COMPUTATIONAL MECHANICS
New Trends and Applications
E. Oñate and S. R. Idelsohn (Eds.)

'MULTI-PATH' MOTION OF DEPLOYABLE STRUCTURES

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Key words: Deployable Structure, Antenna, Equilibrium Path, Limit Point, Snap-buckling.

Abstract. *A recently developed, rigid-panel deployable antenna has shown, unexpectedly, a kind of multi-path behaviour. A physical model of the antenna deploys automatically without problems but, during retraction, it attempts to follow an alternative path and gradually locks up. Manual intervention is necessary to take the model back to its nominal retraction path, and this successfully completes the retraction process. A simple mechanical model of the antenna is analysed, and it is found that a similar type of 'multi-path' behaviour occurs for certain parameter values. Thus, by exploring how changes in the system parameters affect the multi-path behaviour, a way of improving the antenna design is obtained.*

1 INTRODUCTION

A rigid-panel deployable antenna design recently developed at Cambridge University [4] has shown, unexpectedly, different behaviour during deployment and retraction. The new antenna consists of six wings which are divided into five panels, respectively, and forming a symmetric paraboloidal reflector. Each wing is connected to the next by a bar. The connections between adjacent panels, between the first panel of each wing and a central hub, and between the bars and the wings are made by revolute joints. The antenna is deployed by driving simultaneously the six joints between the connecting bar and the last panel of each wing. This produces a complex motion, see Fig. 1, in which the wings first unwrap in a six-fold symmetric fashion, and then rotate about the hub. Details about the way in which the various panels are connected are shown in Fig. 2 and Fig. 3.

Figure 1: Antenna deployment

A physical model of this antenna has shown, unexpectedly, loss of symmetry about half-way through deployment. Detailed measurements have revealed that the angle between the hub and the first panel changes rapidly when the angle driven by the motors reaches a certain value. This corresponds to a sudden motion of the antenna where the wings begin to rotate as rigid bodies around the hub. During retraction, at exactly the same point the model starts to follow a path which is different from the deployment path and the model does not return to its initial configuration without manual intervention.

This kind of multi-path behaviour, where a deployable structure follows different paths during deployment and retraction, is not uncommon. In some cases, there are only two end configurations but more than one path between them, which may be perfectly acceptable for practical applications. In the present case, though, the alternative retraction path gradually locks up, as interference between the panels prevents further motion.

Figure 2: Top view of deployable antenna. (a) Fully deployed; (b) Fully folded, with the panels shown flat for clarity

Figure 3: Al-alloy bracket between connecting bar and panel 4

This paper presents a preliminary study of this type of behaviour, using a simple mechanical model of one wing of the antenna. Although a rather crude representation of the real system, for certain parameter values this model shows a similar type of ‘multi-path’ behaviour. Thus, by exploring how changes in the system parameters affect the multi-path behaviour, one can attempt to answer the key question: how can we improve the antenna design, so that when it is retracted it will return to its initial configuration without manual intervention?

The paper is laid out as follows. Section 2 describes some key experimental observations that were made on the physical model. Section 3 introduces the simplified mechanical model, whose motion is simulated in Section 4 by tracing its equilibrium path. Experimental and computational results are compared, and the physical phenomenon leading to the ‘multi-path’ motion of the simple model is explained. A discussion of a possible way of improving the antenna design concludes the paper.

2 MEASUREMENTS OF DEPLOYMENT BEHAVIOUR

Measurements of the position and orientation of each panel of the antenna during deployment were taken by Gardiner [3], using a six-degree-of-freedom measurement system called the Flock of Birds (FOB) [1]. This system consists of a series of small sensors, which were attached to the panels of the antenna, and of a transmitter that sends out a pulsed DC magnetic field. Measurements of the characteristics of the magnetic field are taken by each sensor, and a central processor then determines the position and orientation of all sensors. From the output of the FOB the hinge angles in the antenna were computed.

Figure 4 shows the results obtained for one wing of the antenna. Angle 1 is the angle between the hub and panel 1, angle 2 is the angle between panels 1 and 2 and so on, see Fig. 2. Angle 6 is the angle between panel 5 and the connecting bar, and is driven by a motor. All angles are defined to be zero in the fully-deployed configuration.

The initial value of angle 6 is about 210°. As angle 6 decreases to about 140°, Fig. 4 shows that there are large changes in angles 3, 4, and 5 as the model deploys smoothly and symmetrically. Between 140° and 110° only angles 1 and 2 vary. First, angle 1 shows a sudden change, by about 5°, and then angle 2 suddenly varies by about 10°, as their graphs become practically vertical. During this phase one observes sudden movements of the antenna model where each wing rotates as a rigid body, towards a more horizontal position. During retraction, the model follows the deployment path in the reverse direction until angle 6 is about 130°, but then starts following an anomalous path, and retraction has to be stopped to avoid damaging the model. At this point, one has to push the model by hand, to return to the correct path and continue the retraction process.

Figure 4: Hinge angles of one wing of the antenna, during deployment

3 A SIMPLE MECHANICAL MODEL OF ONE WING OF THE ANTENNA

Consider the simple mechanical model of Fig. 5, comprising two rigid links of length l_2 connected by a pin joint and tied by an extensional spring of stiffness k_2 as shown. A second spring of stiffness k_1 is attached to pin joint Q at one end and at its other end to a guide bearing allowing only vertical displacements. We think of spring k_1 representing the connecting bar and substitute the arch comprising the two rigid links for panels 1 to 4 of the antenna. The elasticity of the panels as well as the connections between adjacent panels and between panel 1 and the hub is globally taken into account by spring k_2.

The passive revolute joint connecting the bar of the antenna to panel 4 is a simple hinge. This hinge is connected to panel 4 by a bracket, Fig. 3, which has low stiffness in the direction of the bar axis, and thus acts as a soft extensional spring. Compared with the bracket, all joints and panels are stiff. For this reason, spring k_2 in our simple model should be much stiffer than spring k_1. We choose k_2/k_1=2300.

We assume that the initial configuration of this system is defined by

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and corresponds to the folded antenna. In this configuration both springs are undeformed because the real antenna is strain free when folded. The distance between the motorized revolute joint and the point on panel 4 where the bracket is attached corresponds in our simple model to the distance between the guide bearing and the pin joint Q. These distances vary with time during deployment when decreasing angle 6 or correspondingly increasing angle *p* of the simple model.

Figure 5: Simple mechanical model for one wing of the antenna

4 TRACING THE EQUILIBRIUM PATH

Since the deployment behaviour of the antenna is quasi-static up to the point where a sudden motion is observed, it is sufficient for a simulation to determine the static equilibrium of the simplified system for varying values of the driven angle *p*. In the literature, this way of proceeding is referred to as tracing the equilibrium path of a system [2]. The equilibrium path in the θ-*p*-space can be expressed with the aid of a parameter *s*:

, . (1)

A suitable choice for *s* would be the arc-length of the path. We can also choose the control parameter *p* itself. In the neighbourhood of a limit point, however, *p* is not suitable and we might switch to the generalized coordinate θ. Introducing the potential energy *U* of the system the equation of equilibrium can be written as

(2)

or evaluated at a point (, *p*) of the equilibrium path

, (3)

where indicates the partial derivative with respect to θ.

The equilibrium path can be traced by applying a continuation method. The path parameter is incremented by finite steps and the nonlinear algebraic equations (3) are solved for the current value of the path parameter. In every incrementation step the solution of the preceding step is chosen as an initial guess. An improved initial guess can be determined by expanding (1) in a Taylor series at the solution point of the preceding incrementation step and evaluating the series for the current value of *s*. If we terminate the series after the linear term, only the first order path derivatives , are required, where the dot denotes total differentiation with respect to *s*. The improved scheme is similar to the integration of the first order equilibrium equations with the aid of a predictor-corrector algorithm, which is an alternative method for tracing the equilibrium path. In order to obtain the first order equilibrium equations, condition (2) is differentiated with respect to *s* and evaluated at the state of equilibrium

, (4)

where the prime denotes partial differentiation with respect to the control parameter *p*. If we choose *p* itself as path parameter, the corresponding equation reads

(5)

and solved for the path derivative

. (7)

When advancing on the equilibrium path the denominator of Eq. (7) can become zero. In this case, we might be at a point of bifurcation or at a limit point. A limit point where the equilibrium path reaches a maximum or minimum in the p-θ space is characterized by the well-known snapping condition [5, 6]

, (7)

which holds when

. (8)

Hence, if we choose as path parameter at a limit point and in its neighbourhood, we can evaluate the corresponding path derivative

. (9)

4.1 Equilibrium path of the simple model

The total potential energy *U* of the system is equal to the strain energy stored in the two springs

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where

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and

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Differentiation yields the first order derivative

with

, .

The second order derivatives are

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with

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and

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Substituting the expressions for and into Eq. (7) or Eq. (9) and integrating the differential equations we can trace the equilibrium path shown in Fig. 6.

If we monotonically and quasi-statically increase p from -60^o to +60^o, the system follows the equilibrium path from point A to the limit point B where it dynamically snaps to point C. Further increasing p, we advance on the equilibrium path until we reach the final configuration at point D. Now, decreasing p the system simply follows the equilibrium path from D to E where p=–60^o and thus, does not return to its initial configuration. The system configurations corresponding to the points A-E of the equilibrium path are shown in Fig. 7. Snap-buckling at a limit point is the physical phenomenon responsible for the described 'multi-path' motion of our simple model.

Figure 6 : Equilibrium path for *k2*=2300

Figure 7: System configuration during deployment and retraction

A snap-through corresponds to a jump from one point of the equilibrium path to another. In Fig. 6 the jump is in the horizontal direction whereas in Fig. 4 the jump would be in the vertical direction, because the control parameter is plotted on the x-axis. Such a jump under a prescribed value of the control parameter requires a quasi-static change of this parameter. Clearly, a quasi-static process is an idealization, which can be realized only approximately in the experiments. Taking this into account, the measurements described in Section 2, and the plot in Fig. 4, showing a rapid change of angle 1 as well as angle 2 when the control parameter (angle 6) is about 130° might indeed indicate a snap-buckling of the real antenna.

For a relatively soft spring k_2=200, the simple system returns to its initial configuration, although snap-buckling still takes place. This is possible due to the changed shape of the equilibrium path. The system reaches a limit point B not only for increasing *p*, but also at E for decreasing *p*, Fig. 8. At point E it snaps back to point F, from where the system returns to A. If we choose k_2=10, there is no limit point at all and, hence, 'multi-path' motion does not occur either. This means that we may be able to improve the real antenna design by introducing additional elasticity, for example, close to the hub. Clearly, we would have to ensure, for example using a ratchet mechanism, that the deployed antenna remains so rigid that vibrations due to external excitation do not become too large.

Figure 8: Equilibrium path for *k2*=200

5 CONCLUSIONS

A simple mechanical model of a rigid-panel deployable antenna recently developed at Cambridge University has been presented. It has been shown that snap-buckling can cause different behaviour during deployment and retraction, as observed in a physical model of the antenna. On the basis of the computational results obtained for the simple model it would appear that an improved antenna design would require more flexible connections between the wings and the hub.

We plan to set up a more sophisticated and less idealised mechanical model of the antenna, in order to verify that snap-buckling continues to lead to 'multi-path' motion. This model will also take into account unilateral contact between panels of different wings, which is observed during deployment of the antenna and appears to be an important feature of this system, as it provides a means of synchronising the motion of different wings.

ACKNOWLEDGEMENTS

Support from the European Commission, in the form of a TMR Marie Curie Research Training Grant for M. Schulz, and from the Royal Academy of Engineering, in the form of a Foresight Award for S. Pellegrino, is gratefully acknowledged.

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