On the Equivalence of Shannon Capacity and Stable Capacity in Networks with Memoryless Channels

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***Abstract*—An equivalence result is established between the Shannon capacity and the stable capacity of communication networks. Given a discrete-time network with memoryless, time-invariant, discrete-output channels, it is proved that the Shannon capacity equals the stable capacity. The results treat general demands (e.g., multiple unicast demands) and apply even when neither the Shannon capacity nor the stable capacity is known for the given demands. The result also generalize from discrete-alphabet channels to Gaussian channels.**

I. Introduction

Shannon's information theory provides a fundamental framework for studying network communications. Under this paradigm, the sources are assumed to be saturated, and thus the source nodes can rely on long vectors of source symbols that are available before encoding begins, and receivers decode complete messages after all channel outputs are received. The **Shannon capacity** is defined as the average (over the blocklength) amount of information that can be decoded at the receiver when the source is saturated in this way.

For many applications, the source messages arrive at source nodes statistically, resulting in both idle periods and periods with many arriving messages at the source nodes. In a communication network, we use "stable network solutions" to denote balanced input-output relationships. To be precise, in a stable network solution each receiver node can eventually decode the desired source messages, and the state (*e.g.*, the queue size of each network node) of the network approaches a stable distribution as time goes by [8]. We notice that the decoding delay may vary due to the random source arrival processes. This differs from the block codes used for the Shannon capacity, where messages are decoded at the end of each block. The **stable capacity** is defined to be the set of all source arrival rates that can be achieved by a stable network solution.

The Shannon capacity does not equal the stable capacity in general. An example is shown in Figure 1. In the example, the channel C takes one bit from source S and outputs one bit to receiver R at each time slot. Let x_i and y_i denote the input bit and output bit respectively of channel C in time slot i. The channel function is $y_i = x_i \oplus x_{\lceil i/2 \rceil}$. By the definition of Shannon capacity (more details can be found in Section III), the Shannon capacity of the network is 1 bit per slot, which means the source can transmit n bits of information to the receiver by using a block network code with blocklength n. However, since the receiver needs to store an increasing

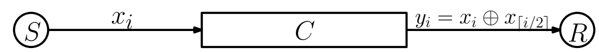

Fig. 1. A network with Shannon capacity (1 bit per slot) strictly larger than the Stable capacity (0 bit per slot).

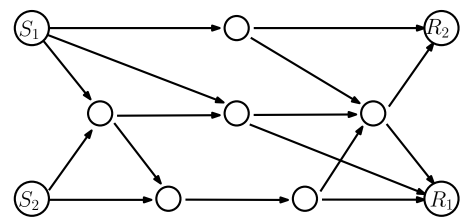

Fig. 2. A two-unicast network.

number of bits for the decoding of future bits, the queue size of the receiver goes to infinity unless the source arrival rate is zero. Thus, the stable capacity of the network is 0. We notice that in Figure 1 the channel C has memory, which turns out to be the reason that the Shannon capacity differs from the stable capacity. In the following, given a discrete time network with memoryless, discrete-output channels and a arbitrary collection of demands (e.g., multiple unicast demands), we prove that the Shannon capacity equals the stable capacity. This result applies even when neither the Shannon capacity nor the stable capacity is known for the given demands. It also generalizes to network with Gaussian noise channels.

I-A. Applications of the equivalence result

Shannon capacity and stable capacity are studied as fundamental concepts in different research fields. The equivalence result in this paper implies that the complexity of computing Shannon capacity equals that of stable capacity. In particular, we can apply the techniques developed for computing Shannon capacity to compute the stable capacity, and vise versa. For example, consider the two-unicast network $\mathcal{N}$ in Figure 2. For each $i = 1, 2$, source S_i wants to communicate with receiver R_i. Assume that for S_1 and S_2 the bit arrivals follow Poisson processes[1] with rate α_1 and α_2, respectively, and that each link is a binary erasure channel (BEC) [1] with erasure probability 0.5. Our objective is determining whether a given rate pair (α_1, α_2) is in the stable capacity region. Using the equivalence result proved in this paper, the original

[1]We assume each time slot is a continuous time interval and a bit arrival can happen anytime.

problem is equivalent to computing the Shannon capacity of the same network $\mathcal{N}$. Using the Shannon capacity equivalence result between point-to-point noisy channels and point-to-point noiseless channels [5], it is equivalent to compute the Shannon capacity of a network $\mathcal{N}'$ with the same topology but assuming each link is a noiseless channel with capacity 0.5. Applying the result for computing the Shannon capacity of two-unicast acyclic noiseless networks [12], we can determine whether (α_1, α_2) is in the Shannon capacity region of $\mathcal{N}'$ and therefore answer the original question, whether (α_1, α_2) is in the stable capacity region of $\mathcal{N}$. A flowchart of these techniques is shown in Figure 3.

I-B. More details on the equivalence result

So far we have not discussed the source arrival statistics. It is a natural question whether the stable capacity of a network depends on the source arrival process. By Loynes Theorem [7], the stable capacity of a network that only permits routing does not change as long as the source arrivals are stationary and ergodic. In this paper, our equivalence result applies as long as the source arrivals are stationary and ergodic, and therefore generalizes Loynes's Theorem to networks with network coding.

Another issue is the definition of network stability. In particular, in this paper we define network stable capacity in terms of queue size and delay, respectively, and show that both are equal to the Shannon capacity of the network. We note that the relationship between queue size and delay is not obvious in the setting of network coding. For the case of routing only, the average decoding delay and average total size of network queues are linearly related (Little's Law) [6], and, the stability of routing networks is usually defined in terms of network queue size by convention [8].

I-C. Related work

The work in [11] shows that queue channels can use "timing information" to deliver more information to the receiver nodes than is carried in the bits alone. Thus, the "actual" Shannon capacity of the queue channel is strictly larger than its "traditional" Shannon capacity *i.e.*, the service rate of the queue multiplied by bits per channel use. In this work, both "Shannon capacity" and "stable capacity" are defined in the setup where the network nodes can use any network resource (e.g., the "timing information") for the purpose of communication. Reference [10] compares the stable throughput region with the saturated throughput region for a random multiple access channel, but the models differ in some assumptions about the protocol and feedback information. Differing from the definition of "capacity", the definition of "throughput" counts the number of information units, such as packets, that can be delivered to the destinations [8]. For statistical sources, [2] shows that extra protocol information needs to be transmitted in order to achieve a desired communication delay constraint. In practice this protocol information may even dominate the "real" information traffic. In this paper, no fixed communication delay constraint is assumed for either Shannon capacity or stable capacity.

Shannon capacity for a multiple access communication system with neither time synchronization nor feedback was derived in [9], [4]. By [8], the stable throughput region for the multiple access communication system with a packet erasure channel model is equal to the Shannon capacity provided that a conjectured property (the sensitivity monotonicity property) holds. For a single source multicast demand network, [3] proves that the Shannon capacity is equal to the stable capacity.

The idea of using "equivalence relationship" to study network information theory was first investigated in [5], which shows the Shannon capacity of a network is unchanged when each noisy point-to-point channel is replaced by a noiseless bit pipe with throughput equal to the noisy channel capacity. The result in [5] applies for general network demands.

I-D. The organization of the paper

The rest of the paper is organized as follows. We formulate the network model in Section II. The Shannon capacity is defined in Section III, and the stable capacity is defined in Section IV. The main result of the paper is shown in Section V. The proof of the main theorem can be found in our technical report [13]..

II. Network Model

II-A. Discrete time discrete alphabet networks

We apply the standard definitions (e.g., [5]) for discrete time discrete alphabet network model. Consider a discrete-time network in which time is slotted. Let $\mathcal{V} = \{1, 2, \ldots, m\}$ be the set of network nodes. In the t'th time slot, each network node $v \in \mathcal{V}$ transmits a random variable $X_t^{(v)} \in \mathcal{X}^{(v)}$ and receives a random variable $Y_t^{(v)} \in \mathcal{Y}^{(v)}$. Both $\mathcal{X}^{(v)}$ and $\mathcal{Y}^{(v)}$ are discrete and countable. Let

$$\mathbf{X}_t \triangleq \left(X_t^{(v)} : v \in \mathcal{V} \right)$$

be the collection of network channel inputs in time slot t and

$$\mathbf{Y}_t \triangleq \left(Y_t^{(v)} : v \in \mathcal{V} \right)$$

be the collection of network channel outputs in time slot t.

Assuming a memoryless and time-invariant network, in each time slot t the network behavior is characterized by a conditional probability density distribution

$$p\left(\mathbf{y}_t | \mathbf{x}_t\right) = p\left(\mathbf{y} | \mathbf{x}\right). \qquad (1)$$

Thus, any network $\mathcal{N}$ is defined by its corresponding triple

$$\mathcal{N} \triangleq \left(\prod_{v=1}^{m} \mathcal{X}^{(v)}, p\left(\mathbf{y}|\mathbf{x}\right), \prod_{v=1}^{m} \mathcal{Y}^{(v)} \right)$$

and the causality constraint that $X_t^{(v)}$ is a function only of past channel outputs $(Y_1^{(v)}, Y_2^{(v)}, \ldots, Y_{t-1}^{(v)})$ at v and outgoing messages originating at node v. Messages are defined formally in Sections III and IV.

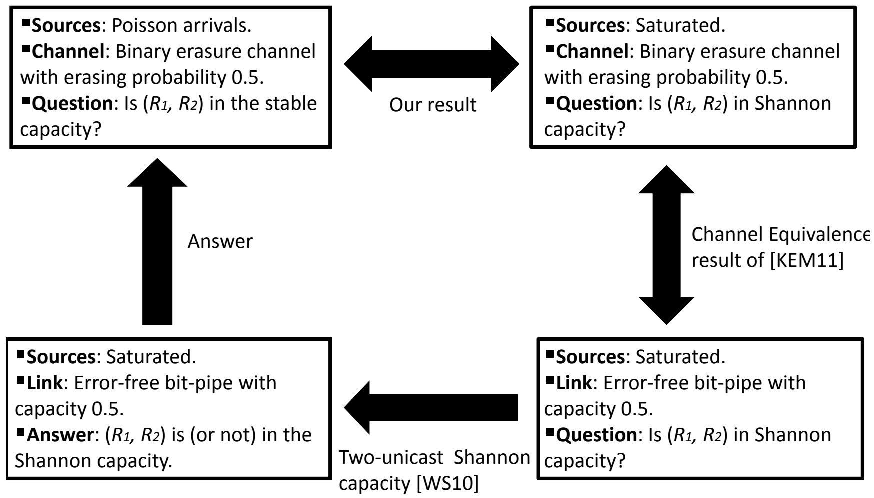

Fig. 3. The diagram of computing the stable capacity of the network in Figure 2.

II-B. Discrete time Gaussian networks

Let $\mathcal{V} = \{1, 2, \ldots, m\}$ be the set of network nodes. In the t'th time slot, each network node $v \in \mathcal{V}$ transmits a random variable $X_t^{(v)} \in \mathcal{X}^{(v)}$ and receives a random variable $Y_t^{(v)} \in \mathcal{Y}^{(v)}$. Assume that node v has $N_X^{(v)}$ transmit antennas and $N_Y^{(v)}$ receive antennas. The input alphabet $\mathcal{X}^{(v)}$ of node v is

$$\mathcal{X}^{(v)} = [-P_1^{(v)}, -P_1^{(v)}] \times \ldots \times [-P_{N_X^{(v)}}^{(v)}, -P_{N_X^{(v)}}^{(v)}],$$

where $P_i^{(v)}$ is the transmit power constraint for the i'th transmit antenna of node v. The output alphabet $\mathcal{Y}^{(v)}$ is

$$\mathcal{Y}^{(v)} = \mathbb{R}^{N_Y^{(v)}}.$$

Similarly, let

$$\mathbf{X}_t \triangleq \left(X_t^{(v)} : v \in \mathcal{V}\right)$$

be the collection of network channel inputs in time slot t and

$$\mathbf{Y}_t \triangleq \left(Y_t^{(v)} : v \in \mathcal{V}\right)$$

be the collection of network channel outputs in time slot t. For each node $v \in \mathcal{V}$, $Y_t^{(v)}$ is given by

$$Y_t^{(v)} = h^{(v)}\left(\mathbf{X}_t\right) + Z_t^{(v)}, \tag{2}$$

where $h^{(v)}(\cdot)$ is a deterministic function and $Z_t^{(v)} \in \mathbb{R}^{N_Y^{(v)}}$ is a noise vector with each component i.i.d. chosen from $\mathcal{N}(0,1)$. For clarity, we assume each node normalizes its received signals such that the Gaussian noise has variance 1. We use

$$\mathbf{h} \triangleq \left(h^{(v)}(\mathbf{X}_t) : v \in \mathcal{V}\right)$$

to denote the collection of all network output functions. Thus, any Gaussian network $\mathcal{N}$ is defined by its corresponding triple

$$\mathcal{N} \triangleq \left(\prod_{v=1}^{m} \mathcal{X}^{(v)}, \mathbf{h}, \prod_{v=1}^{m} \mathcal{Y}^{(v)}\right)$$

and the causality constraint that $X_t^{(v)}$ is a function only of past channel outputs $\left(Y_1^{(v)}, Y_2^{(v)}, \ldots, Y_{t-1}^{(v)}\right)$ at node v and messages originating at v.

III. Saturated source model and network Shannon capacity

For the network models defined in Section II, we apply standard definitions (e.g., [5]) for the saturated source model and Shannon capacity. In particular, let

$$\mathcal{M} \triangleq \{(v, U) : v \in \mathcal{V}, U \subseteq \mathcal{V}\}$$

denote the set of all possible multicast connections across the network. A network code of block length n operates the network over n time slots for the purpose of communicating, for each $(v, U) \in \mathcal{M}$, message

$$W^{(v \to U)} \in \mathcal{W}^{(v \to U)} \triangleq \left\{1, 2, \ldots, 2^{n\mathcal{R}^{(v \to U)}}\right\}$$

from source node v to all the sink nodes $u \in U$. The messages $\left(W^{(v \to U)} : (v, U) \in \mathcal{M}\right)$ are independent and uniformly distributed by assumption. We use $W^{(v \to *)} \in \mathcal{W}^{(v \to *)}$ to denote the vector of messages transmitted from node v and $W \in \mathcal{W}$ to denote the vector of all messages. That is,

$$W^{(v \to *)} \triangleq \left(W^{(v \to U)} : (v, U) \in \mathcal{M}\right)$$

$$\mathcal{W}^{(v \to *)} \triangleq \prod_{(v,U) \in \mathcal{M}} \mathcal{W}^{(v \to U)}$$

$$W \triangleq \left(W^{(v\to*)} : v \in \mathcal{V}\right)$$

$$\mathcal{W} \triangleq \prod_{v\in\mathcal{V}} \mathcal{W}^{(v\to*)}.$$

The constant $\mathcal{R}^{v\to U}$ is called the multicast rate from v to U. Let

$$\mathcal{R} \triangleq \left(\mathcal{R}^{v\to U} : (v,U) \in \mathcal{M}\right) \in \mathbb{R}^{m2^{m-1}}$$

be the dimension-$\left(m2^{m-1}\right)$ rate vector.

Definition 1. *Let the network $\mathcal{N}$ be given. A network block code $\mathcal{C}(\mathcal{N})$ with blocklength n is a set of channel encoding functions*

$$X_t^{(u)} : \left(\mathcal{Y}^{(u)}\right)^{t-1} \times \mathcal{W}^{(u\to*)} \to \mathcal{X}^{(u)}$$

mapping $\left(Y_1^{(u)}, Y_2^{(u)}, \dots, Y_{t-1}^{(u)}, W^{(u\to*)}\right)$ to $X_t^{(u)}$ for each $u \in \mathcal{V}$ and $t \in \{1,2,\dots,n\}$ and a set of decoding functions

$$\hat{W}^{(u\to V,v)} : \left(\mathcal{Y}^{(v)}\right)^{n} \times \mathcal{W}^{(v\to*)} \to \mathcal{W}^{(u\to V)}$$

mapping $\left(Y_1^{(v)}, Y_2^{(v)}, \dots, Y_n^{(v)}, W^{(v\to*)}\right)$ to $\hat{W}^{(u\to V,v)}$ for each (u,V,v) with $(u,V)\in\mathcal{M}$ and $v\in\mathcal{V}$.

Definition 2. *The network block code $\mathcal{C}(\mathcal{N})$ with blocklength n is called a $(\lambda,\mathcal{R})$-block code, if*

$$\left(\log|\mathcal{W}^{(u\to V)}|\right)/n = \mathcal{R}^{(u\to V)}$$

for all $(u,V)\in\mathcal{M}$ and[2]

$$\Pr\left(\hat{W}^{(u\to V,v)} \neq W^{(u\to V)}\right) < \lambda$$

for all (u,V,v) with $(u,V)\in\mathcal{M}$ and $v\in V$. The Shannon capacity $\Psi(\mathcal{N})$ of network $\mathcal{N}$ is the closure of all rate vectors $\mathcal{R}$ such that for any $\lambda>0$ and all n sufficiently large, there exists a $(\lambda,\mathcal{R})$-block code $\mathcal{C}(\mathcal{N})$ of blocklength n.

IV. Statistical source model and network stable capacity

For the network models defined in Section II, we define the statistical source model and network stable capacity as follows. We use

$$\mathcal{M} \triangleq \{(v,U) : v\in\mathcal{V}, U\subseteq\mathcal{V}\}$$

to denote the set of all possible multicast connections across the network. For each $(u,V)\in\mathcal{M}$ and $t>0$, let $A_t^{(u\to V)}\in\mathbb{N}$ denote the message that arrives at node u in time slot t, where $\mathbb{N}$ is the set of all positive integers. For each time slot t, we use notation $A_t^{(u\to*)}$ to denote the collection of all messages arrived at node u. Thus

$$A_t^{(u\to*)} \triangleq \left(A_t^{(u\to V)} : (u,V)\in\mathcal{M}\right).$$

[2]Note that the error probability defined in [5] is the probability λ' that there exists a node that decodes in error. Since 1) $\lambda \leqslant \lambda' \leqslant m2^m\lambda$ and 2) both λ and λ' must be able to approach zero for an achievable rate vector in the Shannon capacity region, there is no difference between these two definitions.

For each $(u,V)\in\mathcal{M}$, the random process $\mathcal{P}^{(u\to V)}$ for $\left\{A_t^{(u\to V)} : t=1,2,\dots\right\}$ is stationary and ergodic and independent of

$$\left\{\mathcal{P}^{(u'\to V')} : (u',V')\in\mathcal{M}, (u',V')\neq(u,V)\right\}.$$

Let

$$\mathcal{P} = \left(\mathcal{P}^{(u\to V)} : (u,V)\in\mathcal{M}\right)$$

be the vector of source arrival processes of all multicast connections, and $\mathscr{P}$ be the set of all possible $\mathcal{P}$ that meet the requirements of stationarity, ergodicity, and independence.

For each $(u,V)\in\mathcal{M}$, the source arrival rate of $\mathcal{P}^{(u\to V)}$ is defined by

$$\alpha^{(u\to V)} = H_\infty\left(A^{(u\to V)}\right),$$

where

$$H_\infty\left(A^{(u\to V)}\right) = \lim_{T\to\infty} H\left(A_1^{(u\to V)},\dots,A_T^{(u\to V)}\right)/T.$$

We use notation

$$\alpha = \alpha(\mathcal{P}) = \left(\alpha^{(u\to V)} : (u,V)\in\mathcal{M}\right) \in \mathbb{R}^{m2^{m-1}}$$

to denote the collection of all source arrival rates for $\mathcal{P}$.

Each node $u\in\mathcal{V}$ has a queue of infinite capacity by assumption. In time slot t, let $Q_t^{(u)}$ denote the bits in the queue of node u and let $q_t^{(u)}$ be the number of bits in $Q_t^{(u)}$. In time slot 0 the queue of each network node is empty by assumption.

Definition 3. *Let the network $\mathcal{N}$ be given. For each time slot $t>0$, a network transmit solution $\mathcal{S}(\mathcal{N})$ is a set of channel encoding functions*

$$X^{(v)} : \{0,1\}^* \to \mathcal{X}^{(v)}$$

mapping $Q_{t-1}^{(v)}$ to $X_t^{(v)}$ for each node $v\in\mathcal{V}$, a set of queue updating functions

$$Q^{(v)} : \{0,1\}^* \times \{0,1\}^* \times \mathcal{Y}^{(v)} \to \{0,1\}^*$$

mapping $\left(Q_{t-1}^{(v)}, A_t^{(v\to*)}, Y_t^{(v)}\right)$ to $Q_t^{(v)}$ for each node $v\in\mathcal{V}$, and a set of message output functions

$$\hat{\mathcal{A}}^{(u\to V,v)} : \{0,1\}^* \to \mathbb{N}^*$$

mapping $Q_t^{(v)}$ to a collection of integers in $\mathbb{N}$ for each (u,V,v) with $\alpha^{(u\to V)}>0$ and $v\in V$.

We notice that $\hat{\mathcal{A}}^{(u\to V,v)}\left(Q_t^{(v)}\right)$ can output an empty set $\emptyset$, *i.e.*, the length of $\hat{\mathcal{A}}^{(u\to V,v)}\left(Q_t^{(v)}\right)$ is zero. In the following, we use $\hat{A}_i^{(u\to V,v)}$ to denote the i'th integer in the set

$$\left\{\hat{\mathcal{A}}^{(u\to V,v)}\left(Q_1^{(v)}\right), \hat{\mathcal{A}}^{(u\to V,v)}\left(Q_2^{(v)}\right)\dots\right\},$$

and $\hat{t}_i^{(u\to V,v)}$ to denote the time slot in which $\hat{A}_i^{(u\to V,v)}$ is output, *i.e.*, $\hat{A}_i^{(u\to V,v)} \in \hat{\mathcal{A}}^{(u\to V,v)}\left(Q_{\hat{t}_i^{(u\to V,v)}}^{(v)}\right)$. For node $v\in$

$\mathcal{V}$, $\hat{A}_i^{(u\to V,v)}$ is the decoding output of $A_i^{(u\to V,v)}$, $\hat{t}_i^{(u\to V,v)}$ is the decoding time of $A_i^{(u\to V,v)}$, and $\hat{t}_i^{(u\to V,v)} - i$ is therefore the decoding delay for $A_i^{(u\to V,v)}$.

In the following we provide the definitions of network stable capacity in terms of network queue size and decoding delay, respectively.

Definition 4. *For any $\lambda > 0$ and $\mathcal{P} \in \mathscr{P}$, a network solution $\mathcal{S}(\mathcal{N})$ is said to be a $(\lambda, \mathcal{P})$-queue stable solution if and only if the following two conditions are met.*

- **Queue stability condition***:*

$$\lim_{t\to\infty} \Pr(q_t < \ell) = F(\ell) \quad \textit{and} \quad \lim_{\ell\to\infty} F(\ell) = 1. \qquad (3)$$

- **Message decodability condition***:*
For each (u, V, v) with $\alpha^{(u\to V)} > 0$ and $v \in V$,

$$\Pr\left(\hat{t}_i^{(u\to V,v)} < \infty\right) = 1$$

and

$$\Pr\left(\hat{A}_i^{(u\to V,v)} \neq A_i^{(u\to V)}\right) < \lambda$$

for each $i \in \mathbb{N}$.

The queue stable region $\mathscr{P}_Q(\mathcal{N})$ of network $\mathcal{N}$ is the set of all arrival process vectors $\mathcal{P} \in \mathscr{P}$ such that for any $\lambda > 0$, there exists a $(\lambda, \mathcal{P})$-queue stable solution $\mathcal{S}(\mathcal{N})$. Let $\Upsilon_Q(\mathcal{N})$ be the closure of the set

$$\{\alpha^* : \mathcal{P} \in \mathscr{P}_Q(\mathcal{N}) \ \ \textit{when } \mathcal{P} \in \mathscr{P} \textit{ and } \alpha(\mathcal{P}) = \alpha^*\},$$

and $\Upsilon'_Q(\mathcal{N})$ be the closure of the set

$$\{\alpha^* : \ \ \textit{there exists } \mathcal{P} \in \mathscr{P}_Q \textit{ such that } \alpha(\mathcal{P}) = \alpha^*\}.$$

Specifically, $\Upsilon_Q(\mathcal{N})$ can be interpreted as the intersection of the queue stable rate regions that corresponds to different source arrival types, and $\Upsilon'_Q(\mathcal{N})$ can be interpreted as the union of these regions. Therefore, it is straightforward that $\Upsilon_Q(\mathcal{N}) \subseteq \Upsilon'_Q(\mathcal{N})$. In the following section, Theorem 1 states that $\Upsilon_Q(\mathcal{N}) = \Upsilon'_Q(\mathcal{N})$, which means the queue stable rate region of a network is invariant with different source arrival types as long as the stationary, ergodic, and independence conditions are met. Thus, both $\Upsilon_Q(\mathcal{N})$ and $\Upsilon'_Q(\mathcal{N})$ define the queue stable capacity of network $\mathcal{N}$.

We define the delay stable capacity as follows.

Definition 5. *For any $\lambda > 0$ and $\mathcal{P} \in \mathscr{P}$, the network solution $\mathcal{S}(\mathcal{N})$ is said to be a $(\lambda, \mathcal{P})$-delay stable solution if and only if $\mathcal{S}(\mathcal{N})$ satisfies the message decodability condition (see Definition 4) and the following delay stability condition.*

- **Delay stability condition***:*

$$\lim_{i\to\infty} \Pr\left(\hat{t}_i - i < \ell\right) = F(\ell) \quad \textit{and} \quad \lim_{\ell\to\infty} F(\ell) = 1 \qquad (4)$$

for each $i \in \mathbb{N}$, where $\hat{t}_i = \max(\hat{t}_i^{(u\to V,v)} : (u, V) \in \mathcal{M}, \alpha^{(u\to V)} > 0, v \in V)$ is the maximum decoding time for the messages arriving in time slot i.

The delay stable region $\mathscr{P}_D(\mathcal{N})$ of network $\mathcal{N}$ is the set of all arrival process vectors $\mathcal{P} \in \mathscr{P}$ such that for any $\lambda > 0$, there exists a $(\lambda, \mathcal{P})$-delay stable solution $\mathcal{S}(\mathcal{N})$. Let $\Upsilon_D(\mathcal{N})$ be the closure of the set

$$\{\alpha^* : \mathcal{P} \in \mathscr{P}_D(\mathcal{N}) \ \ \textit{when } \mathcal{P} \in \mathscr{P} \textit{ and } \alpha(\mathcal{P}) = \alpha^*\},$$

and $\Upsilon'_D(\mathcal{N})$ be the closure of the set

$$\{\alpha^* : \ \ \textit{there exists } \mathcal{P} \in \mathscr{P}_D \textit{ such that } \alpha(\mathcal{P}) = \alpha^*\}.$$

Similarly, $\Upsilon_D(\mathcal{N})$ (or $\Upsilon'_D(\mathcal{N})$) can be interpreted as the intersection (or union) of the delay stable rate regions that corresponds to different source arrival types, and we have $\Upsilon_D(\mathcal{N}) \subseteq \Upsilon'_D(\mathcal{N})$ as a straightforward result. Again, since Theorem 1 shows that $\Upsilon_D(\mathcal{N}) = \Upsilon'_D(\mathcal{N})$, both $\Upsilon_D(\mathcal{N})$ and $\Upsilon'_D(\mathcal{N})$ define the delay stable capacity of network $\mathcal{N}$.

V. Main result

For any network $\mathcal{N}$ as defined in Section II, we have the following theorem.

Theorem 1. $\Psi(\mathcal{N}) = \Upsilon_Q(\mathcal{N}) = \Upsilon'_Q(\mathcal{N}) = \Upsilon_D(\mathcal{N}) = \Upsilon'_D(\mathcal{N})$.

The proof of the theorem can be found in our technical report [13].

VI. Acknowledgment

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