Retractable roof structures

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This paper presents a new concept for retractable roof structures. The new structures consist of a foldable lattice of beams connected by cylindrical joints, to which covering panels or membranes are attached. These structures fold towards their perimeter and there is practically no limit to their shape. Solutions to the key problems that have to be solved in the course of the kinematic design of this new type of structure are presented, including two different ways of connecting them to fixed foundation points while maintaining their internal degree of mobility, and how to determine the shapes of the covering panels to avoid interference during retraction.

Keywords: buildings, structure and design; structural frameworks

Introduction

Retractable roofs are playing an increasingly important role in the development of flexible sports facilities that can be operated in 'optimal conditions', i.e. with the roof open as long as allowed by the weather, throughout the year. This requires roofs that can be opened and closed in a few minutes, at the push of a button. A recent report by a working group of the International Association for Shell and Spatial Structures[1] lists 23 retractable roofs, where the movable part typically spans in excess of 50 m, of which 14 were built in the 1990s. The most commonly used folding schemes involve large, rigid elements undergoing translation or rotation (Fig. 1). To reduce the amount of ground that remains covered by the roof in the retracted configuration, these elements are often overlapped (Figs 1(a) and 1(b)) or folded (Figs 1(c) and 1(d)). Many variants on these concepts have been devised.[1,2]

2. Among the alternative approaches, Fig. 1(e) shows a solution in which the retractable cover is a flexible membrane, whose attachments to a cable net can be moved automatically. This scheme has been used a number of times, mainly in Europe, but has proved less robust and durable than the earlier schemes based on more massive elements.

3. A less conventional structural concept was pioneered by the Spanish engineer and inventor Pinero, who patented a system for movable theatres that could be erected and dismantled in only a few hours and by a small number of people.[3] Pinero's concept was based on expandable trusses, forming a kind of three-dimensional lazy tongs. Fig. 2 shows a recent implementation of this concept in a design for a dismountable roof cover for a swimming pool.[4] This structure consists of a membrane, attached to three identical expandable trusses made from aluminium-alloy tubes joined at the ends and approximately in the middle by 'scissor joints', which are delivered to the site compactly packaged and with the membrane already attached to them. Fig. 3 shows two photographs taken during the erection process. Each packaged truss is suspended from a single point, using a crane, and is pulled open from the ground; when it is fully open, it is attached to fixed supports. Once all the trusses have been deployed, side by side, they are bolted together.

4. A structure of this type is, of course, not automatically retractable. Escrig and his associates have devised various schemes for reducing the amount of manual intervention required for deployment and retraction, such as reducing the number of expandable trusses to only one, hanging from a permanent arch structure. However, the overall span is then quite limited, and manual intervention is still required to connect or disconnect the structure to or from the ground.

5. The main problem with the application of this particular concept to retractable roofs is that it practically rules out the existence of any permanent connections to a foundation. It is, however, very well suited to large structures that have to be packaged very small, and this feature has been exploited by Zeigler[5] for the design of pop-up displays.

6. A breakthrough in the development of concepts for retractable roofs that work as a whole, like Pinero's and Escrig's structures, but which can be permanently attached to their supports was the Iris Dome concept (Fig. 4), invented by Hoberman.[6–8] An important feature of this invention is that the pantograph elements from which it is made are not of the type used by Pinero and Escrig, but are made from non-straight elements, usually known as *angulated elements*, whose properties will be explained in the next section. These pantograph elements support either a membrane cover or partly overlapping, rigid covering elements that can move with respect to one another like the scales

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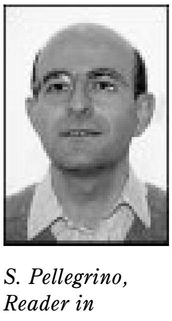

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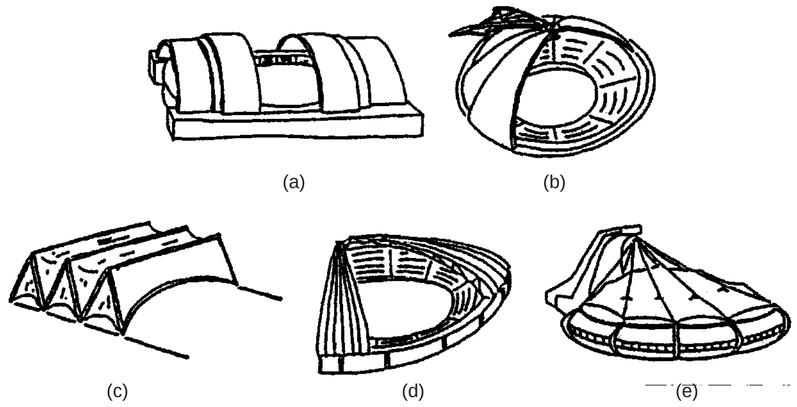


Fig. 1. Folding schemes for retractable roofs (courtesy of K. Ishii)

of a fish. The Iris Dome model shown in Fig. 4 consists of five layers of angulated elements and each layer forms a complete ring lying on a conical surface. Note that the bottom layer is connected to the ground by a series of hinged elements, which allow the small, radial edge expansion/contraction that accompanies retraction/deployment of the dome.

7. Further development of this approach became possible with the discovery[9] of a large family of deployable bar structures which, for any plan shape, can fold towards their perimeter. In plan, these structures are formed by a tessellation of parallelograms, which, in practice, can be made by a series of continuous beams with multiple kinks, which we call ***multi-angulated elements***, connected by cylindrical joints. These elements are directed from the perimeter towards the centre of the structure.

8. Figure 5 shows a 2 m diameter model of such a retractable roof structure consisting of twelve folded plates (plus twelve more, partially hidden under them) supported by a highly redundant structure whose internal degree of freedom resembles that of the Iris Dome. The shape and motion of this support structure are best understood from Fig. 6, showing the retraction of a flat model based on a plan shape of twelve regularly spaced points on a circle. This particular model is made from a series of identical aluminium-alloy elements. Fig. 7 shows a similar model whose joints are regularly spaced on an ellipse, instead of a circle. The elements that make up this structure are not all identical, because of the lower order of symmetry of the ellipse, but the structure can still be folded.

9. Each of these models consists of two sets of multi-angulated elements; those running from the perimeter towards the centre in a clockwise sense are called *clockwise elements*, and those running from the perimeter towards the centre in an anticlockwise sense are called *anticlockwise elements*. Note that in the circular example there are twelve clockwise and twelve anti-clockwise identical elements. Note also that the multi-angulated elements form two concentric 'rings' of rhombuses, which are easiest to see in Fig. 6(b).

10. Both of these structures can be thought of as rather complicated variants of the simple foldable structure shown in Fig. 8, consisting of two sets of parallel, straight rods connected by cylindrical joints. Clearly, most structures obtained by 'kinking' these straight rods will not be foldable, and the present paper will show ways of designing foldable structures formed by interconnected kinked rods.

11. The retractable roof structure shown in Fig. 5 is not flat, but was obtained simply by raising each inner ring by a chosen amount with respect to the previous ring (in this particular case, the initial, flat layout consisted of three rings). The view remains the same in plan but a dome shape is formed in elevation.

12. This paper presents solutions to the three key problems that have to be solved in the course of the kinematic design of this new type of retractable roof structure. The next section explains the kinematic properties of angulated elements, which are the basis for the design of two-dimensional foldable structures of general shapes described in the following section. The way in which these two-dimensions solutions are used to design three-dimensional foldable structures is also explained. The section on 'supports' presents two different ways of connecting these structures to fixed foundation points while maintaining their internal degree

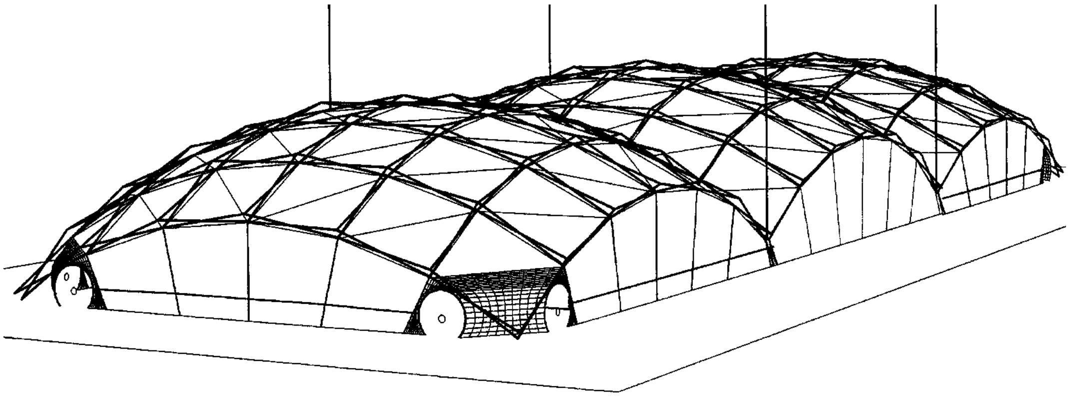


Fig. 2. Dismountable cover for a swimming pool (courtesy of F. Escrig)

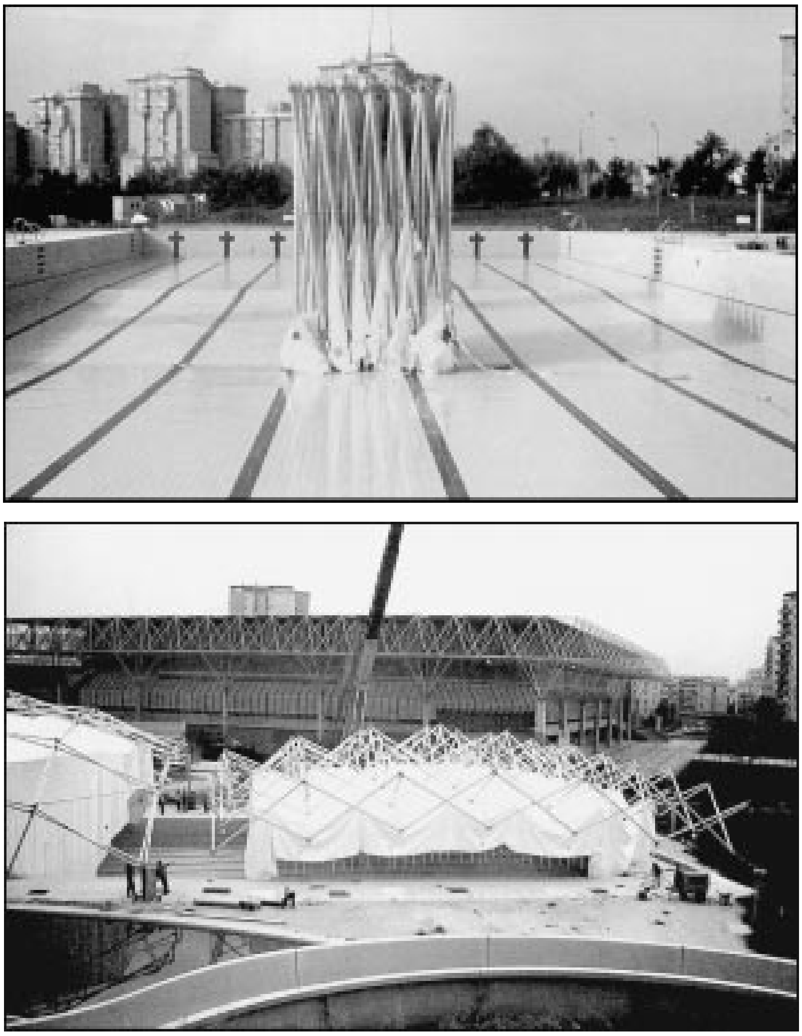


Fig. 3. Deployment of roof elements over a swimming pool (courtesy of F. Escrig): two photographs taken during erection process

of mobility. The section on 'covering elements' presents the results of a kinematic study to determine the shapes of the rigid panels that can be attached to these foldable structures. These panels provide complete cover in the fully deployed configuration without any interference during retraction. A discussion concludes the paper.

Angulated elements

13. An angulated element is a pair of kinked, coplanar rods connected by a cylindrical joint, as shown in Fig. 9. It has been shown by You and Pellegrino[9] that the angle α subtended by the end connectors A, B, C, D of an angulated element does not change when the rods AEC and BED are rotated relative to one another, if either

$$\mathrm{AE} = \mathrm{DE}, \quad \mathrm{BE} = \mathrm{CE} \tag{1}$$

(Type I), where in general $\psi \neq \phi$, i.e. the triangles AED and BEC are isosceles triangles, or

$$\frac{\mathrm{AE}}{\mathrm{DE}} = \frac{\mathrm{CE}}{\mathrm{BE}} \quad \text{and} \quad \psi = \phi \tag{2}$$

(Type II), i.e. the triangles AED and BEC are similar. The special case where the two types of element coincide, i.e. AE = DE, BE = CE and $\psi = \phi$, was discovered by Hoberman,[7] and it is the element used in the Iris Dome.

14. More general foldable elements which also have the property of subtending a constant angle α are obtained by 'cutting' the element shown in Fig. 9 at the scissor hinge E and inserting any number of parallelograms between the triangles AED and BEC. More precisely, a *generalized angulated element* (GAE) is defined as a set of interconnected angulated rods that form a chain of any number of parallelograms with either isosceles triangles (Type I GAE) or similar triangles (Type II GAE) at either end; see the examples shown in Fig. 10.

15. Foldable structures with many different shapes can be made by forming chains of GAEs, provided that certain interface conditions are satisfied between adjacent GAEs (see You and Pellegrino[9] for further details). Also, if certain global conditions are satisfied, a chain of GAEs that forms a closed loop is still a foldable structure. Finally, a structure obtained by connecting a tessellation of parallelograms to a foldable

chain—where any pair of edges of the tessellation that have a point in common and are parallel to an angulated rod of the chain are rigidly connected—is also a foldable structure.

Layout design

16. The only layouts that will be considered here are generated by adding tessellations of parallelograms to (i) a ring formed by two or more GAEs with at least one axis of symmetry, or (ii) a ring of similar rhombuses. According to You and Pellegrino,[9] all structures of this type have an internal degree of mobility. Hence, provided that they are designed so that different parts do not interfere, within a sufficient range of motion, they can provide the basis for the design of retractable-roof layouts.

17. Consider, for example, the layout shown in Fig. 11. Its inner ring has been generated by reflecting the Type I GAE ABCD—which has isosceles triangles at the ends—through the symmetry axes mm and nn. Then, a ring of parallelograms has been connected to the outside of this ring (it would have been equally possible to add one to the inside). Note that the shape of any foldable structure designed by the above method is controlled by the shape of the first ring, as the shape of any parallelogram that is added is determined by the direction of the members of the first ring. The only option available for the parallelograms is whether or not one puts them in.

18. Once the complete layout has been defined, all adjacent edges that run from the perimeter towards the centre, in either a clockwise or anticlockwise sense, are made from continuous, multi-angulated beams, which are connected by cylindrical joints at all kink points. In Fig. 11 the clockwise elements are represented by broken lines and the anticlockwise elements by solid lines. The existence of an internal degree of freedom in this structure has already been verified, as its layout coincides with the intermediate configuration of the model shown in Fig. 7.

19. Note that the process described above does not have a single solution, as there are many different ways of choosing an inner ring that satisfies the folding conditions stated above. Also, in general the shape of the first ring will vary during the retraction process and, thus, a further choice that is available is the stage of deployment at which the shape of the first ring is defined. For example, the model shown in Fig. 7 was designed to have an elliptical first ring in the fully retracted configuration. Note that its outer edge becomes almost a circle in the fully deployed configuration, shown at the top of Fig. 7. A different criterion that could be used, for example, is the minimization of the distance travelled by a set of selected joints which are to be connected to foundation elements.

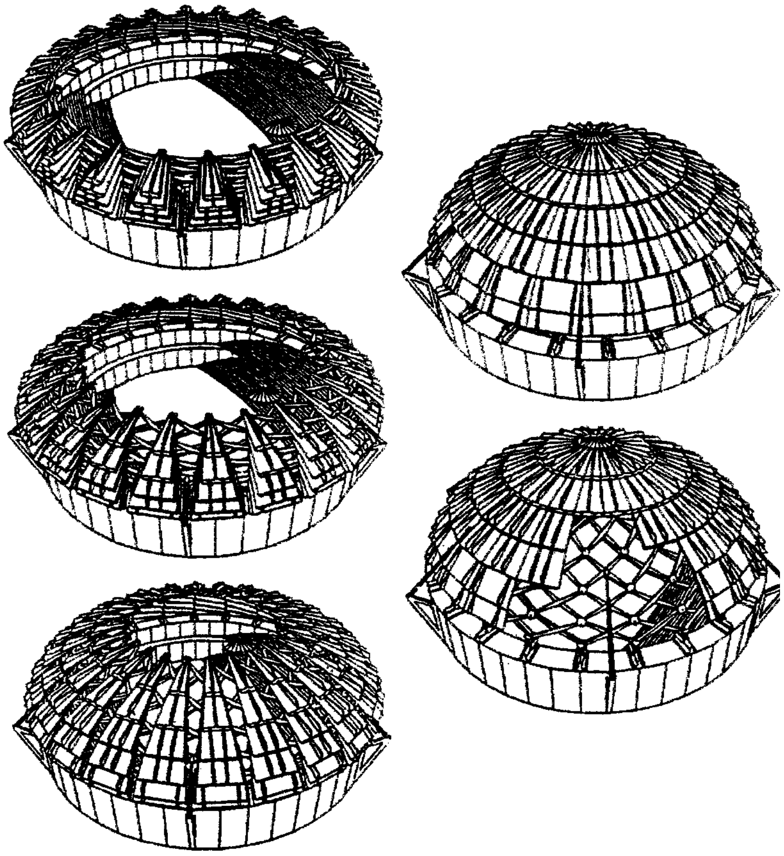


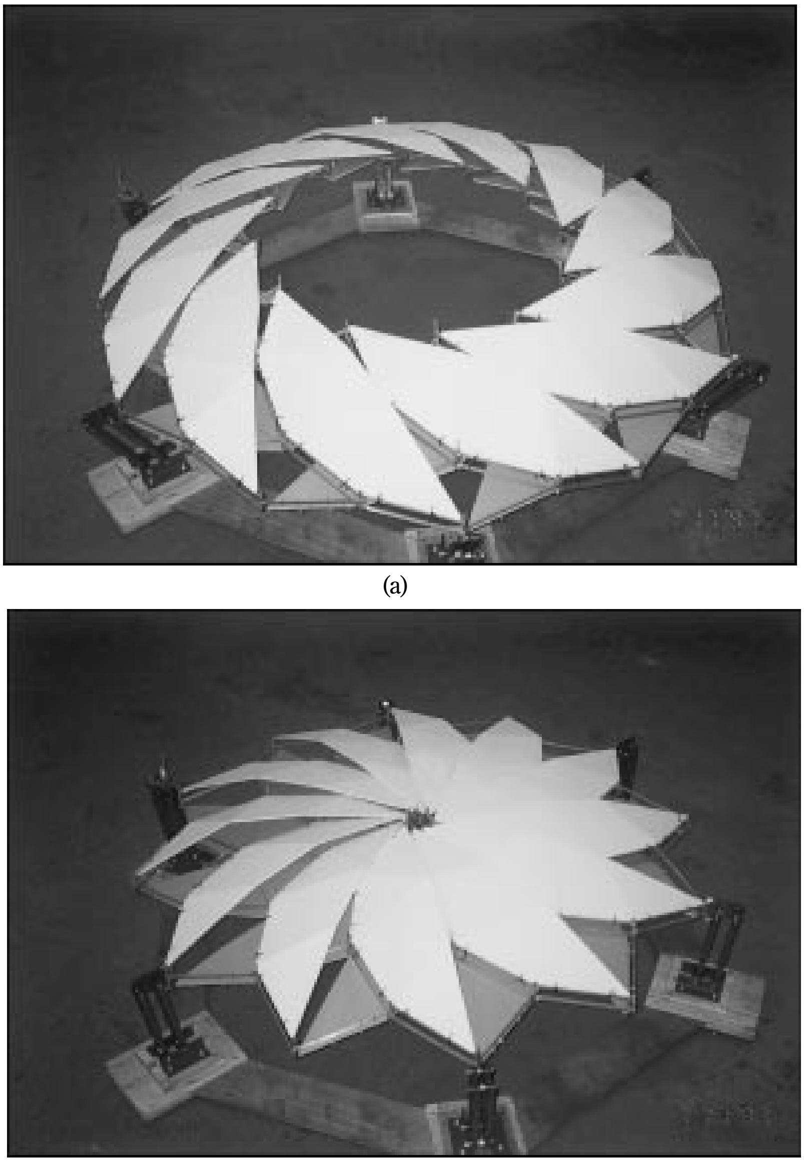
Fig. 4. Iris Dome (courtesy of C. Hoberman)

20. The two-dimensional solutions generated by the method described above are easily extended to three-dimensional structures, by projecting any two-dimensional solution onto a surface with the required shape; see Fig. 12. Note that during this process each multi-angulated rod becomes curved out of its plane but, of course, all connectors between multi-angulated beams must be parallel to the direction of projection, in order to maintain the same degree of kinematic freedom as in the two-dimensional structure.

Supports

21. There are two different ways of connecting to the ground the kinds of retractable structure discussed above, without removing their internal degree of freedom.

22. Structures based on a symmetric layout can be connected to supports that permit translation within the plane of symmetry. Thus, the 2 m diameter model structure shown in Fig. 5 is supported along its perimeter on six supports which, during retraction, translate radially without rotating. The radial deployment of the circular structure is shown in Fig. 13. The magnitude of the edge translation is small in comparison with the translation of the inner joints, because each ring distorts less than the next inner ring. Hence, in Fig. 5, each support consists of two parallel columns connected by cylindrical hinges to the


(a)

(b)

Fig. 5. Model of deployable roof (courtesy of Taiyo Kogyo Co.): (a) open, (b) closed

ground and to the roof structure. During retraction there is a small vertical motion of the whole structure.

23. The second, less intuitive way of supporting any structure whose first ring consists of similar rhombuses is to connect its elements to fixed points, which allow rotation but not translation. The existence and location of such special fixed points are easiest to show for regular, circular layouts, whose inner ring consists of identical rhombuses.

24. To begin with, consider the same type of radial motion as that which would be allowed by radially movable supports. Fig. 14 shows a plot of the motion of a single clockwise element as it deploys. The plot also shows the successive positions taken by the instantaneous centres of rotation as this motion takes place. As can be seen, these instantaneous centres do not remain at a fixed point but instead lie on a circle whose centre is at the origin of the plot. This suggests that, by combining radial deployment with an appropriate rigid-body rotation of the whole structure about its centre, it is possible to keep at fixed points the instantaneous centres of motion of all multi-angulated elements that run in the same sense.

25. This combined motion has different effects on the clockwise and anticlockwise elements that make up the structure, which need to be examined separately. First, consider a clockwise element A_1, A_2, etc. (Fig. 15(a)):

$$\mathrm{OA}_1 = L \sin(\theta - \alpha)/\sin\alpha \tag{3}$$

$$\mathrm{OA}_2 = L \sin\theta/\sin\alpha \tag{4}$$

etc., and, differentiating the above expressions, the radial displacement components of A_1, A_2, etc. due to a small rigid-body rotation $d\theta$ associated with radial deployment are

$$\delta r_{\mathrm{A1}} = L\frac{\cos(\theta - \alpha)}{\sin\alpha}\mathrm{d}\theta \tag{5}$$

$$\delta r_{\mathrm{A2}} = L\frac{\cos\theta}{\sin\alpha}\mathrm{d}\theta \tag{6}$$

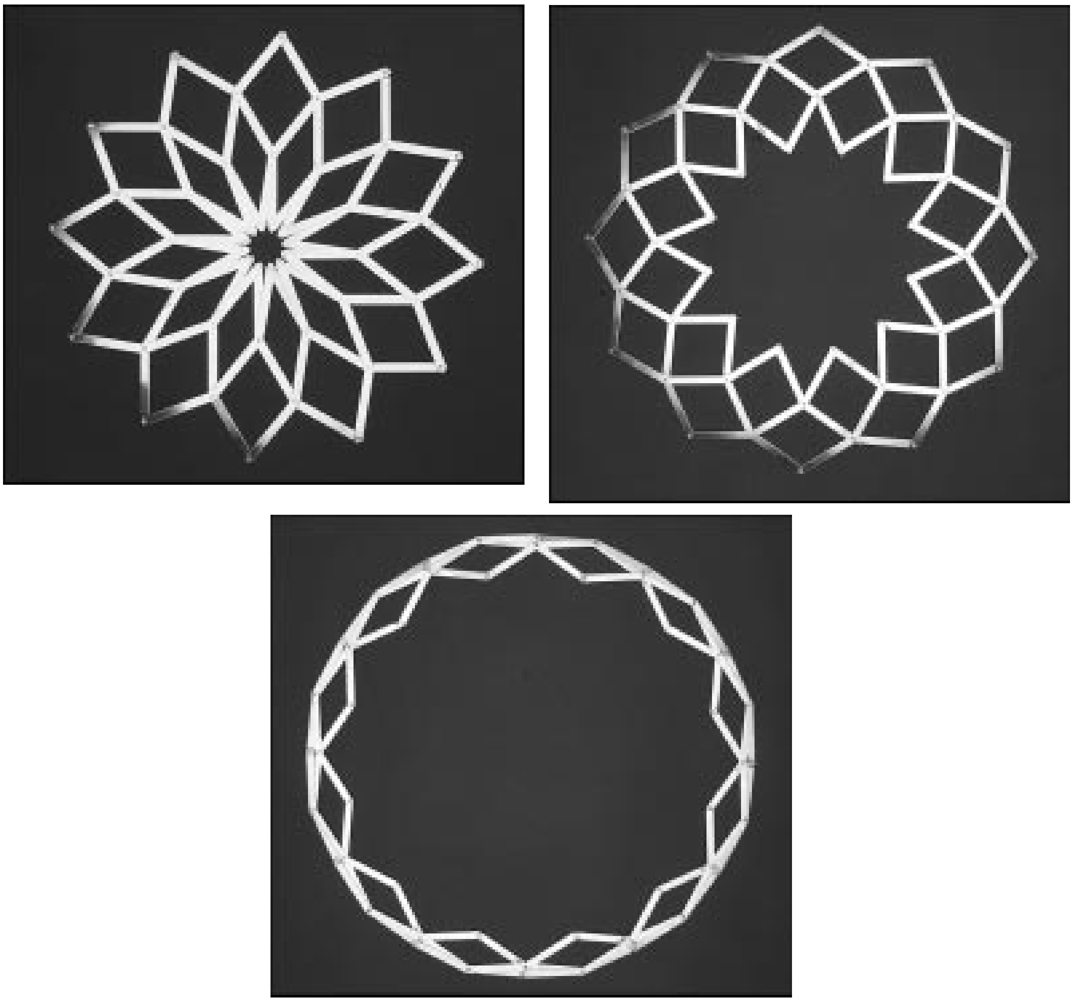


Fig. 6. Model structure with twelve-noded circular layout

etc. The tangential components of displacement of these points, due to a rigid-body rotation about O through the same angle $d\theta$, are

$$\delta t_{A1} = L\frac{\sin(\theta-\alpha)}{\sin\alpha}d\theta \tag{7}$$

$$\delta t_{A2} = L\frac{\sin\theta}{\sin\alpha}d\theta \tag{8}$$

etc.

26. The remaining displacement vectors have magnitude

$$\delta_{A1} = \sqrt{\delta r_{A1}^2 + \delta t_{A1}^2} = L\,d\theta/\sin\alpha \tag{9}$$

$$\delta_{A2} = \sqrt{\delta r_{A2}^2 + \delta t_{A2}^2} = L\,d\theta/\sin\alpha \tag{10}$$

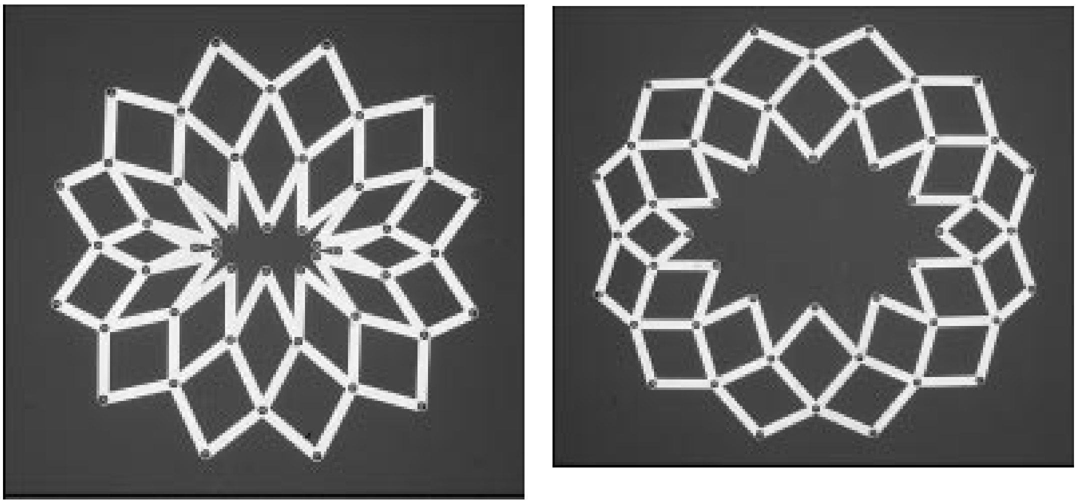
etc., and directions at angles $\theta-\alpha$, θ, etc., respectively, with the radii through A_1, A_2, etc. as shown in Fig. 15(a). Thus, it can be concluded that each multi-angulated element A_1, A_2, etc. rotates about the fixed point C shown in the figure. Note that $R = L/(2\sin\alpha)$. Next, consider an anticlockwise element (Fig. 15(b)). Since $OB_1 = OA_1$, etc., both the radial and the tangential components of displacement of the points B_1, etc. are the same as for the corresponding points A_1, etc. Therefore, the magnitude of the displacement vectors is also unchanged, but their directions are such that these displacements are now parallel. In other words, this element *translates*.

27. Thus, the final result is surprisingly simple: the kink points on each of the clockwise elements of the structure lie on a circle, and during deployment/retraction they rotate about the centre of this circle. The corresponding motion for each of the anticlockwise elements is a pure translation; see Fig. 16.

28. Further analysis shows that fixed points exist for any foldable structure whose first ring is a chain of similar rhombuses whose edges form equal angles θ with the sides of a polygon of any shape. These fixed points lie on a scaled version of this polygon, rotated by θ. The location of the fixed point C for the clockwise element A_1, A_2, A_3 of a general plan shape is shown in Fig. 17.

Covering elements

29. Practical roof structures require, in addition to a retractable bar structure formed by multi-angulated elements, as described above, some forms of covering superstructure. This



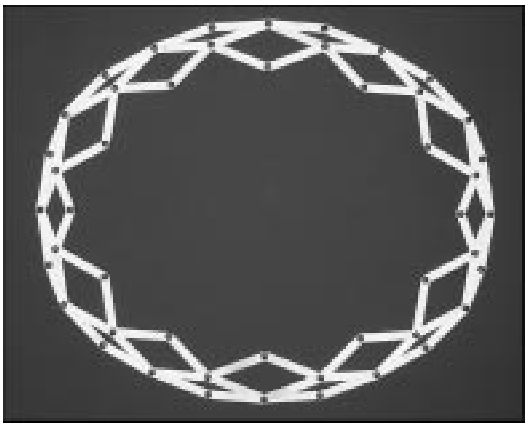


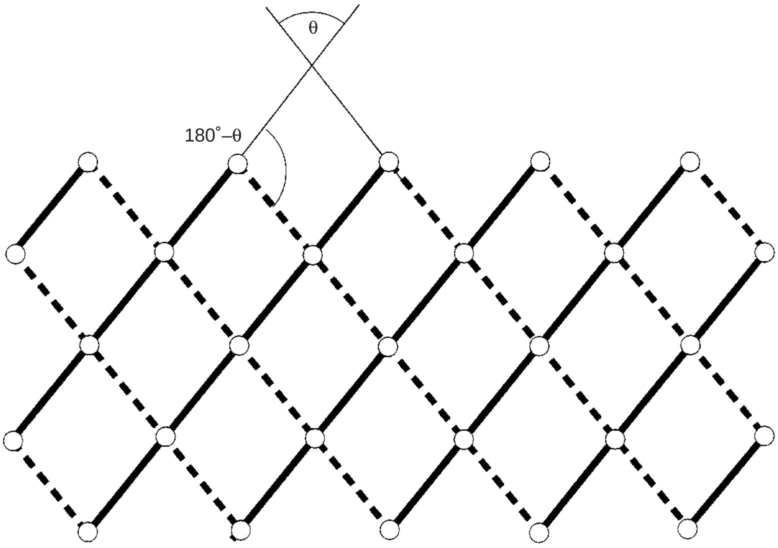
Fig. 7. Model structure with twelve-noded elliptical layout

covering structure can be designed in several different ways. For example, one might cover the whole roof with a single, flexible membrane attached to some or all of the multi-angulated elements. However, in view of the durability problems that have been experienced with previous retractable structures where membranes are subject to repeated stressing and destressing, as mentioned previously, it is likely that this type of solution would suffer from fatigue problems, because the membrane would lose tension and form many creases when the supporting structure was retracted.

30. The alternative solution that is presented here is to divide the cover into separate panels, each of fixed shape. These panels could be prestressed membrane elements, each attached to a single multi-angulated element, or stiff plates of suitable shape, as shown in Fig. 5. Indeed, the plates might even replace some of the multi-angulated elements.

31. A method for determining the plan shapes of the covering elements, so that they leave no gaps when the structure is fully closed and do not interfere with other elements when the structure retracts, will be presented. A simple, preliminary approach to this problem is to require that, regardless of the three-dimensional shape of the panels, there should be no interference between their plan projections. This approach reduces the complexity of the kinematic analysis and makes it easy to carry out interference checks but, of course, poses unnecessary restrictions on the solution. However, once the shapes of a set of panels that satisfy these constraints have been determined, it is often possible to enlarge the

Fig. 8. Simple, trellis-type foldable structure



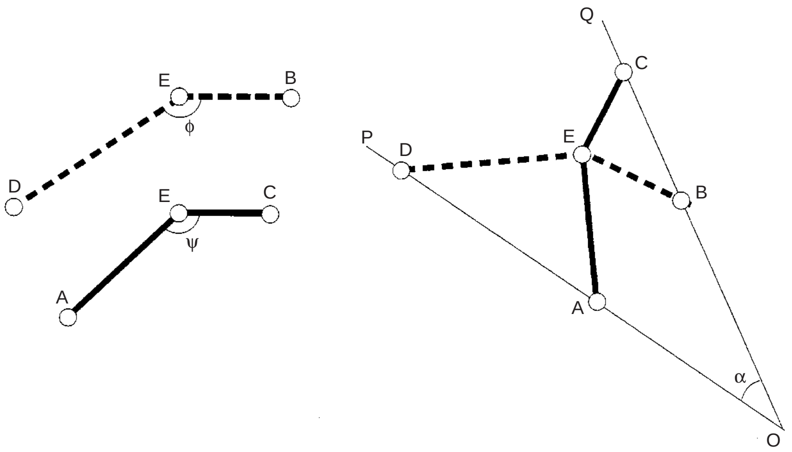


Fig. 9. Angulated element

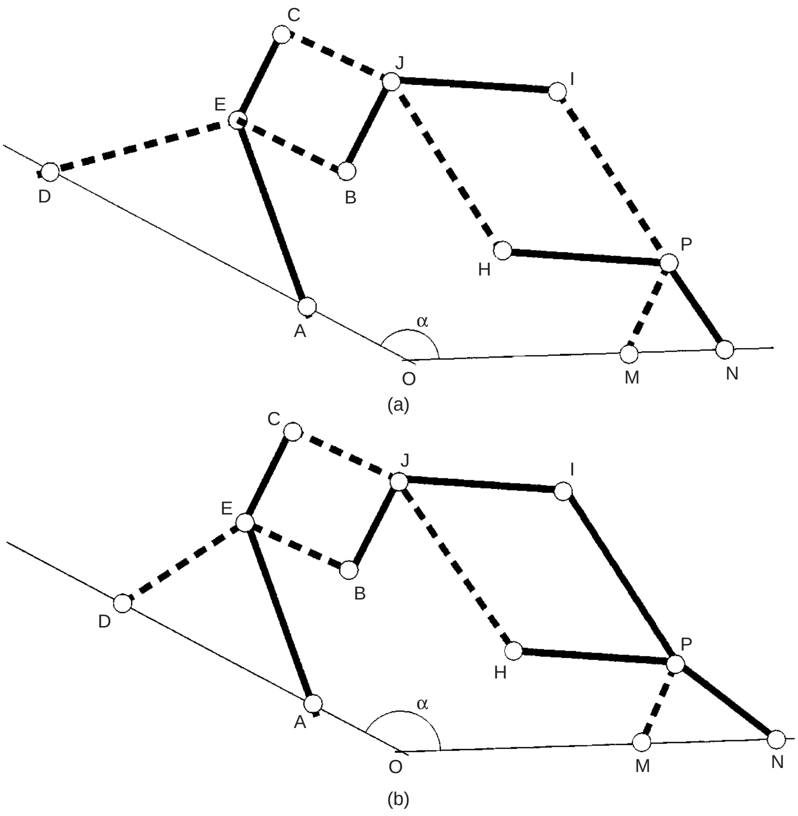


Fig. 10. Generalized angulated elements: (a) Type I and (b) Type II

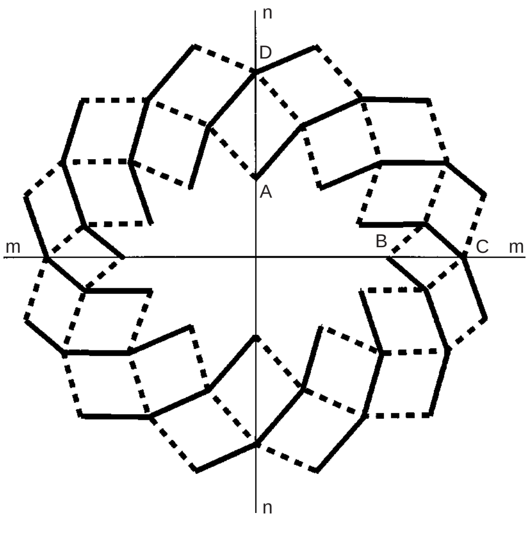


Fig. 11. Foldable structure based on a symmetric ring of elliptical shape

panels by a small amount, provided that one carries out local interference checks which allow for the actual, three-dimensional shape of the panels.

32. A general two-dimensional covering element that is connected to a multi-angulated element of known shape and with k kinks can be defined by $2(k+2)$ length parameters, L_i, plus $2(k+2)$ angles, α_i (see Fig. 18). Limits on these parameters can be set by considering the fully open and fully closed configurations of the underlying multi-angulated bar structure. Then, a kinematic simulation of the deployment/retraction of the bar structure with panels attached to it is carried out and checked visually for no interference between the panels.

33. It has been found that a general solution, valid for structures with any plan shape, is to choose covering panels with a triangular plan shape. To define the layout of these triangles one considers the fully closed, i.e. fully deployed, configuration of the bar structure and defines a series of adjoining triangles with a common vertex at the centre of the structure. The sides of the triangles lie in radial directions, and their base corners coincide with the perimeter nodes of the bar structure. Figs 19(a) and 19(c) show the resulting shapes for a circular structure and an elliptical structure, respectively. Note that the covering elements for the circular structure are twelve identical isosceles triangles. Note also that, in the elliptical structure, some of the innermost points never reach the centre, as other inner pionts, at the apogees of the ellipse, 'collide' first.

34. An alternative cover design obtained from the kinematic analysis, which satisfies the kinematic requirements and produces a more dynamic visual effect when deployed, is shown in Fig. 19(b).

Discussion

35. A new concept for retractable roof structures has been presented. The new structures consist of a foldable lattice of multi-angulated beams connected by cylindrical joints, to which covering panels or membranes are attached. These structures fold along their perimeter and there is practically no limit to the range of shapes that can be achieved.

36. A key property of these roof structures is that they have an internal degree of mobility that allows them to fold without any deformation of their members. Therefore, there is no inherent limitation on the stiffness of their members, which can be designed to be as stiff as required to avoid excessive deformation under the range of loads that will be applied to the roof during operation. For long-span construction, it is likely that three-dimensional truss elements would be used, instead of multi-angulated beams. All that would be required in order to meet the foldability conditions

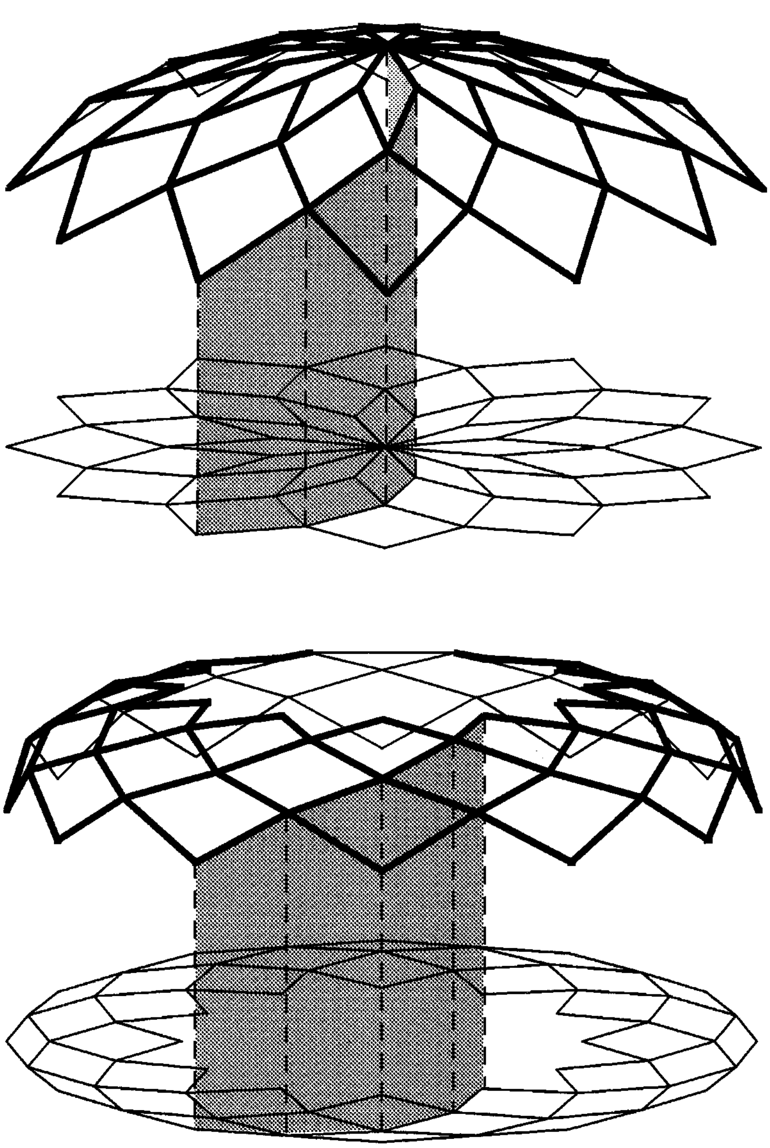


Fig. 12. Two configurations of a two-dimensional foldable structure, projected onto a curved surface

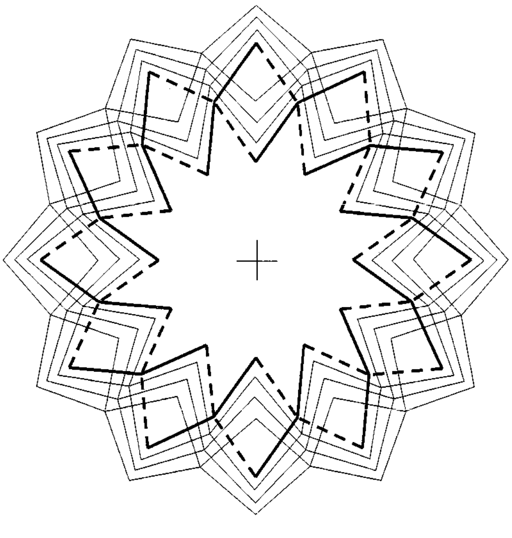


Fig. 13. Radial retraction of twelve-noded circular structure

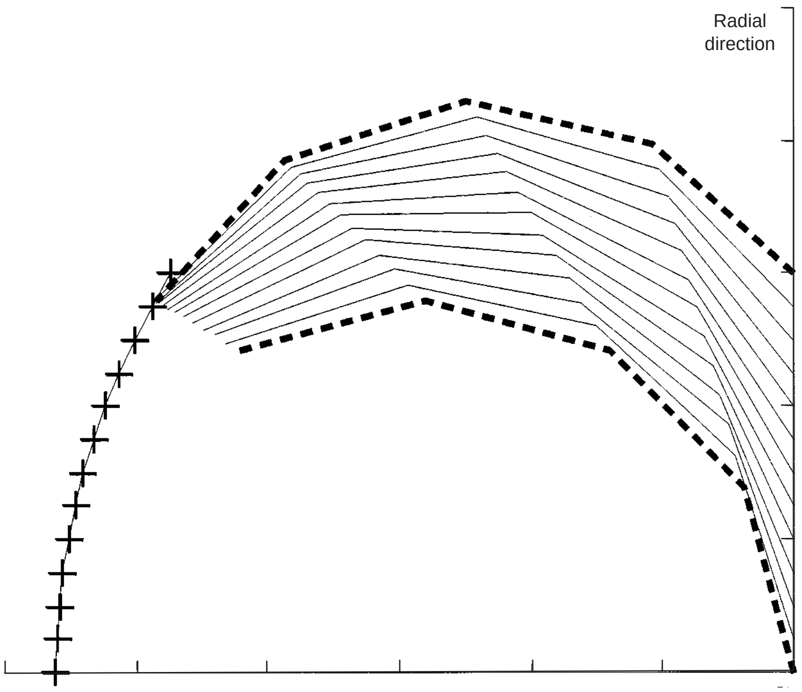


Fig. 14. Position of instantaneous centres of motion during radial retraction

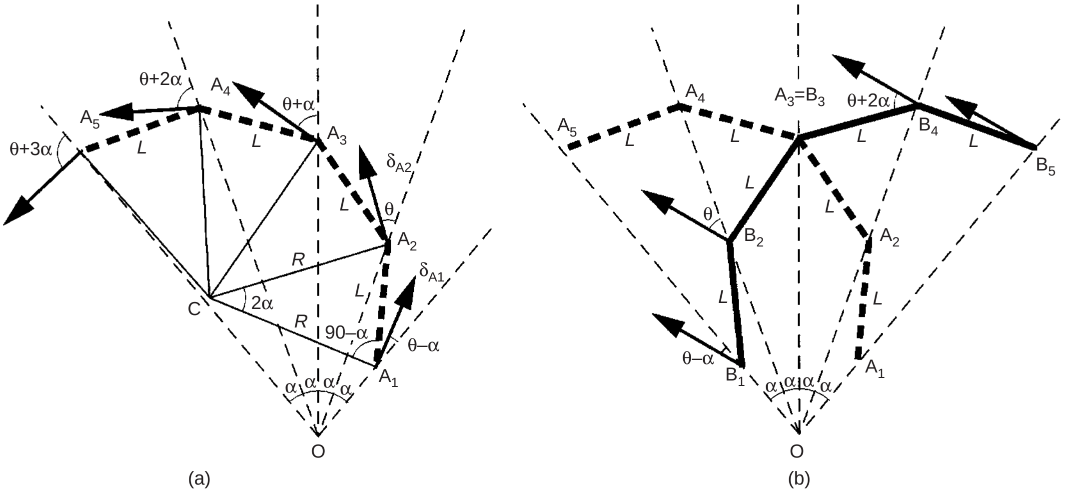


Fig. 15. Motion of (a) clockwise and (b) anticlockwise elements

discussed earlier would be that the truss elements were interconnected by means of cylindrical joints located at the same positions as where the joints of the multi-angulated elements would have been.

37. A preliminary study of the gravity-induced deflections of the type of structure described in this paper, including simulations with the finite-element package ABAQUS and experiments on a physical model similar to that shown in Fig. 5, has been carried out.[10] The study showed that this type of structure behaves in a way similar to a grillage of beams connected by momentless joints. The gravity-induced deflections vary during the expansion of the structure, as each beam is subject to twisting moments of increasing magnitude as it rotates towards the centre. Thus, the innermost points

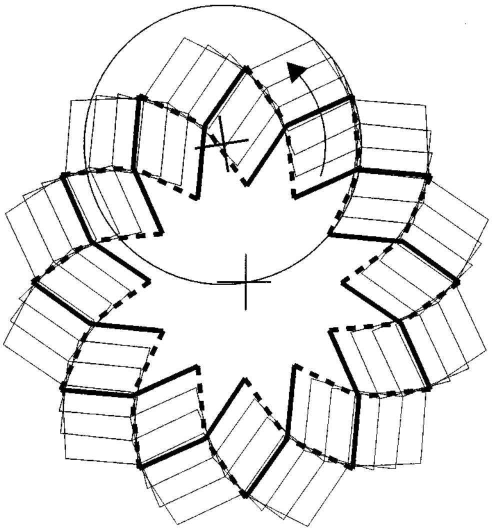


Fig. 16. Motion of circular structure with fixed points: each clockwise element rotates about the centres of the circle through its joints, and each anticlockwise element translates

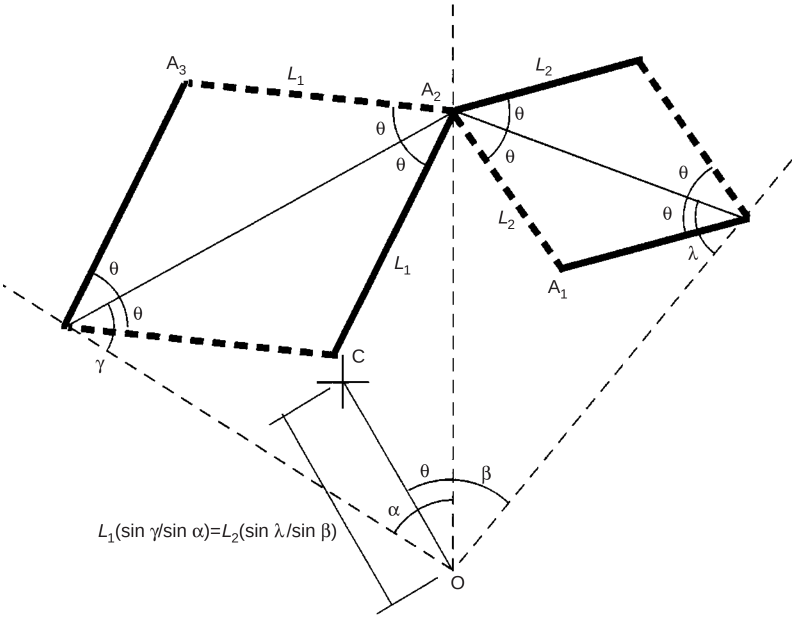


Fig. 17. Point C is a fixed point for angulated element A_1, A_2, A_3

of the structure deflect downwards. Once the structure is fully expanded, various strategies can be adopted to make it secure under operational loads. For example, some of the joints can be latched, or the deployment actuators can be driven beyond the point of first 'collision', in order to preload the whole structure and remove the backlash from the joints. Work on these issues is currently under way.

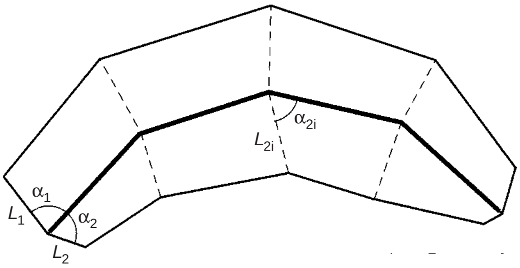
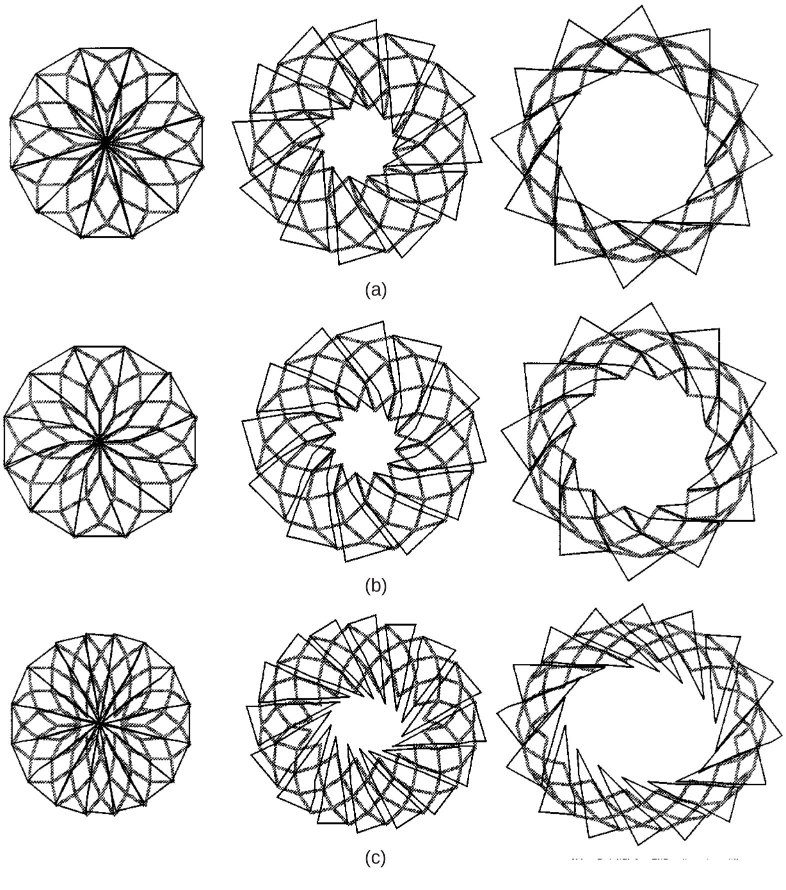


Fig. 18. Plan shape of covering element


(a)

(b)

(c)

Fig. 19. Covering elements for (a), (b) circular and (c) elliptical structures

38. A general method for the conceptual design of this new type of retractable roof structure has been presented. The three key steps are as follows.

- Step 1: design of foldable support structure. This is done first in two dimensions, by considering either a ring of two or more generalized angulated elements, as defined above, with symmetry axes, or a ring of similar rhombuses, and by adding to either of these any tessellation of parallelograms. Kinematic simulations of the motion of trial layouts are carried out, to find a layout that meets the requirements of a particular application. Once a satisfactory two-dimensional layout has been found, it can be projected onto any curved surface without changing its kinematic behaviour.
- Step 2: design of supports. Depending on which type of ring was chosen in step 1, the foldable support structure can be connected to the ground either by a set of 'tilting' supports that allow radial translation within the symmetry planes or by a set of fixed supports that allow rotation only. These fixed supports are located within the perimeter of the structure, at special locations defined by the shape of the first ring used in step 1, whereas the tilting supports can be located right at the edge of the structure, to suit different applications.
- Step 3: covering elements. The plan shapes of the covering elements that are attached to the foldable bar structure have to satisfy the kinematic requirements during deployment. A simple, general solution has been identified, consisting of triangular panels. It has been shown that other cover shapes are also possible, although each specific case needs to be analysed in detail.

39. A limitation of the approach pursued in step 3 above is that no kinematic interference is allowed between the plan projections of the covering panels, rather than the panels themselves. Further work will be required to consider the actual shape of the panels and, indeed, of the foldable support structure itself, in order to obtain designs that are optimal in terms of both shape and structural efficiency.

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41. The inventions presented in this paper are protected by two patent applications.[11] Part of the research presented in this paper was carried out by P. Kassabian during the course of his MEng project, and was awarded the first prize in the ICE Student Paper Competition held in June 1997.

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References

1. International Association for Shell and Spatial Structures, Working Group 16. *Structural Design of Retractable Roof Structures* 1996, Draft 3.
2. Karni E. Retractable spatial structures for swimming pool enclosures. *International Journal of Space Structures*, 1995, **10,** No. 4, 231–236.
3. Calatrava S., Escrig F. and Valcarcel J. P. Las estructuras de Emilio Perez Pinero. In *Arquitectura Transformable* (F. Candele, E. P. Pimero, S. Colatreve, F. Escrig and J. P. Valcarcel (eds)). Escuela Tecnica Superior de Arquitectura de Sevilla, 1993, 11–32.
4. Escrig F., Valcarcel J. P. and Sanchez J. Deployable cover on a swimming pool in Seville. *Journal of the International Association for Shell and Spatial Structures*, 1996, **37,** No. 1, 39–70.
5. Zeigler T. R. *Collapsible/Expandable Structural Frameworks*. US Pat. 4 761 929, 1988.
6. Hoberman C. *Reversibly Expandable Doubly-Curved Truss Structure*. US Pat. 4 942 700, 1990.
7. Hoberman C. *Radial Expansion/Retraction Truss Structures*. US Pat. 5 024 031, 1991.
8. Waters T. The unfolding world of C. Hoberman. *Discover*, March 1992, 70–78.
9. You Z. and Pellegrino S. Foldable bar structures. *International Journal of Solids and Structures*, 1997, **34,** No. 15, 1825–1847.
10. Teall M. J. *Deployable Roof Structure*. MEng Project Report, Department of Engineering, University of Cambridge, 1996.
11. You Z. and Pellegrino S. *Expandable/Collapsible Structures*. UK Pat. 9 601 450.1, 1996; International Patent Application PCT/GB97/00224, 1997.

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