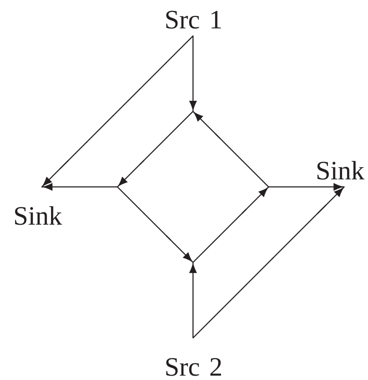

Fig. 1. An example of a multicast connection problem in which capacity can be achieved only if information is transmitted in a cycle. To multicast both sources simultaneously to both sinks requires information to be transmitted around the square.

in average queue length, but the increase is finite, except in cases where the source rates are achievable only if information is transmitted in a cycle, such as in the example of Figure 1, from [13]. The altered queue lengths bias the routing and scheduling decisions, but by reasoning similar to that in [21], this does not affect the stability region.

Still better decoding complexity can be obtained by restricting network coding to occur only among packets of the same batch, as in [7]. However, this further decreases the capacity region for networks with variable sources or links, since packets near the batch boundaries may have to be transmitted without coding. An illustration is given in Figure 2. Such capacity loss decreases with increasing batch size.

For simplicity, the detailed descriptions and analysis of our policies in the following sections are for the case without batch restrictions; an analysis of the case with multiple finite-sized batches would require more detailed source and channel statistics.

C. Stability for multicast

Intuitively, a network is stable if all its queues remain bounded for all time. Let $U_i^c(t)$ be the amount of session c data queued at node i at time t. We define stability as in [21], which considers the "overflow" functions

$$\gamma_i^c(M) = \limsup_{t\to\infty} \frac{1}{t}\int_0^t 1_{[U_i^c(\tau)>M]} d\tau \tag{1}$$

$$\gamma_{sum}(M) = \limsup_{t\to\infty} \frac{1}{t}\int_0^t 1_{[\sum_{i,c} U_i^c(\tau)>M]} d\tau. \tag{2}$$

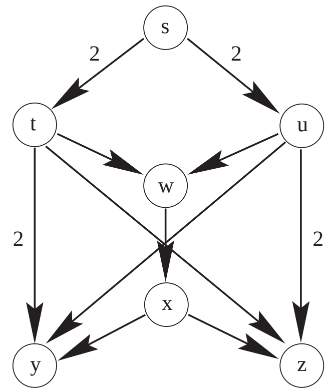

Fig. 2. An example illustrating throughput loss caused by restricting coding to occur only among packets of a batch. Source node s is multicasting to sink nodes y and z. All the links have average capacity 1, except for the four labeled links which have average capacity 2. The optimal solution requires link (w, x) to carry coded information for both receivers. However, variability in the instantaneous capacities of links (u, w), (u, y), (t, w) and (t, z) may cause the number of sink y packets in a batch arriving at node w to differ from the number of sink z packets of that batch arriving at w.

The session c queue at node i is considered stable if $\gamma_i^c(M) \to 0$ as $M \to \infty$. A network of queues is considered stable if each individual queue is stable. The following lemma from [21] gives an equivalent condition for stability of a network:

Lemma 1: As $M \to \infty$, $\gamma_i^c(M) \to 0 \; \forall \, i, c$ if and only if $\gamma_{sum}(M) \to 0$. If the network is stable, there exists a finite value M such that arbitrarily large times $\tilde{t}$ can be found for which $U_i^c(\tilde{t}) < M$ for all i, c simultaneously.

Our approach for the multicast case considers virtual queues of data intended for different sinks. The queues are called virtual as different virtual queues at the same node may correspond to the same actual data. All data originating at the sources of a session c is intended for all sinks in $\mathcal{T}_c$. Data intended for a set $\mathcal{T} \subset \mathcal{T}_c$ of sinks may be replicated or coded at an intermediate node and transmitted on multiple outgoing links; the set $\mathcal{T}$ of intended sinks is partitioned among these links as described in the following sections.

Let $U_i^{c\beta}(t)$ be the amount of session c data intended for sink β queued at node i at time t, and define

$$\gamma_{virtualsum}(M) = \limsup_{t \to \infty} \frac{1}{t} \int_0^t 1_{[\sum_{i,c,\beta} U_i^{c\beta}(\tau) > M]} d\tau.$$

Lemma 2: (a) As $M \to \infty$, $\gamma_i^c(M) \to 0 \; \forall \, i, c$ if and only if $\gamma_{virtualsum}(M) \to 0$.

(b) If the network is stable, there exists a finite value M such that arbitrarily large times $\tilde{t}$ can be found for which $U_i^{c\beta}(\tilde{t}) < M$ for all i, c, β simultaneously.

Proof: (a) Note that $\sum_{i,c} U_i^c \leq \sum_{i,c,\beta} U_i^{c\beta} \leq \tau_{max} \sum_{i,c} U_i^c \; \forall \; \beta \in \mathcal{T}_c$. Thus, we have

$$1_{[\sum_{i,c} U_i^c > M]} \leq 1_{[\sum_{i,c,\beta} U_i^{c\beta} > M]} \leq 1_{[\tau_{max} \sum_{i,c} U_i^c > M]}$$
$$\gamma_{sum}(M) \leq \gamma_{virtualsum}(M) \leq \gamma_{sum}\left(\frac{M}{\tau_{max}}\right).$$

where τ_{max} is the maximum number of sinks in a single multicast session. The result follows from taking limits as $M \to \infty$, and using Lemma 1.

(b) From part (a), if the network is stable, there exists some finite M such that

$$\gamma_{virtualsum}(M) = \limsup_{t\to\infty} \frac{1}{t} \int_0^t 1_{[\sum_{i,c,\beta} U_i^{c\beta}(\tau) > M]} d\tau < \delta$$

for arbitrary δ. If there exists some $\bar{t}$ such that for all $\tau > \bar{t}$, $U_i^{c\beta}(\tau) \geq M$ for some i, c, β, then $\sum_{i,c,\beta} U_i^{c\beta}(\tau) \geq M$ for all $\tau > \bar{t}$, which gives a contradiction. Thus, for any $\bar{t}$, there exists some $\tilde{t} > \bar{t}$ such that $U_i^{c\beta} < M$ for all i, c, β simultaneously. ■

III. Wired networks

We first describe the capacity region and our corresponding back-pressure algorithm for independent sources on wired networks, deferring proofs of the results to the next section, which generalizes these results to the wireless case. We present these results separately for the wired case as they are simpler and hence easier to describe. The main difference between the wired and wireless scenarios is that in a wired network all links are point-to-point links with fixed transmission rates, whereas in a wireless network, links could be point-to-multipoint or vice versa, with mutually dependent transmission rates.

Let Λ be the set of all input rate matrices (λ_i^c) such that there exist variables $\{f_{ab}^{c\beta}, g_{ab}^c\}$ satisfying:

$$f_{ab}^{c\beta} \geq 0 \quad \forall \; a, b, c, \beta \in \mathcal{T}_c$$
$$\lambda_i^c = \sum_b f_{ib}^{c\beta} - \sum_a f_{ai}^{c\beta} \quad \forall \; i, c, \beta \neq i$$
$$\sum_i \lambda_i^c = \sum_a f_{a\beta}^{c\beta} \quad \forall \; c, \beta$$
$$f_{ab}^{c\beta} \leq g_{ab}^c \quad \forall \; a, b, c, \beta$$
$$\sum_c g_{ab}^c \leq R_{ab}$$

where R_{ab} is the capacity of link (a, b). Rates $\lambda_i^c, f_{ab}^{c\beta}, g_{ab}^c$ may be given in any consistent units, such as bits or packets per second.

The variables $\{f_{ab}^{c\beta}\}$ for a (session, sink) pair $(c, \beta \in \mathcal{T}_c)$, where $\mathcal{T}_c$ is the set of session c sinks, define a flow solution from the session c sources to β. Network coding allows flows for different sinks of a common multicast session to share capacity by being coded together [3], so the total usage g_{ab}^c of link (a, b) by session c need only be as large as the maximum usage $f_{ab}^{c\beta}$ by individual sinks $\beta \in \mathcal{T}_c$ of the session. The flow constraints given above provide a characterization of the capacity region as shown in Theorem 1 below.

Theorem 1: (a) A necessary condition for stability of multiple multicast sessions with intra-session network coding is $(\lambda_i^c) \in \Lambda$.

(b) A sufficient condition for stability is that (λ_i^c) is strictly interior to Λ.

The sufficiency part of the above theorem is proved by exhibiting a randomized policy that depends on the knowledge of the statistics of the arrival process and the channel variations. We next present a policy that is based on the back-pressure approach and show that it stabilizes the network for all input rates within the capacity region.

Back-pressure policy for wired networks

At each time slot $[t, t+T)$, the following are carried out:

Scheduling: For each link (a, b), one session

$$c_{ab}^* = \arg\max_c \left\{ \sum_{\beta \in \mathcal{T}_c} \max\left(U_a^{c\beta} - U_b^{c\beta}, 0 \right) \right\}$$

is chosen.

Network coding: For each link (a, b), a random linear combination of data from nonempty queues at a corresponding to all pairs $(c_{ab}^*, \beta \in \mathcal{T}_{c_{ab}^*})$ for which $U_a^{c_{ab}^*\beta} - U_b^{c_{ab}^*\beta} > 0$ is sent at the capacity of link (a, b).

Theorem 2: If input rates (λ_i^c) are such that $(\lambda_i^c + \epsilon) \in \Lambda$, the back-pressure policy stabilizes the system and guarantees an average total buffer occupancy upper bounded by $\frac{TBN}{\epsilon}$, where

$$B = \frac{\tau_{max}}{2} \left(\frac{1}{N} \sum_{i,c} E\left\{ \left(\frac{A_i^c(t)}{T} \right)^2 \right\} + (\mu_{max}^{out} + \mu_{max}^{in})^2 \right)$$

where $A_i^c(t)$ is the amount of session c data arriving at node i in the time slot starting at t, and μ_{max}^{out} and μ_{max}^{in} are the maximum rates into and out of a node respectively.

The above theorem follows as a special case of the corresponding result for wireless networks that is stated and proved in the next section. While the above result shows that each sink receives packets at the source rate, the packets received are packets containing data that is network coded using random linear combinations. In order to retrieve the actual information, each sink decodes the information from the coded packets that it receives. The following theorem shows that the probability that all sinks are not able to decode the information tends to zero exponentially in the length of the code.

Theorem 3: Suppose the source data rates are $(\lambda_i^c - \epsilon')$ for any $\epsilon' > 0$, and that the sources generate random linear combinations of their own data at rates (λ_i^c), which are the input to the network. If (λ_i^c) is strictly interior to Λ, for sufficiently large time t, the probability that not all sinks are able to decode their respective information decreases exponentially in the length of the code.

Proof: Let the randomly chosen network coding coefficients associated with the session-c packets be represented by a vector $\underline{\xi} = (\xi_1, \ldots, \xi_\nu)$. Since only a finite number of packets remain in virtual queues corresponding to any sink β, after a sufficiently large time t, β receives $(\lambda_i^c - \epsilon')t$ packets from the batch. Since the algorithm only codes together packets from different receivers' queues, each of these $(\lambda_i^c - \epsilon')t$ packets corresponds to a different input packet generated at a source, possibly linearly combined with other session-c packets.

Consider the matrix whose rows represent the coefficient vectors of these $(\lambda_i^c - \epsilon')t$ packets. Let its determinant be denoted by $f(\underline{\xi})$, and consider the value $\underline{\tilde{\xi}}$ of $\underline{\xi}$ corresponding to the case where these $(\lambda_i^c - \epsilon')t$ packets each contain different source data and do not undergo network coding, i.e. the code coefficients corresponding to transmissions between β's virtual queues are all 1, and all other code coefficients are 0. In this case, β receives $(\lambda_i^c - \epsilon')t$ independent uncoded packets, and $f(\underline{\tilde{\xi}}) = 1$. Thus, $f(\underline{\xi})$ is not identically zero.

The same holds for each session c': each sink in $\mathcal{T}_{c'}$ receives a set of $(\lambda_i^{c'} - \epsilon')t$ packets whose associated determinant polynomial, as a function of the network code coefficients, is not identically zero.

Since the product of these determinant polynomials as a function of the network code coefficients is not identically zero, by the Schwartz-Zippel theorem, choosing the code coefficients uniformly at random from a finite field of size q yields a zero value with probability inversely proportional to q. The result follows since q is exponential in the length of the code. ■

If no restrictions are placed on where packets may be transmitted, some of the initial packets may be transmitted in the wrong direction and may not reach the sink until the end of the entire session. If the topology of the network is known, this can be avoided by restricting transmission of virtual packets to paths that lead to the corresponding sink. By reasoning similar to that in [21], such restrictions do not affect the stability region; performance can also be improved by biasing towards selection of short paths. Otherwise, each source can start by transmitting dummy packets to fill the virtual queues of nodes with no paths to the corresponding sinks; these set up gradients preventing further packets from being transmitted to those nodes.

IV. Wireless Networks

A. Wireless model

The wireless case is considerably more complicated, as we take into account interference among signals transmitted simultaneously by multiple nodes and the fact that a single node's transmission can be received by multiple nodes, referred to as wireless multicast advantage. Furthermore, it is possible for a receiving node to combine information received from multiple nodes that have transmitted the same signal.

We model wireless transmissions by generalized links, denoted by (a, Z), where a is the originating node and Z is the set of receiving nodes. Each generalized link can correspond to

- a single transmission from node a to neighboring destination nodes at which the SINR of the transmission is greater than some threshold, or
- a multi-step transmission in which a first transmits to one or more destination nodes, some of which, possibly together with a, repeat the transmission in the next time slot, and so on. Combining receptions from multiple transmissions of the same information has been investigated for broadcast in [1], [16]. This potentially improves efficiency as partial receptions may be combined at destination nodes; the transmission rates in a transmission scenario containing such links are averaged over the number of time slots over which such forwarding takes place. For example, say the first transmission from node 1 is received by nodes 2 and 3, with SINR 0.6 and 1.6 respectively. In the next slot, nodes 1 and 3 transmit the same information, which is received by node 2 with combined SINR 0.8. The total SINR received by nodes 2 and 3 over the two time slots is 1.4 and 1.6 respectively. Thus, the compound rate (for the case when the rate function is simply equal to the SINR) is $\min(0.7, 0.8) = 0.7$.

Link rates $\underline{\mu}(\underline{P}, \underline{S}) = (\mu_{aZ}(\underline{P}, \underline{S}))$ are determined by the vector of transmit powers $\underline{P}(t) = (P_{aZ}(t))$ and a channel state vector $\underline{S}(t)$. $\underline{S}(t)$ is assumed to be constant over each time slot, i.e., state transitions occur only on slot boundaries $t = kT$, k integer. We also assume that $\underline{S}(t)$ takes values from a finite set and is ergodic; we denote by $\pi_{\underline{S}}$ the time average probability of state $\underline{S}$. $\underline{P}(t)$ is also held constant over each time slot, and is chosen from a compact set Π of power allocations representing limits on transmit power per node and/or across nodes.

B. Capacity region with intra-session network coding

Let Λ be the set of all input rate matrices (λ_i^c) such that there exist variables $\{f_{abZ}^{c\beta}, g_{aZ}^c\}$ satisfying:

$$f_{abZ}^{c\beta} \geq 0 \quad \forall\ a, b, c, \beta \in \mathcal{T}_c, Z \tag{3}$$

$$\lambda_i^c = \sum_{b,Z} f_{ibZ}^{c\beta} - \sum_{a,Z} f_{aiZ}^{c\beta} \quad \forall\ i, c, \beta \neq i \tag{4}$$

$$\sum_i \lambda_i^c = \sum_{a,Z} f_{a\beta Z}^{c\beta} \quad \forall\ c, \beta \tag{5}$$

$$\sum_{b \in Z} f_{abZ}^{c\beta} \leq g_{aZ}^c \quad \forall\ a, b, c, \beta \in \mathcal{T}_c, Z \tag{6}$$

$$\left(\sum_c g_{aZ}^c \right) \leq (R_{aZ}) \quad \text{for some} \quad (R_{aZ}) \in \Gamma \tag{7}$$

where we define, as in [21], the network *graph family*

$$\Gamma = \sum_{\underline{S}} \pi_{\underline{S}} ConvexHull\{\underline{\mu}(\underline{P}, \underline{S}) | \underline{P} \in \Pi\}, \tag{8}$$

which is shown subsequently to represent the set of all long-term transmission rates (R_{aZ}) supportable by the network. Analogously to the wired case, variables $\{f_{abZ}^{c\beta}\}$ define a flow solution from the session c sources to β; in the wireless case, the total usage of link (a, Z) by (c, β) is the sum over all nodes $b \in Z$ of the variables $f_{abZ}^{c\beta}$.

Theorem 4: (a) A necessary condition for stability of multiple multicast sessions with intra-session network coding is $(\lambda_i^c) \in \Lambda$.

(b) A sufficient condition for stability is that (λ_i^c) is strictly interior to Λ.

Proof: (a) Let $X_i^c(t)$ be the total amount of session c data that has entered the network at node i up to time t.

Suppose the system is stabilizable with some power control and network coding policy that does not involve coding across different multicast sessions. Let $F_{abZ}^{c\beta}(t)$ be the total traffic intended for sink β in session c transmitted from node a to b over link (a, Z) up to time t, and let $G_{aZ}^c(t)$ be the total session c traffic transmitted over link (a, Z) up to time t.

We can derive from this policy one which satisfies

$$F_{abZ}^{c\beta}(t) \geq 0 \quad \forall\ a, b, c, \beta \in \mathcal{T}_c, Z \tag{9}$$

$$X_i^c(t) = \sum_{b,Z} F_{ibZ}^{c\beta}(t) - \sum_{a,Z} F_{aiZ}^{c\beta}(t) + U_i^{c\beta}(t) \quad \forall\ i, c, \beta \neq i \tag{10}$$

$$\sum_i X_i^c(t) = \sum_{a,Z} F_{a\beta Z}^{c\beta}(t) + \sum_i U_i^{c\beta}(t) \quad \forall\ c, \beta \tag{11}$$

$$\sum_{b \in Z} F_{abZ}^{c\beta}(t) \le G_{aZ}^c(t) \quad \forall\ a, b, c, \beta \in \mathcal{T}_c, Z \tag{12}$$

$$\left(\sum_c G_{aZ}^c(t) \right) \le \left(\int_0^t \mu_{aZ}(\underline{P}(\tau), \underline{S}(\tau)) d\tau \right) \quad \forall\ a, Z \tag{13}$$

where the matrix inequalities are considered entrywise.

For stability, by Lemma 2, there must exist some finite M such that $U_i^{c\beta}(t) \le M$ simultaneously for all i, c, β for arbitrarily large t. Thus, there exists a time $\tilde{t}$ for which

$$U_i^{c\beta}(\tilde{t}) \le M \ \forall\ i, c, \beta \tag{14}$$

$$\frac{M}{\tilde{t}} \le \epsilon \tag{15}$$

$$\left| \frac{X_i^c(\tilde{t})}{\tilde{t}} - \lambda_i^c \right| \le \epsilon \tag{16}$$

$$\frac{||T_{\underline{S}}(\tilde{t})||}{\tilde{t}} \le \pi_{\underline{S}} + \epsilon \tag{17}$$

where $||T_{\underline{S}}(t)||$ is the total time in $[0, t]$ during which the channel is in state $\underline{S}$.

Consider $f_{abZ}^{c\beta} = F_{abZ}^{c\beta}(\tilde{t})/\tilde{t}$, $g_{aZ}^c = G_{aZ}^c(\tilde{t})/\tilde{t}$. Substituting into (13) at $t = \tilde{t}$ and dividing by $\tilde{t}$, we obtain

$$\begin{aligned} \left(\sum_c g_{aZ}^c \right) &\le \left(\frac{1}{\tilde{t}} \int_0^{\tilde{t}} \mu_{aZ}(\underline{P}(\tau), \underline{S}(\tau)) d\tau \right) \ \forall\ a, Z \\ &\le \sum_{\underline{S}} \frac{||T_{\underline{S}}(\tilde{t})||}{\tilde{t}} \left(\mu_{aZ}^{\underline{S}} \right) \end{aligned} \tag{18}$$

where matrices $\left(\mu_{aZ}^{\underline{S}} \right)$ are elements of the convex hull ConvexHull$\{(\mu(\underline{P}, \underline{S}))\, | \underline{P} \in \Pi\}$ [21]. Substituting (17) into (18) gives

$$\left(\sum_c g_{aZ}^c(t) \right) \le \sum_{\underline{S}} \pi_{\underline{S}} \left(\mu_{aZ}^{\underline{S}} \right) + \epsilon \left(\mu_{aZ}^{max} \right) \text{card}\{\underline{S}\} \tag{19}$$

where card$\{\underline{S}\}$ is the number of channel states and (μ_{aZ}^{max}) is the maximum transmission rate of link (a, Z).

Similarly, considering (9)-(12) at $\tilde{t}$ and dividing by $\tilde{t}$, and using (14)-(16), we obtain

$$f_{abZ}^{c\beta} \geq 0 \ \forall \ a,b,c,\beta \in \mathcal{T}_c, Z \tag{20}$$

$$\sum_{b,Z} f_{ibZ}^{c\beta} - \sum_{a,Z} f_{aiZ}^{c\beta} - \epsilon \leq \lambda_i^c \leq \sum_{b,Z} f_{ibZ}^{c\beta} - \sum_{a,Z} f_{aiZ}^{c\beta} + 2\epsilon \ \forall \ i,c,\beta \neq i \tag{21}$$

$$\sum_{a,Z} f_{a\beta Z}^{c\beta} - N\epsilon \leq \sum_i \lambda_i^c \leq \sum_{a,Z} f_{a\beta Z}^{c\beta} + 2N\epsilon \ \forall \ c,\beta \tag{22}$$

$$\sum_{b\in Z} f_{abZ}^{c\beta} \leq g_{aZ}^c \ \forall \ a,b,c,\beta \in \mathcal{T}_c, Z \tag{23}$$

Considering (19)-(23) in the limit as $\epsilon \to 0$, we see that (λ_i^c) is a limit point of Λ, which is compact and thus contains its limit points.

(b) is proved constructively in the following sections. ■

C. *Randomized policy*

We first consider a randomized policy requiring prior knowledge of or computation of flow variables $\{f_{abZ}^{c\beta}, g_{aZ}^{c\beta}\}$ from the long-term input and channel statistics.

Assume we have a rate matrix (R_{aZ}) and flow variables $\{f_{abZ}^{c\beta}, g_{aZ}^{c\beta}\}$ satisfying:

$$f_{abZ}^{c\beta} \geq 0 \quad \forall \ a,b,c,\beta \in \mathcal{T}_c, Z \tag{24}$$

$$\lambda_i^c + \epsilon = \sum_{b,Z} f_{ibZ}^{c\beta} - \sum_{a,Z} f_{aiZ}^{c\beta} \quad \forall \ i,c,\beta \neq i \tag{25}$$

$$\sum_i (\lambda_i^c + \epsilon) = \sum_{a,Z} f_{a\beta Z}^{c\beta} \quad \forall \ c,\beta \tag{26}$$

$$\sum_{b\in Z} f_{abZ}^{c\beta} \leq g_{aZ}^c \quad \forall \ a,Z,c,\beta \in \mathcal{T}_c, Z \tag{27}$$

$$\left(\sum_c g_{aZ}^c\right) \leq (R_{aZ}) \quad \text{for some} \ \ (R_{aZ}) \in \Gamma \tag{28}$$

For each time slot $[t, t+T)$, the following are carried out:

Power allocation: The channel state $\underline{S}(t)$ is observed, and power is allocated according to the algorithm of [21], giving instantaneous link rates $\mu_{aZ}(t)$ and long-term average rates R_{aZ}.

Scheduling and network coding: For each link (a,Z), one session $c = c_{aZ}$ is chosen randomly with probability $\frac{g_{aZ}^c}{\sum_c g_{aZ}^c}$. Each of its sinks β is chosen with probability $\frac{\sum_b f_{abZ}^{c\beta}}{g_{aZ}^c}$. For each chosen sink, one destination node $b \in Z$ is chosen with probability $\frac{f_{abZ}^{c\beta}}{\sum_b f_{abZ}^{c\beta}}$ and a random linear combination of data corresponding to the chosen (sink, destination) pairs, if any, is sent on (a,Z) at a fraction $\frac{\sum_c g_{aZ}^c}{R_{aZ}}$ of the instantaneous link rate $\mu_{aZ}(t)$.

Theorem 5: Suppose input rates (λ_i^c) are such that $(\lambda_i^c + \epsilon) \in \Lambda$. The randomized policy stabilizes the system and guarantees a average total buffer occupancy upper bounded by $\frac{TBN}{\epsilon}$, where

$$B = \frac{\tau_{max}}{2} \left(\frac{1}{N} \sum_{i,c} E\left\{ \left(\frac{A_i^c}{T} \right)^2 \right\} + (\mu_{max}^{out} + \mu_{max}^{in})^2 \right)$$

Proof: Let $\mu_{abZ}^{c\beta}(t)$ be the rate offered to sink β of session c for destination b over link (a, Z). The unfinished work evolves according to:

$$\begin{aligned} U_i^{c\beta}(t+T) \quad \leq \quad & \max\left\{ U_i^{c\beta}(t) - T \sum_{b,Z} \mu_{ibZ}^{c\beta}(t) \ , \ 0 \right\} \\ & + T \sum_{a,Z} \mu_{aiZ}^{c\beta}(t) + A_i^c(t) \end{aligned} \tag{29}$$

Define the Lyapunov function $L(\underline{U}) = \sum_{i,c,\beta} (U_i^{c\beta})^2$. Squaring (29) and dropping some negative terms from the right hand side, we obtain

$$\begin{aligned} &[U_i^{c\beta}(t+T)]^2 \leq [U_i^{c\beta}(t)]^2 + T^2 \left[\left(\frac{A_i^c}{T} \right)^2 \right. \\ &\left. + \left(\sum_{b,Z} \mu_{ibZ}^{c\beta} \right)^2 + \left(\sum_{a,Z} \mu_{aiZ}^{c\beta} \right)^2 + 2 \frac{A_i^c}{T} \left(\sum_{a,Z} \mu_{aiZ}^{c\beta} \right) \right] \\ &-2TU_i^{c\beta}(t) \left[\sum_{b,Z} \mu_{ibZ}^{c\beta} - \sum_{a,Z} \mu_{aiZ}^{c\beta} - \frac{A_i^c}{T} \right] \end{aligned} \tag{30}$$

where the time dependencies of $\mu_{abZ}^{c\beta}$ and A_i^c are not shown for ease of notation, since these remain constant over the considered time slot.

Taking expectations of the sum of (30) over all i, c, β, we obtain

$$\begin{aligned} &E\{L(\underline{U}(t+T)) - L(\underline{U}(t)) | \underline{U}(t)\} \leq 2T^2 BN - \\ &2T \sum_{i,c,\beta} U_i^{c\beta}(t) \left[E\left\{ \sum_{b,Z} \mu_{ibZ}^{c\beta} - \sum_{a,Z} \mu_{aiZ}^{c\beta} \middle| \underline{U}(t) \right\} - \lambda_i^c \right] . \end{aligned} \tag{31}$$

by noting, using the Cauchy-Schwarz inequality, that

$$\begin{aligned} \sum_{i,c,\beta} \left(\sum_{b,Z} \mu_{ibZ}^{c\beta} \right)^2 \quad & \leq \quad \sum_{i,c} \tau_{max} \left(\max_{\beta \in \mathcal{T}_c} \sum_{b,Z} \mu_{ibZ}^{c\beta} \right)^2 \\ & \leq \quad \sum_{i} \tau_{max} \left(\sum_{c} \left[\max_{\beta \in \mathcal{T}_c} \sum_{b,Z} \mu_{ibZ}^{c\beta} \right] \right)^2 \\ & \leq \quad N \tau_{max} \left(\mu_{max}^{out} \right)^2 , \end{aligned}$$

$$\sum_{i,c,\beta}\left(\sum_{a,Z}\mu_{aiZ}^{c\beta}\right)^2 \leq N\tau_{max}\left(\mu_{max}^{in}\right)^2,$$

and

$$\begin{aligned}\sum_{i,c,\beta}\left(\frac{A_i^c}{T}\sum_{a,Z}\mu_{aiZ}^{c\beta}\right) &\leq \sum_i \tau_{max}\sum_c \max_{\beta\in\mathcal{T}_c}\left(\frac{A_i^c}{T}\sum_{a,Z}\mu_{aiZ}^{c\beta}\right)\\ &\leq \sum_i \tau_{max}\left(\sum_c \frac{A_i^c}{T}\right)\left(\sum_c \max_{\beta\in\mathcal{T}_c}\sum_{a,Z}\mu_{aiZ}^{c\beta}\right)\\ &\leq N\tau_{max}\mu_{max}^{in}\mu_{max}^{out}\end{aligned}$$

Now

$$E\{\mu_{abZ}^{c\beta}(t)\} = f_{abZ}^{c\beta} \tag{32}$$

for the randomized policy. Substituting (25) and (32) into (31) gives

$$E\{L(\underline{U}(t+T)) - L(\underline{U}(t))|\underline{U}(t)\} \leq 2T^2BN - 2T\epsilon\sum_{i,c,\beta}U_i^{c\beta}(t). \tag{33}$$

Applying the Lyapunov Drift Lemma of [21] gives

$$\sum_{i,c,\beta}\overline{U}_i^{c\beta} \leq \frac{TBN}{\epsilon}. \tag{34}$$

The result follows from noting that $\sum_{i,c}\overline{U}_i^c \leq \sum_{i,c,\beta}\overline{U}_i^{c\beta}$. ■

This proves also Theorem 4(b).

D. Dynamic back-pressure policy

Here we describe a dynamic policy that uses queue state information to make network coding and scheduling decisions, without requiring any knowledge of the input or channel statistics.

Back-pressure policy

In each time slot $[t, t+T)$, the following are carried out:

Scheduling: For each link (a, Z), one session

$$c_{aZ}^* = \arg\max_c\left\{\sum_{\beta\in\mathcal{T}_c}\max\left(\max_{b\in Z}\left(U_a^{c\beta} - U_b^{c\beta}\right), 0\right)\right\}$$

is chosen. Let

$$w_{aZ}^* = \sum_{\beta\in\mathcal{T}_{c_{aZ}^*}}\max\left(\max_{b\in Z}\left(U_a^{c_{aZ}^*\beta} - U_b^{c_{aZ}^*\beta}\right),\ 0\right). \tag{35}$$

Power control: The state $\underline{S}(t)$ is observed, and a power allocation

$$\underline{P}(t) = \arg\max_{\underline{P}\in\Pi} \sum_{a,Z} \mu_{aZ}(\underline{P}, \underline{S}(t)) w^*_{aZ} \tag{36}$$

is chosen.

Network coding: For each link (a, Z), a random linear combination of data corresponding to each (session, sink) pair $(c^*_{aZ}, \beta \in \mathcal{T}_{c^*_{aZ}})$ for which $\max_{b\in Z}\left(U_a^{c^*_{aZ}\beta} - U_b^{c^*_{aZ}\beta}\right) > 0$ is sent at the rate offered by the power allocation. Each destination node $d \in Z$ associates the received information with the virtual buffers corresponding to sinks $\beta \in \mathcal{T}_{c^*_{aZ}}$ for which $d = \arg\max_{b\in Z}\left(U_a^{c^*_{aZ}\beta} - U_b^{c^*_{aZ}\beta}\right)$.

In a network where simultaneous transmissions interfere, optimizing (36) requires a centralized solution. In practice, the optimization (36) can be done heuristically using a greedy approach similar to that in [17], [27] but with the added guidance of weights w^*_{aZ} for prioritization among candidate links (a, Z). If there are enough channels for independent transmissions, the optimization can be done independently for each transmitter.

Theorem 6: If input rates (λ_i^c) satisfy $(\lambda_i^c + \epsilon) \in \Lambda$, the back-pressure policy stabilizes the system and guarantees an average total buffer occupancy upper bounded by $\frac{TBN}{\epsilon}$, where

$$B = \frac{\tau_{max}}{2}\left(\frac{1}{N}\sum_{i,c} E\left\{\left(\frac{A_i^c}{T}\right)^2\right\} + (\mu^{out}_{max} + \mu^{in}_{max})^2\right)$$

Proof: As in the proof of Theorem 5, we denote by $\mu^{c\beta}_{abZ}(t)$ the rate offered to sink β in session c for destination b over link (a, Z). The derivations in the proof of Theorem 5 up to (31) apply also to the back-pressure policy. However, unlike in the randomized policy where $E\{\mu^{c\beta}_{abZ}(t)|\underline{U}(t)\} = f^{c\beta}_{abZ}$ independently of $\underline{U}(t)$, in the back pressure policy, $E\{\mu^{c\beta}_{abZ}(t)|\underline{U}(t)\}$ is dependent on $\underline{U}(t)$. The portion of the drift term that depends on the policy can be rewritten as

$$\begin{aligned} D &= \sum_{i,c,\beta} U_i^{c\beta}(t)\left[E\left\{\sum_{b,Z}\mu^{c\beta}_{ibZ} - \sum_{a,Z}\mu^{c\beta}_{aiZ}\middle|\underline{U}(t)\right\}\right] \\ &= \sum_{a,b,Z}\sum_{c,\beta} E\left\{\mu^{c\beta}_{abZ}|\underline{U}(t)\right\}\left(U_a^{c\beta}(t) - U_b^{c\beta}(t)\right) \end{aligned} \tag{37}$$

We compare the values of (37) for the two policies, giving

$$
\begin{aligned}
D_{rand} &= \sum_{a,b,Z}\sum_{c,\beta} f_{abZ}^{c\beta}\left(U_a^{c\beta}-U_b^{c\beta}\right) && (38)\\
&\le \sum_{a,Z}\sum_{c} g_{aZ}^{c}\sum_{\beta}\max_{b\in Z}\left(U_a^{c\beta}-U_b^{c\beta}\right) && (39)\\
&\le \sum_{a,Z}\sum_{c} g_{aZ}^{c} w_{aZ}^{*} && (40)\\
&\le \sum_{a,Z} R_{aZ} w_{aZ}^{*} && (41)\\
&= \sum_{a,Z}\left(\sum_{\underline{S}} \pi_{\underline{S}} R_{aZ}^{\underline{S}}\right) w_{aZ}^{*} && (42)\\
&\le \sum_{\underline{S}} \pi_{\underline{S}} \max_{\underline{P}\in\Pi}\sum_{a,Z}\mu_{aZ}(\underline{P},\underline{S}) w_{aZ}^{*} && (43)\\
&= D_{backpressure}. && (44)
\end{aligned}
$$

Since the Lyapunov drift for the back-pressure policy is more negative than the drift for the randomized policy, the bound (34) also applies for the back-pressure policy. This completes the proof. ■

Proof of Theorems 1 and 2: Specializing Theorems 4 and 6 to the case where each link (a, Z) has a destination set Z of size 1 and a capacity that does not depend on $\underline{P}$ or $\underline{S}$ gives the result. ■

Theorem 3 on decoding in the wired case applies also to the wireless case.

V. CORRELATED SOURCES

A. Capacity region with intra-session network coding

In this section, we consider transmission problems where, for each session c, the exogenous data arriving at sources $\alpha \in \mathcal{S}_c$ in each unit time period are drawn i.i.d. from some joint distribution Q_c. We assume that the exogenous session c arrival rate λ_α^c of each source $\alpha \in \mathcal{S}_c$ is less than or equal to the maximum outflow rate μ_{max}^{out} of a node, and note that

$$\sum_{\alpha\in\mathcal{S}'}\lambda_\alpha^c \ge \sum_{\alpha\in\mathcal{S}'} H(\alpha) \ge H(\mathcal{S}') \ge H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}')) \;\; \forall \;\; c, \mathcal{S}' \subseteq \mathcal{S}_c.$$

A transmission problem with correlated sources is considered *achievable* with intra-session network coding if there exists a sequence of codes such that the probability of decoding any session c source symbol in error at any sink in $\mathcal{T}_c$ tends to zero. Decoding can be done by a variety of methods such as typical set decoding or minimum entropy decoding.

Let Λ be the set of all values of $(\{H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}'))|\mathcal{S}'\subset\mathcal{S}_c, c\in\mathcal{C}\},\{\lambda_\alpha^c|\alpha\in\mathcal{S}_c, c\in\mathcal{C}\})$ such that there exist variables (R_{aZ}) and $\{f_{abZ}^{c\alpha\beta}, g_{aZ}^c, \lambda^{c\alpha\beta}\}$ satisfying:

$$f_{abZ}^{c\alpha\beta} \geq 0 \quad \forall \quad a,b,Z,c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c \tag{45}$$

$$\sum_{Z,a\in Z} f_{aiZ}^{c\alpha\beta} - \sum_{Z,b\in Z} f_{ibZ}^{c\alpha\beta} = \begin{cases} -\lambda^{c\alpha\beta} & \forall \quad c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c, i=\alpha \\ \lambda^{c\alpha\beta} & \forall \quad c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c, i=\beta \\ 0 & \forall \quad c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c, i\notin\{\alpha,\beta\} \end{cases} \tag{46}$$

$$\sum_{\alpha\in\mathcal{S}_c,b\in Z} f_{abZ}^{c\alpha\beta} \leq g_{aZ}^c \quad \forall \quad a,Z,c,\beta\in\mathcal{T}_c \tag{47}$$

$$\left(\sum_c g_{aZ}^c\right) \leq (R_{aZ}) \quad \text{for some} \quad (R_{aZ})\in\Gamma \tag{48}$$

$$\lambda^{c\alpha\beta} \leq \lambda_\alpha^c \ \forall \ c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c \tag{49}$$

$$\sum_{\alpha\in\mathcal{S}'} \lambda^{c\alpha\beta} > H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}')) \ \forall \ c,\mathcal{S}'\subseteq\mathcal{S}_c,\beta\in\mathcal{T}_c \tag{50}$$

where Γ is defined as in (8).

Analogously to the independent sources case, variables $\{f_{abZ}^{c\alpha\beta}\}$ define a session c flow of size $\lambda^{c\alpha\beta}$ from source α to sink β. For each (c,β), (50) defines the Slepian-Wolf region [23].

Theorem 7: A necessary condition for achievability of a transmission problem is that the source statistics satisfy $(\{H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}'))|\mathcal{S}'\subset\mathcal{S}_c, c\in\mathcal{C}\},\{\lambda_\alpha^c|\alpha\in\mathcal{S}_c, c\in\mathcal{C}\})\in\Lambda$.

Proof: Let $X^{c\alpha}(t)$ be the total amount of exogenous session c data that has arrived at source α up to time t.

Suppose the transmission problem is achievable with some stable power control and network coding policy that does not involve coding across different multicast sessions. For each session c, the average rates from the sources in $\mathcal{S}_c$ to each sink $\beta\in\mathcal{T}_c$ must be in the Slepian-Wolf region. Let $Y_{out}^{c\alpha\beta}(t)$ be the total amount of (session c, source α, sink β) data received by β, and let $G_{aZ}^c(t)$ be the total session c traffic transmitted over link (a,Z) up to time t. We also define quantities $Y_{in}^{c\alpha\beta}(t)$ and $F_{abZ}^{c\alpha\beta}(t)$ that can be viewed, respectively, as the total amount of session c data from source α that is intended for sink β, and the total session c data intended for sink β transmitted from node a to b over link (a,Z) up to time t.

We can derive from this policy one which satisfies

$$F_{abZ}^{c\alpha\beta}(t) \geq 0 \ \forall \ a,b,c,\alpha\in\mathcal{S}_c,\beta\in\mathcal{T}_c,Z \tag{51}$$

$$
\begin{aligned}
Y_{in}^{c\alpha\beta}(t) &= \sum_{b,Z} F_{\alpha bZ}^{c\alpha\beta}(t) - \sum_{a,Z} F_{a\alpha Z}^{c\alpha\beta}(t) + U_{\alpha}^{c\alpha\beta}(t) \ \ \forall \ \ c,\alpha,\beta && (52)\\
-Y_{out}^{c\alpha\beta}(t) &= \sum_{b,Z} F_{\beta bZ}^{c\alpha\beta}(t) - \sum_{a,Z} F_{a\beta Z}^{c\alpha\beta}(t) + U_{\beta}^{c\alpha\beta}(t) \ \ \forall \ \ c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c && (53)\\
0 &= \sum_{b,Z} F_{ibZ}^{c\alpha\beta}(t) - \sum_{a,Z} F_{aiZ}^{c\alpha\beta}(t) + U_{i}^{c\alpha\beta}(t) \ \ \forall \ \ c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c, i \notin \{\alpha,\beta\} && (54)\\
Y_{in}^{c\alpha\beta}(t) - Y_{out}^{c\alpha\beta}(t) &= \sum_{i} U_{i}^{c\alpha\beta}(t) \ \ \forall \ \ c,\alpha,\beta && (55)\\
\sum_{\alpha\in\mathcal{S}_c, b\in Z} F_{abZ}^{c\alpha\beta}(t) &\leq G_{aZ}^{c}(t) \ \ \forall \ \ a,c,\beta \in \mathcal{T}_c, Z && (56)\\
\left(\sum_{c} G_{aZ}^{c}(t)\right) &\leq \left(\int_0^t \mu_{aZ}(\underline{P}(\tau),\underline{S}(\tau))d\tau\right) && (57)\\
Y_{in}^{c\alpha\beta}(t) &\leq X^{c\alpha}(t) \ \forall \ c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c && (58)\\
\sum_{\alpha\in\mathcal{S}'} \frac{Y_{out}^{c\alpha\beta}(t)}{t} &> H(\mathcal{S}'|(\mathcal{S}_c \backslash \mathcal{S}')) \ \forall \ c, \mathcal{S}' \subseteq \mathcal{S}_c, \beta \in \mathcal{T}_c && (59)
\end{aligned}
$$

where the matrix inequalities are considered entrywise.

For stability, by Lemma 2, there must exist some finite M such that $U_i^{c\alpha\beta}(t) \leq M$ simultaneously for all $i, c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c$ for arbitrarily large t. Thus, there exists a time $\tilde{t}$ for which

$$
\begin{aligned}
U_i^{c\alpha\beta}(\tilde{t}) &\leq M \ \forall \ i,c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c && (60)\\
\frac{M}{\tilde{t}} &\leq \epsilon && (61)\\
\left|\frac{X^{c\alpha}(\tilde{t})}{\tilde{t}} - \tilde{\lambda}^{c\alpha}\right| &\leq \epsilon \ \forall && (62)\\
\frac{||T_{\underline{S}}(\tilde{t})||}{\tilde{t}} &\leq \pi_{\underline{S}} + \epsilon && (63)
\end{aligned}
$$

where $||T_{\underline{S}}(t)||$ is the total time in $[0,t]$ during which the channel is in state $\underline{S}$.

Consider

$$
\begin{aligned}
f_{abZ}^{c\alpha\beta} &= F_{abZ}^{c\alpha\beta}(\tilde{t})/\tilde{t} && (64)\\
g_{aZ}^{c} &= G_{aZ}^{c}(\tilde{t})/\tilde{t} && (65)\\
\lambda^{c\alpha\beta} &= Y_{in}^{c\alpha\beta}(\tilde{t})/\tilde{t} && (66)
\end{aligned}
$$

As in the case of independent sources, (57), (63) and (65) give

$$
\left(\sum_{c} g_{aZ}^{c}(t)\right) \leq \sum_{\underline{S}} \pi_{\underline{S}} \left(\mu_{aZ}^{\underline{S}}\right) + \epsilon \left(\mu_{aZ}^{max}\right) \text{card}\{\underline{S}\} \qquad (67)
$$

where matrices $\left(\mu_{aZ}^{\underline{S}}\right)$ are elements of the convex hull ConvexHull$\{(\mu(\underline{P},\underline{S}))\,|\underline{P} \in \Pi\}$, card$\{\underline{S}\}$ is the number of channel states and (μ_{aZ}^{max}) is the maximum transmission rate of link (a, Z).

Considering (51)-(56) and (58) at $\tilde{t}$ and dividing by $\tilde{t}$, and using (60)-(62) and (64)-(66), we obtain

$$\begin{aligned}
f_{abZ}^{c\alpha\beta} &\geq 0 \quad \forall \quad a,b,c,\beta \in \mathcal{T}_c, Z && (68)\\
\sum_{b,Z} f_{\alpha bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\alpha Z}^{c\alpha\beta} \leq \lambda^{c\alpha\beta} &\leq \sum_{b,Z} f_{\alpha bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\alpha Z}^{c\alpha\beta} + \epsilon \quad \forall \quad c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c && (69)\\
\sum_{b,Z} f_{\beta bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\beta Z}^{c\alpha\beta} \leq -\frac{Y_{out}^{c\alpha\beta}(\tilde{t})}{\tilde{t}} &\leq \sum_{b,Z} f_{\beta bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\beta Z}^{c\alpha\beta} + \epsilon \quad \forall \quad c,\alpha,\beta && (70)\\
\sum_{b,Z} f_{\beta bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\beta Z}^{c\alpha\beta} \leq 0 &\leq \sum_{b,Z} f_{\beta bZ}^{c\alpha\beta} - \sum_{a,Z} f_{a\beta Z}^{c\alpha\beta} + \epsilon \quad \forall \quad c,\alpha,\beta, i \notin \{\alpha,\beta\} && (71)\\
0 \leq \lambda^{c\alpha\beta} - \frac{Y_{out}^{c\alpha\beta}(\tilde{t})}{\tilde{t}} &\leq N\epsilon && (72)\\
\sum_{b\in Z} f_{abZ}^{c\alpha\beta} &\leq g_{aZ}^{c} \quad \forall \quad a,b,c,\beta \in \mathcal{T}_c, Z && (73)\\
\lambda^{c\alpha\beta} &\leq \lambda_{\alpha}^{c} + \epsilon && (74)\\
\sum_{\alpha\in\mathcal{S}'} \frac{Y_{out}^{c\alpha\beta}(\tilde{t})}{\tilde{t}} &> H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}')) \;\; \forall \;\; c, \mathcal{S}' \subseteq \mathcal{S}_c, \beta \in \mathcal{T}_c && (75)
\end{aligned}$$

Considering (67)-(75) in the limit as $\epsilon \to 0$, we see that (Q_c) is a limit point of Λ, which is compact and thus contains its limit points. ■

B. Policies and achievability

We next give sufficient conditions and a back-pressure policy for achievability. Similarly to the case of independent sources, these conditions involve, informally speaking, the network capacity and source data rates being slightly higher than the joint source entropy rates.

The back-pressure policy for correlated sources differs from that for independent sources primarily in the operation at the sinks and the sources. The rates at which packets are injected into the network by the different sources of a session may have to be traded off against each other as the total information rate from all the sources can exceed the joint entropy rate.

We propose a mechanism in which the different sinks monitor the amount of information received from each of the sources and provide feedback implicity through back-pressure to throttle the source rates. This is accomplished by maintaining virtual queues on a per source basis at each of the sinks and emptying these queues at appropriate rates. The information in these virtual queues creates the necessary gradient in queue sizes that then propagates back to the sources. The sources compress the information stream and transmit packets into the network at rates limited by the gradients and thus each source in the set of correlated sources transmits at the appropriate rate.

Specifically, suppose we have a rate matrix (R_{aZ}) and flow variables $\{f_{abZ}^{c\alpha\beta}, g_{aZ}^{c}, \lambda^{c\alpha\beta}\}$ satisfying, for some $\epsilon > 0$:

$$f_{abZ}^{c\alpha\beta} \geq 0 \quad \forall \quad a, b, Z, c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c \tag{76}$$

$$\sum_{Z, a \in Z} f_{aiZ}^{c\alpha\beta} - \sum_{Z, b \in Z} f_{ibZ}^{c\alpha\beta} = \begin{cases} -\lambda^{c\alpha\beta} & \forall \quad c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c, i = \alpha \\ \lambda^{c\alpha\beta} & \forall \quad c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c, i = \beta \\ 0 & \forall \quad c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c, i \notin \{\alpha, \beta\} \end{cases} \tag{77}$$

$$\sum_{\alpha \in \mathcal{S}_c, b \in Z} f_{abZ}^{c\alpha\beta} \leq g_{aZ}^{c} \quad \forall \quad a, Z, c, \beta \in \mathcal{T}_c \tag{78}$$

$$\left(\sum_{c} g_{aZ}^{c}\right) \leq (R_{aZ}) \quad \text{for some} \quad (R_{aZ}) \in \Gamma \tag{79}$$

$$\lambda^{c\alpha\beta} \leq \lambda_{\alpha}^{c} \ \forall \ c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c \tag{80}$$

$$\sum_{\alpha \in \mathcal{S}'} \lambda^{c\alpha\beta} \geq H(\mathcal{S}' | (\mathcal{S}_c \backslash \mathcal{S}')) + \epsilon \ \forall \ c, \mathcal{S}' \subseteq \mathcal{S}_c, \beta \in \mathcal{T}_c \tag{81}$$

Each source has a *source buffer* which stores random linear combinations of exogenous data. Specifically, suppose that the link and source data rates are in bits per unit time, where the unit of time is chosen such that all rates are integers. A source $\alpha \in \mathcal{S}_c$ groups its exogenous data bits into length-n blocks, which are viewed as symbols in the finite field $\mathbb{F}_{2^n}$. These symbols are in turn collected into groups of $\tilde{t}\lambda_{\alpha}^{c}$ symbols. For each such group, the source buffer stores an equal number of symbols that are random linear combinations in $\mathbb{F}_{2^n}$ of the exogenous data symbols in the group. The decoding error probability decreases with the block length n. To reduce the code description overhead, we can collect a number of groups into a batch, put one symbol from each group of a batch into a packet, and apply the same random combinations to all symbols in a packet; the overhead decreases with the number of groups in a batch. As in the previous sections, our policy descriptions and analysis below are for the case of a single batch.However, the coding delay increases with the amount of data on each batch, as all exogenous data bits in a batch must be available at the sources before any packets are transmitted.

Each node i maintains, for each (session, source, sink) triple $(c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c)$, a virtual *transmission queue* $Q_i^{c\alpha\beta}$ whose length at time t is denoted $U_i^{c\alpha\beta}(t)$, and whose maximum length is M. We define $V_i^{c\alpha\beta}(t) = M - U_i^{c\alpha\beta}(t)$.

Each sink $\beta \in \mathcal{T}_c$ has a *token queue* of length $W^{c\alpha\beta}(t)$ for each $\alpha \in \mathcal{S}_c$. $W^{c\alpha\beta}(t)$ keeps track of the net amount of data "owed" when there is insufficient data to remove from $Q_\beta^{c\alpha\beta}$. This has parallels with the use of overflow buffers for positive flow in [6].

The following reverse back-pressure algorithm uses the state of the virtual queues described above to allow receiver nodes to draw appropriate rates from among various correlated sources, and to allow network coding and scheduling decisions to be made without any knowledge of the input or channel statistics.

Reverse back-pressure policy

In each time slot $[t, t+T)$, the following are carried out:

Initialization: For each $(c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c)$, let

$$\hat{W}^{c\alpha\beta}(t) = \min\left(V - V_\beta^{c\alpha\beta}(t^-), W^{c\alpha\beta}(t^-)\right)$$

If this is positive, a corresponding amount of data is removed from $Q_\beta^{c\alpha\beta}$ and from the token queue at β; the queues at β are updated as:

$$\begin{aligned} V_\beta^{c\alpha\beta}(t^+) &= V_\beta^{c\alpha\beta}(t^-) + \hat{W}^{c\alpha\beta}(t) \\ W^{c\alpha\beta}(t^+) &= W^{c\alpha\beta}(t^-) - \hat{W}^{c\alpha\beta}(t) \end{aligned}$$

Outflow rate allocation: Let $\hat{\epsilon}$ be any positive constant less than ϵ. Each sink $\beta \in \mathcal{T}_c \ \forall\ c$ chooses outflow variables $\{A_{out}^{c\alpha\beta}(t) | \alpha \in \mathcal{S}_c\}$ to minimize

$$\sum_\alpha V_\beta^{c\alpha\beta}(t^+) A_{out}^{c\alpha\beta}(t)$$

subject to

$$\sum_{\alpha \in \mathcal{S}'} \frac{A_{out}^{c\alpha\beta}(t)}{T} \geq H(\mathcal{S}' | (\mathcal{S}_c \backslash \mathcal{S}')) + \epsilon - \hat{\epsilon} \ \forall\ c, \mathcal{S}' \subseteq \mathcal{S}_c \tag{82}$$

$$\frac{A_{out}^{c\alpha\beta}(t)}{T} \leq \lambda_\alpha^c \ \forall\ c, \alpha \in \mathcal{S}_c. \tag{83}$$

It then removes an amount

$$\hat{A}_{out}^{c\alpha\beta}(t) = \min\left[V - V_\beta^{c\alpha\beta}(t^+),\ A_{out}^{c\alpha\beta}(t)\right] \tag{84}$$

of data from queue $Q_\beta^{c\alpha\beta}$ and an amount $A_{out}^{c\alpha\beta}(t) - \hat{A}_{out}^{c\alpha\beta}(t)$ is added to the token queue at β.

Inflow rate control: Each source $\alpha \in \mathcal{S}_c \ \forall\ c$ adds an amount $\min\left\{A_{in}^{c\alpha\beta}(t), V_\alpha^{c\alpha\beta}(t)\right\}$ of data from its source buffer to queue $Q_\alpha^{c\alpha\beta}$ for each sink $\beta \in \mathcal{T}_c$, where $A_{in}^{c\alpha\beta}(t) = T\lambda_\alpha^c$.

Scheduling: For each link (a, Z), one session

$$c_{aZ}^* = \arg\max_c \left\{ \sum_{\beta \in \mathcal{T}_c} \max\left(\max_{\alpha \in \mathcal{S}_c} \left(\max_{b \in Z} \left(V_b^{c\alpha\beta} - V_a^{c\alpha\beta} \right) \right),\ 0 \right) \right\}$$

is chosen. For each $\beta \in \mathcal{T}_{c_{aZ}^*}$, let

$$\begin{aligned} \alpha_{aZ}^{\beta*} &= \arg\max_{\alpha \in \mathcal{S}_{c_{aZ}^*}} \left(\max_{b \in Z} \left(V_b^{c_{aZ}^*\alpha\beta} - V_a^{c_{aZ}^*\alpha\beta} \right) \right) \\ b_{aZ}^{\beta*} &= \arg\max_{b \in Z} \left(V_b^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta} - V_a^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta} \right) \\ w_{aZ}^* &= \sum_{\beta \in \mathcal{T}_{c_{aZ}^*}} \max\left(V_{b_{aZ}^{\beta*}}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta} - V_a^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta},\ 0 \right) \end{aligned}$$

Power control: The state $\underline{S}(t)$ is observed, and a power allocation

$$\underline{P}(t) = \arg\max_{\underline{P} \in \Pi} \sum_{a,Z} \mu_{aZ}(\underline{P}, \underline{S}(t)) w_{aZ}^* \tag{85}$$

is chosen.

Network coding: For each link (a, Z), a random linear combination of data from queues $Q_a^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta}$ for all sinks $\beta \in \mathcal{T}_{c_{aZ}^*}$ for which $V_{b_{aZ}^{\beta*}}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta} - V_a^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta} > 0$, up to an amount

$$\begin{cases} V_{b_{aZ}^{\beta*}}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta}(t^+) & \text{for} \quad b_{aZ}^{\beta*} \neq \alpha_{aZ}^{\beta*} \\ \max\left\{ V_{b_{aZ}^{\beta*}}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta}(t^+) - A_{in}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta}(t),\ 0 \right\} & \text{for} \quad b_{aZ}^{\beta*} = \alpha_{aZ}^{\beta*}, \end{cases}$$

is sent on (a, Z) at the rate offered by the power allocation to the corresponding destination queues $Q_{b_{aZ}^{\beta*}}^{c_{aZ}^*\alpha_{aZ}^{\beta*}\beta}$.

In analyzing our reverse back-pressure algorithm, we compare it with the following randomized policy, which maintains virtual queues in the same way as for the reverse back-pressure policy, but bases its control decisions on the input and channel statistics rather than the queue states.

Randomized policy

Initialization: The step is identical to the initialization step in the reverse back-pressure algorithm.

Outflow rate allocation: For each $(c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c)$, let

$$A_{out}^{c\alpha\beta}(t) \quad = \quad T\hat{\lambda}^{c\alpha\beta}$$

where $\hat{\lambda}^{c\alpha\beta} = \lambda^{c\alpha\beta} - \hat{\epsilon}$. An amount

$$\hat{A}_{out}^{c\alpha\beta}(t) = \min\left[V - V_\beta^{c\alpha\beta}(t^+),\ A_{out}^{c\alpha\beta}(t) \right] \tag{86}$$

of data is removed from queue $Q_\beta^{c\alpha\beta}$ and an amount $A_{out}^{c\alpha\beta}(t) - \hat{A}_{out}^{c\alpha\beta}(t)$ is added to the token queue at β.

Inflow rate control: Each source $\alpha \in \mathcal{S}_c \; \forall \; c$ adds an amount $\min\left\{A_{in}^{c\alpha\beta}(t), V_\alpha^{c\alpha\beta}(t)\right\}$ of data from its source buffer to queue $Q_\alpha^{c\alpha\beta}$ for each sink $\beta \in \mathcal{T}_c$, where $A_{in}^{c\alpha\beta}(t) = T\lambda^{c\alpha\beta}$.

Power allocation: The channel state $\underline{S}$ is observed, and power is allocated according to the algorithm of [21], giving instantaneous link rates $\mu_{aZ}(t)$ and long-term average rates R_{aZ}.

Scheduling and network coding: For each link (a, Z), one session c is chosen randomly with probability $\frac{g_{aZ}^c}{\sum_c g_{aZ}^c}$. Each of its sinks β is independently chosen with probability $\frac{\sum_{\alpha,b} f_{abZ}^{c\alpha\beta}}{g_{aZ}^c}$. For each chosen sink, one (source, destination node) pair $(\alpha \in \mathcal{S}_c, b \in Z)$ is chosen with probability $\frac{f_{abZ}^{c\alpha\beta}}{\sum_{\alpha,b} f_{abZ}^{c\alpha\beta}}$ and a random linear combination of data corresponding to the chosen (session, source, sink, destination)-tuples (c, α, β, d), up to an amount

$$\begin{cases} V_b^{c\alpha\beta}(t^+) & \text{for} \quad b \neq \alpha \\ \max\left\{V_b^{c\alpha\beta}(t^+) - A_{in}^{c\alpha\beta}(t),\ 0\right\} & \text{for} \quad b = \alpha, \end{cases}$$

is sent on (a, Z) at a fraction $\frac{\sum_c g_{aZ}^c}{R_{aZ}}$ of the instantaneous link rate $\mu_{aZ}(t)$.

Let σ_{max} and τ_{max} be the maximum number of sources and sinks respectively of a multicast session.

Theorem 8: Suppose the source statistics satisfy $(\{H(\mathcal{S}') + \epsilon | \mathcal{S}' \subset \mathcal{S}_c, c \in \mathcal{C}\}, \{\lambda_\alpha^c | \alpha \in \mathcal{S}_c, c \in \mathcal{C}\}) \in \Lambda$. The reverse back-pressure algorithm with $V = \frac{TBN}{\hat{\epsilon}}$ and $M = V + N\mu_{max}^{out}$, where

$$B = \frac{\tau_{max}}{2}\left(\frac{1}{N}\sum_{i,c} E\left\{\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)^2\right\} + \frac{2}{N}\sigma_{max}\mu_{max}^{out}\mu_{max}^{in} + (\mu_{max}^{out})^2 + (\mu_{max}^{in})^2\right)$$

is stable and asymptotically achieves the desired multicast rates.

The proof of the above theorem is presented in the appendix.

VI. Summary and Future Work

We presented dynamic algorithms with network coding for multicast in time-varying wired and wireless networks. We showed that random network coding can be applied in such a dynamic setting. In our algorithms, feedback to nodes upstream is achieved through back-pressure. In particular, for the correlated sources case, the source rate control is also achieved through back-pressure by control of per-source virtual queues at the sinks modulating the relative gradients between different sources. This in contrast to the Internet where source control is achieved through explicit feedback such as in the transmission control protocol (TCP). Combining network coding with the methods that are currently in widespread use for flow control and scheduling would be an important area for future research.

In wireless networks, network coding results in nodes transmitting different information that may interfere with each other resulting in lower transmission rates compared to the case without network coding where the same information is broadcast by different nodes. Thus there is an inherent trade-off between network coding and reduced interference in combination with larger combining gain in the wireless case. Our approach provides a way to combine both techniques by optimizing over the different transmit scenarios. Understanding the balance between network coding and interference reduction are interesting topics for investigation.

Another line of work concerns heuristics for efficient construction of good transmit scenarios. One possibility is a greedy approach similar to that in [17], [27], but weighting the choice of links by the product of link rate and queue size difference, rather than by link rate alone.

APPENDIX

Proof of Theorem 8: Consider a time slot $[t, t+T)$, where $t = kT, k$ integer. Let $\mu_{abZ}^{c\alpha\beta}(t)$ be the rate offered to (source, sink) pair $(\alpha, \beta) \in \mathcal{S}_c \times \mathcal{T}_c$ of session c for transfer of data from $Q_a^{c\alpha\beta}$ to $Q_b^{c\alpha\beta}$,$b \in Z$, over link (a, Z). In both policies, note that

$$V_i^{c\alpha\beta}(t^-) = V_i^{c\alpha\beta}(t^+) = V_i^{c\alpha\beta}(t) \; \forall \; t, c, \alpha, \beta, i \neq \beta.$$

For each $(c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c)$, define the Lyapunov function

$$L^{c\alpha\beta}(\underline{V}^{c\alpha\beta}, W^{c\alpha\beta}) = \sum_i (V_i^{c\alpha\beta})^2 + 2VW^{c\alpha\beta}$$

and let

$$L^{c\alpha\beta}(t) = L^{c\alpha\beta}(\underline{V}^{c\alpha\beta}(t), W^{c\alpha\beta}(t)).$$

In the initialization step, the change in $L^{c\alpha\beta}(t)$ is

$$\begin{aligned} L^{c\alpha\beta}(t^+) - L^{c\alpha\beta}(t^-) &= \left(V_\beta^{c\alpha\beta}(t^-)w + \hat{W}^{c\alpha\beta}(t)\right)^2 - \left(V_\beta^{c\alpha\beta}(t^-)\right)^2 - 2V\hat{W}^{c\alpha\beta}(t) \\ &= \hat{W}^{c\alpha\beta}(t)\left(\hat{W}^{c\alpha\beta}(t) + 2V_\beta^{c\alpha\beta}(t^-) - 2V\right) \end{aligned}$$

If $V_\beta^{c\alpha\beta}(t^-) \leq V$, then $\hat{W}^{c\alpha\beta}(t) \geq 0$ and $V_\beta^{c\alpha\beta}(t^-) + \hat{W}^{c\alpha\beta}(t) \leq V$, so $\hat{W}^{c\alpha\beta}(t) + 2V_\beta^{c\alpha\beta}(t^-) - 2V \leq 0$. If $V_\beta^{c\alpha\beta}(t^-) > V$, then $\hat{W}^{c\alpha\beta}(t) < 0$ and $V_\beta^{c\alpha\beta}(t^-) + \hat{W}^{c\alpha\beta}(t) = V$, so $\hat{W}^{c\alpha\beta}(t) + 2V_\beta^{c\alpha\beta}(t^-) - 2V > 0$. Thus,

$$L^{c\alpha\beta}(t^+) - L^{c\alpha\beta}(t^-) \leq 0. \tag{87}$$

In the rest of the time slot $(t, t+T)$, $V_i^{c\alpha\beta}(t)$ and $W^{c\alpha\beta}(t)$ evolve according to:

$$
\begin{aligned}
V_\alpha^{c\alpha\beta}(t+T) \;&\leq\; \max\left\{V_\alpha^{c\alpha\beta}(t) - A_{in}^{c\alpha\beta}(t) - T\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta}(t)\;,\;0\right\} \\
&\quad +T\sum_{b,Z}\mu_{\alpha bZ}^{c\alpha\beta}(t) \qquad (88)\\
V_\beta^{c\alpha\beta}((t+T)^-) \;&\leq\; \max\left\{V_\beta^{c\alpha\beta}(t^+) - T\sum_{a,Z}\mu_{a\beta Z}^{c\alpha\beta}(t)\;,\;0\right\} \\
&\quad +T\sum_{b,Z}\mu_{\beta bZ}^{c\alpha\beta}(t) + \hat{A}_{out}^{c\alpha\beta}(t) \qquad (89)\\
V_i^{c\alpha\beta}(t+T) \;&\leq\; \max\left\{V_i^{c\alpha\beta}(t) - T\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}(t)\;,\;0\right\} \\
&\quad +T\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}(t) \qquad \forall\;\; i\notin\{\alpha,\beta\} \qquad (90)\\
W^{c\alpha\beta}((t+T)^-) \;&=\; W^{c\alpha\beta}(t^+) + A_{out}^{c\alpha\beta}(t) - \hat{A}_{out}^{c\alpha\beta}(t) \qquad (91)
\end{aligned}
$$

Squaring (88)-(90) and dropping some negative terms from the right hand sides, we obtain

$$
\begin{aligned}
&[V_\alpha^{c\alpha\beta}(t+T)]^2 \leq [V_\alpha^{c\alpha\beta}(t)]^2 + T^2\left[\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)^2\right.\\
&\quad\left.+\left(\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta}\right)^2 + \left(\sum_{b,Z}\mu_{\alpha bZ}^{c\alpha\beta}\right)^2 + 2\frac{A_{in}^{c\alpha\beta}}{T}\left(\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta}\right)\right]\\
&\quad -2TV_\alpha^{c\alpha\beta}(t)\left[\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta} - \sum_{b,Z}\mu_{\alpha bZ}^{c\alpha\beta} + \frac{A_{in}^{c\alpha\beta}}{T}\right] \qquad (92)\\
&[V_\beta^{c\alpha\beta}((t+T)^-)]^2 \leq [V_\beta^{c\alpha\beta}(t^+)]^2 + T^2\left[\left(\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\right)^2\right.\\
&\quad\left.+\left(\sum_{a,Z}\mu_{a\beta Z}^{c\alpha\beta}\right)^2 + \left(\sum_{b,Z}\mu_{\beta bZ}^{c\alpha\beta}\right)^2 + 2\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)\right]\\
&\quad -2TV_\beta^{c\alpha\beta}(t^+)\left[\sum_{a,Z}\mu_{a\beta Z}^{c\alpha\beta} - \sum_{b,Z}\mu_{\beta bZ}^{c\alpha\beta} - \frac{\hat{A}_{out}^{c\alpha\beta}}{T}\right] \qquad (93)\\
&[V_i^{c\alpha\beta}(t+T)]^2 \leq [V_i^{c\alpha\beta}(t)]^2 + T^2\left[\left(\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}\right)^2 + \left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)^2\right]
\end{aligned}
$$

$$-2TV_i^{c\alpha\beta}(t)\left[\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}-\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right]\forall\ i\notin\{\alpha,\beta\} \tag{94}$$

where the time dependencies of $\mu_{abZ}^{c\alpha\beta}$ and $A_{in}^{c\alpha\beta}$ are not shown for simplicity of notation, since these remain constant over the considered time slot. In both policies, $A_{in}^{c\alpha\beta}\leq T\lambda_\alpha^c$ and $\hat{A}_{out}^{c\alpha\beta}\leq A_{out}^{c\alpha\beta}\leq T\lambda_\alpha^c$.

From (91)-(94), we have

$$\begin{aligned}
L^{c\alpha\beta}((t+T)^-)-L^{c\alpha\beta}(t^+) \leq\ & T^2\left[\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)^2+\left(\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\right)^2+\sum_i\left(\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}\right)^2+\sum_i\left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)^2\right.\\
&\left.+2\frac{A_{in}^{c\alpha\beta}}{T}\left(\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta}\right)+2\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)\right]\\
&-2T\left[\sum_i V_i^{c\alpha\beta}(t)\left(\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}-\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)+V_\alpha^{c\alpha\beta}(t^+)\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)\right.\\
&\left.-V_\beta^{c\alpha\beta}(t^+)\left(\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\right)\right]+2V\left(A_{out}^{c\alpha\beta}-\hat{A}_{out}^{c\alpha\beta}\right)\\
\leq\ & T^2\left[\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)^2+\left(\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\right)^2+\sum_i\left(\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}\right)^2+\sum_i\left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)^2\right.\\
&\left.+2\frac{A_{in}^{c\alpha\beta}}{T}\left(\sum_{a,Z}\mu_{a\alpha Z}^{c\alpha\beta}\right)+2\frac{\hat{A}_{out}^{c\alpha\beta}}{T}\left(\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)\right]\\
&-2T\left[\sum_i V_i^{c\alpha\beta}(t)\left(\sum_{a,Z}\mu_{aiZ}^{c\alpha\beta}-\sum_{b,Z}\mu_{ibZ}^{c\alpha\beta}\right)+V_\alpha^{c\alpha\beta}(t^+)\left(\frac{A_{in}^{c\alpha\beta}}{T}\right)\right.\\
&\left.-V_\beta^{c\alpha\beta}(t^+)\left(\frac{A_{out}^{c\alpha\beta}}{T}\right)\right]+\frac{\left(A_{out}^{c\alpha\beta}\right)^2}{2}
\end{aligned}$$

since, from (84), (86), either $\hat{A}_{out}^{c\alpha\beta}=A_{out}^{c\alpha\beta}$, in which case

$$V_\beta^{c\alpha\beta}(t^+)\hat{A}_{out}^{c\alpha\beta}+V\left(A_{out}^{c\alpha\beta}-\hat{A}_{out}^{c\alpha\beta}\right)=V_\beta^{c\alpha\beta}(t^+)A_{out}^{c\alpha\beta},$$

or $\hat{A}_{out}^{c\alpha\beta}=V-V_\beta^{c\alpha\beta}(t^+)$, in which case

$$\begin{aligned}
V_\beta^{c\alpha\beta}(t^+)\hat{A}_{out}^{c\alpha\beta}+V\left(A_{out}^{c\alpha\beta}-\hat{A}_{out}^{c\alpha\beta}\right) &= V_\beta^{c\alpha\beta}(t^+)A_{out}^{c\alpha\beta}+\left(A_{out}^{c\alpha\beta}-\hat{A}_{out}^{c\alpha\beta}\right)\left(V-V_\beta^{c\alpha\beta}(t^+)\right)\\
&= V_\beta^{c\alpha\beta}(t^+)A_{out}^{c\alpha\beta}+\left(A_{out}^{c\alpha\beta}-\hat{A}_{out}^{c\alpha\beta}\right)\hat{A}_{out}^{c\alpha\beta}\\
&\leq V_\beta^{c\alpha\beta}(t^+)A_{out}^{c\alpha\beta}+\left(\frac{A_{out}^{c\alpha\beta}}{2}\right)^2
\end{aligned}$$

Summing over all $(c, \alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c)$ and taking expectations, we obtain

$$\begin{aligned}
&\sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} E\{L^{c\alpha\beta}((t+T)^-) - L^{c\alpha\beta}(t^+)|\underline{V}(t^+), \underline{W}(t^+)\} \\
\leq\ & 2T^2 BN - 2T \sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left[\sum_i V_i^{c\alpha\beta}(t) E\left\{ \sum_{a,Z} \mu_{aiZ}^{c\alpha\beta} - \sum_{b,Z} \mu_{ibZ}^{c\alpha\beta} \middle| \underline{V}(t^+), \underline{W}(t^+) \right\} \right. \\
& \left. + V_\alpha^{c\alpha\beta}(t) E\left\{ \frac{A_{in}^{c\alpha\beta}}{T} \middle| \underline{V}(t^+), \underline{W}(t^+) \right\} - V_\beta^{c\alpha\beta}(t^+) E\left\{ \frac{A_{out}^{c\alpha\beta}}{T} \middle| \underline{V}(t^+), \underline{W}(t^+) \right\} \right] \\
=\ & D(\underline{V}(t^+), \underline{W}(t^+))
\end{aligned} \tag{95}$$

where

$$B = \frac{\tau_{max}}{2} \left(\frac{5}{2N} \sum_{c,\alpha} (\lambda_\alpha^c)^2 + \frac{4\sigma_{max}\mu_{max}^{in}\mu_{max}^{out}}{N} + (\mu_{max}^{out})^2 + (\mu_{max}^{in})^2 \right)$$

by noting, using the Cauchy-Schwarz inequality, that

$$\begin{aligned}
\sum_{i,c} \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\sum_{a,Z} \mu_{aiZ}^{c\alpha\beta} \right)^2 &\leq \sum_{i,c} \tau_{max} \max_{\beta \in \mathcal{T}_c} \sum_{\alpha \in \mathcal{S}_c} \left(\sum_{a,Z} \mu_{aiZ}^{c\alpha\beta} \right)^2 \\
&\leq \sum_i \tau_{max} \left(\sum_c \left[\max_{\beta \in \mathcal{T}_c} \sum_{\alpha \in \mathcal{S}_c, a, Z} \mu_{aiZ}^{c\alpha\beta} \right] \right)^2 \\
&\leq N\tau_{max} \left(\mu_{max}^{out}\right)^2, \\
\sum_{i,c} \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\sum_{b,Z} \mu_{ibZ}^{c\alpha\beta} \right)^2 &\leq N\tau_{max} \left(\mu_{max}^{in}\right)^2, \\
\sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\frac{A_{in}^{c\alpha\beta}}{T} \sum_{a,Z} \mu_{a\alpha Z}^{c\alpha\beta} \right) &\leq \sum_c \sigma_{max}\tau_{max} \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\frac{A_{in}^{c\alpha\beta}}{T} \sum_{a,Z} \mu_{a\alpha Z}^{c\alpha\beta} \right) \\
&\leq \sigma_{max}\tau_{max} \left(\sum_c \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \frac{A_{in}^{c\alpha\beta}}{T} \right) \left(\sum_c \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \sum_{a,Z} \mu_{a\alpha Z}^{c\alpha\beta} \right) \\
&\leq \sigma_{max}\tau_{max}\mu_{max}^{in}\mu_{max}^{out} \\
\sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\frac{A_{out}^{c\alpha\beta}}{T} \sum_{b,Z} \mu_{\beta bZ}^{c\alpha\beta} \right) &\leq \sum_c \sigma_{max}\tau_{max} \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left(\frac{A_{out}^{c\alpha\beta}}{T} \sum_{b,Z} \mu_{\beta bZ}^{c\alpha\beta} \right) \\
&\leq \sigma_{max}\tau_{max} \left(\sum_c \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \frac{A_{out}^{c\alpha\beta}}{T} \right) \left(\sum_c \max_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \sum_{b,Z} \mu_{\beta bZ}^{c\alpha\beta} \right) \\
&\leq \sigma_{max}\tau_{max}\mu_{max}^{in}\mu_{max}^{out}.
\end{aligned}$$

For the randomized policy,

$$\begin{aligned} E\{\mu_{abZ}^{c\alpha\beta}(t)\} &= f_{abZ}^{c\alpha\beta} \;\; \forall \;\; a,b,Z,c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c \\ A_{in}^{c\alpha\beta} &= T\lambda^{c\alpha\beta} \;\; \forall \;\; c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c \\ A_{out}^{c\alpha\beta} &= T\hat{\lambda}^{c\alpha\beta} = T(\lambda^{c\alpha\beta} - \hat{\epsilon}) \;\; \forall \;\; c,\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c \end{aligned}$$

independently of $\underline{V}(t), \underline{W}(t)$. Substituting this into (95) gives

$$D_{randomized} \leq 2T^2BN - 2T\hat{\epsilon} \sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} V_\beta^{c\alpha\beta}(t)$$

For the back-pressure policy, variables $E\{\mu_{abZ}^{c\alpha\beta}(t)|\underline{V}(t^+), \underline{W}(t^+)\}, E\left\{\frac{A_{in}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\}$ and $E\left\{\frac{A_{out}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\}$ are dependent on $\underline{V}(t^+)$. The portion of (95) that depends on the policy can be rewritten as

$$\begin{aligned} &\sum_{i,c} \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} V_i^{c\alpha\beta}(t) E\left\{\sum_{a,Z} \mu_{aiZ}^{c\alpha\beta} - \sum_{b,Z} \mu_{ibZ}^{c\alpha\beta} \middle| \underline{V}(t^+), \underline{W}(t^+)\right\} \\ &\quad + \sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} \left[V_\alpha^{c\alpha\beta}(t) E\left\{\frac{A_{in}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\} - V_\beta^{c\alpha\beta}(t) E\left\{\frac{A_{out}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\}\right] \\ = \;\; &\sum_{a,b,Z} \sum_{c,\beta} E\left\{\mu_{abZ}^{c\alpha\beta}|\underline{V}(t^+), \underline{W}(t^+)\right\} \left(V_a^{c\alpha\beta}(t) - V_b^{c\alpha\beta}(t)\right) \\ &\quad + \sum_{c,\alpha,\beta} V_\alpha^{c\alpha\beta}(t) E\left\{\frac{A_{in}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\} - \sum_{c,\alpha,\beta} V_\beta^{c\alpha\beta}(t) E\left\{\frac{A_{out}^{c\alpha\beta}}{T}\middle|\underline{V}(t^+), \underline{W}(t^+)\right\} \end{aligned} \quad (96)$$

The three terms of the expression above involve disjoint sets of policy-dependent variables, so the three terms can be considered separately. The reverse back-pressure policy maximizes each of them subject to constraints which are also satisfied by the randomized policy: for the first term this is shown in detail in the proof of Theorem 6, for the second and third terms, this follows directly from the definitions of the policies. Thus,

$$D_{backpressure}(\underline{V}(t^+), \underline{W}(t^+)) \leq D_{randomized}(\underline{V}(t^+), \underline{W}(t^+)) \leq 2T^2BN - 2T\hat{\epsilon} \sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} V_\beta^{c\alpha\beta}(t). \quad (97)$$

The initialization step is identical for both policies, so for given $\underline{V}(t^-), \underline{W}(t^-)$, both policies give the same $\underline{V}(t^+), \underline{W}(t^+)$. From (87) and (97), we have

$$E\left\{L^{c\alpha\beta}((t-T)^-) - L^{c\alpha\beta}(t^-)|\underline{V}(t^-), \underline{W}(t^-)\right\} \leq 2T^2BN - 2T\hat{\epsilon} \sum_c \sum_{\alpha \in \mathcal{S}_c, \beta \in \mathcal{T}_c} V_\beta^{c\alpha\beta}(t). \quad (98)$$

For the back-pressure policy, since flow is transmitted only from a longer to a shorter transmission queue, and since $V_\beta^{c\alpha\beta}(t^+) \le V$, $\forall\, t' \in (t, t+T)$

$$\begin{aligned} V_\beta^{c\alpha\beta}(t') &\le V + \mu_{max}^{out} \\ V_i^{c\alpha\beta}(t') &\le V + N\mu_{max}^{out}\ \forall\, i. \end{aligned}$$

If $W^{c\alpha\beta}(t^+) = 0$ for all (c, α, β),

$$L^{c\alpha\beta}(t') \le NM^2\ \forall\, t' \in (t, t+T). \tag{99}$$

If $W^{c\alpha\beta}(t^+) > 0$ for some (c, α, β), then $V_\beta^{c\alpha\beta}(t^+) = V$, and

$$E\left\{L^{c\alpha\beta}((t-T)^-) - L^{c\alpha\beta}(t^-)\right\} \le 2T^2BN - 2T\hat{\epsilon}V;$$

setting $V = \frac{TBN}{\hat{\epsilon}}$ gives $E\left\{L^{c\alpha\beta}((t-T)^-) - L^{c\alpha\beta}(t^-)\right\} \le 0$. By induction on the number of time slots,

$$\sum_c \sum_{\alpha\in\mathcal{S}_c, \beta\in\mathcal{T}_c} E\{L^{c\alpha\beta}((kT)^-)\} \le \sum_c \sum_{\alpha\in\mathcal{S}_c, \beta\in\mathcal{T}_c} NM^2\ \forall\, k \in \mathbb{Z}.$$

Since $W^{c\alpha\beta}(t') \le W^{c\alpha\beta}((kT)^-) + \mu_{max}^{out}\ \forall\, t' \in ((k-1)T, kT]$, we have

$$\sum_c \sum_{\alpha\in\mathcal{S}_c, \beta\in\mathcal{T}_c} E\{W^{c\alpha\beta}(t')\} \le \sum_c \sum_{\alpha\in\mathcal{S}_c, \beta\in\mathcal{T}_c} \left(\frac{NM^2}{2V} + \mu_{max}^{out}\right)\ \forall\, t' \tag{100}$$

We can set $M = V + N\mu_{max}^{out}$; the transmission queues are stable since $U_i^{c\alpha\beta}(t') \le M\ \forall\, i, c, \alpha, \beta, t'$.

For each (c, α, β), the average rate $\bar{\lambda}^{c\alpha\beta}(\tilde{t})$ of actual data removed at β in the time interval $[0, \tilde{t})$ is at least

$$\frac{1}{\tilde{t}}\left(\int_0^{\tilde{t}} \frac{A_{out}^{c\alpha\beta}(t')}{T}dt' - W^{c\alpha\beta}(\tilde{t})\right).$$

For a sufficiently large value of $\tilde{t}$, $\bar{\lambda}^{c\alpha\beta}(\tilde{t})$ is within $\frac{\epsilon-\hat{\epsilon}}{2}$ of the corresponding average requested rate

$$\frac{\bar{A}_{out}^{c\alpha\beta}(\tilde{t})}{T} = \frac{1}{\tilde{t}}\int_0^{\tilde{t}} \frac{A_{out}^{c\alpha\beta}(t')}{T}dt'.$$

Averaging (82), (83) over $[0, \tilde{t})$, we have

$$\begin{aligned} \sum_{\alpha\in\mathcal{S}'} \frac{\bar{A}_{out}^{c\alpha\beta}(\tilde{t})}{T} &\ge H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}')) + \epsilon - \hat{\epsilon}\ \ \forall\ \ c, \mathcal{S}' \subseteq \mathcal{S}_c \\ \frac{\bar{A}_{out}^{c\alpha\beta}(\tilde{t})}{T} &\le \lambda_\alpha^c\ \ \forall\ \ c, \alpha \in \mathcal{S}_c. \end{aligned}$$

So we have

$$\begin{aligned} \sum_{\alpha\in\mathcal{S}'} \bar{\lambda}^{c\alpha\beta}(t) &\ge H(\mathcal{S}'|(\mathcal{S}_c\backslash\mathcal{S}')) + \frac{\epsilon-\hat{\epsilon}}{2}\ \ \forall\ \ c, \mathcal{S}' \subseteq \mathcal{S}_c \\ \bar{\lambda}^{c\alpha\beta}(t) &\le \lambda_\alpha^c\ \ \forall\ \ c, \alpha \in \mathcal{S}_c. \end{aligned}$$

Our setup is slightly different from that of [10], but a similar proof approach can be applied to show that the error probability tends to zero as the block length n tends to infinity. Let the randomly chosen network coding coefficients associated with the session-c packets up to time $\tilde{t}$ be represented by a vector $\underline{\xi} = (\xi_1, \ldots, \xi_\nu)$. Since the algorithm only codes together packets from different receivers' queues, each of the $\tilde{t}\bar{\lambda}^{c\alpha\beta}(\tilde{t})$ (session c, source α, sink β) packets received by β corresponds to a different input packet generated at α, which may undergo coding as it traverses a path from α to β. Consider a node v at which a set $\mathcal{P}$ of packets are coded together with coefficients $\underline{\xi}_p$ to form a packet p. Given that the values of $\mathcal{P}$ corresponding to a given pair of distinct source values are distinct, with probability $\frac{1}{2^n}$ over the random choice of $\underline{\xi}_p$, the corresponding values of p are also distinct.

For each sink $\beta \in \mathcal{T}_c$, we classify errors by the subset $\mathcal{S}' \in \mathcal{S}_c$ of sources that are incorrectly decoded, and bound the probability of each type of error. The probability of an error corresponding to $\mathcal{S}'$ can be upper bounded in terms of the probability that for a given pair of source values such that the values of corresponding sources in $\mathcal{S}'$ are distinct while the values of corresponding sources in $\mathcal{S}_c \backslash \mathcal{S}'$ are identical, the corresponding values of all packets removed by β are identical. This probability is at most

$$\left(1 - (1 - \frac{1}{2^n})^K\right)^{\tilde{t} \sum_{\alpha \in \mathcal{S}_c} \bar{\lambda}^{c\alpha\beta}(\tilde{t})},$$

where K is the maximum number of nodes traversed by one of these packets. Identifying this expression with the bound on P_i in [10] and proceeding similarly to the proof there gives the result the error probability tends to zero asymptotically with n.

■

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