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Kinematic Bifurcations of Closed-Loop Deployable Frames

Wei W. Gan, Sergio Pellegrino*

*Department of Engineering, University of Cambridge
Trumpington Street, Cambridge, CB2 1PZ, U.K.
pellegrino@eng.cam.ac.uk

Abstract

This paper is concerned with the deployment kinematics of over-constrained mechanical linkages forming an expandable space frame that can be folded into a compact bundle. Structures of this kind are being developed to deploy large apertures in space, where a key requirement is that there should be a single deployment path that takes the structure uniquely from the folded to the deployed configuration. A scheme for analysing the sensitivity to geometric imperfections of any given design is presented.

1 Introduction

Deployable structures have many applications on spacecraft, where their capability to execute large configuration changes is exploited to design structures that are packaged into a small envelope for transportation or storage, and then expanded at the time of operation. The particular bar structures that are analysed in this paper have the special feature that packaged they form a tight bundle and deployed they form a rectangular frame suitable to support, for example, a flexible active surface or a photovoltaic film. Figure 1 shows an example of a six-rod frame with rolling hinges and light-weight carbon fibre rods.

A particular problem with structures of this kind is that their deployment is often non-unique, due to the existence of intermediate configurations where the path bifurcates. In this paper, we aim to develop a simple and yet efficient analysis scheme that enables a designer to test for the existence of bifurcations and to evaluate the sensitivity of deployment to small imperfections.

2 Kinematic analysis

Following Gan and Pellegrino [1], the scheme begins by formulating the constraint equations of the structure in natural coordinates. By assigning each element to a (4×4) transformation matrix, $\mathbf{T}_i$ for the rigid rods and $\mathbf{T}_{\theta_i}$ for the mechanical joints, and then imposing that the complete structure forms a closed loop, we obtain Uicker et al.'s [3] loop-closure equation

$$\mathbf{T}_1\mathbf{T}_{\theta_1}\mathbf{T}_2\mathbf{T}_{\theta_2}\mathbf{T}_3\mathbf{T}_{\theta_3}\ldots\mathbf{T}_n\mathbf{T}_{\theta_n} = \mathbf{I} \tag{1}$$

where $\mathbf{I}$ is the identity matrix. Considering a set of infinitesimal joint rotation changes $\delta\theta_i$'s, the transformation matrix $\mathbf{T}_{\theta_i}$ can be expanded as

$$\mathbf{T}_{\theta_i+\delta\theta_i} = \mathbf{T}_{\theta_i} + (\mathbf{T}'_{\theta_i})(\delta\theta_i) \tag{2}$$

Figure 1: Deployable rectangular frame consisting of 6-rods.

where a ' $'$ ' denotes the matrix of partial derivatives with respect to θ_i. Substituting the expansion (2) into (1) and omitting higher-order terms yields

$$\begin{array}{l}(\mathbf{T}_1 \quad \mathbf{T}'_{\theta_1} \quad \mathbf{T}_2 \quad \mathbf{T}_{\theta_2} \quad \mathbf{T}_3 \quad \mathbf{T}_{\theta_3} \quad \ldots \quad \mathbf{T}_n \quad \mathbf{T}_{\theta_n})\delta\theta_1 + \\ (\mathbf{T}_1 \quad \mathbf{T}_{\theta_1} \quad \mathbf{T}_2 \quad \mathbf{T}'_{\theta_2} \quad \mathbf{T}_3 \quad \mathbf{T}_{\theta_3} \quad \ldots \quad \mathbf{T}_n \quad \mathbf{T}_{\theta_n})\delta\theta_2 + \\ (\mathbf{T}_1 \quad \mathbf{T}_{\theta_1} \quad \mathbf{T}_2 \quad \mathbf{T}_{\theta_2} \quad \mathbf{T}_3 \quad \mathbf{T}'_{\theta_3} \quad \ldots \quad \mathbf{T}_n \quad \mathbf{T}_{\theta_n})\delta\theta_3 + \\ \ldots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \\ (\mathbf{T}_1 \quad \mathbf{T}_{\theta_1} \quad \mathbf{T}_2 \quad \mathbf{T}_{\theta_2} \quad \mathbf{T}_3 \quad \mathbf{T}_{\theta_3} \quad \ldots \quad \mathbf{T}_n \quad \mathbf{T}'_{\theta_n})\delta\theta_n = [0]\end{array}$$

which can be written in form

$$\mathbf{P}\delta\theta_1 + \mathbf{Q}\delta\theta_2 + \mathbf{R}\delta\theta_3 + \mathbf{S}\delta\theta_4 + \ldots = [0] \tag{3}$$

Note that the (3×3) top-left submatrices of $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{S}$ are always skew-symmetric, due to the fact that any arbitrary rotation can be defined by only three elementary rotations. The top-right (3×1) submatrices provide three independent constraints. Hence, we can re-write Equation (3) in the form

$$\begin{bmatrix} p_{1,2} & q_{1,2} & r_{1,2} & s_{1,n} & \cdots \\ p_{1,3} & q_{1,3} & r_{1,3} & s_{1,3} & \cdots \\ p_{2,3} & q_{2,3} & r_{2,3} & s_{2,3} & \cdots \\ p_{1,4} & q_{1,4} & r_{1,4} & s_{1,4} & \cdots \\ p_{2,4} & q_{2,4} & r_{2,4} & s_{2,4} & \cdots \\ p_{3,4} & q_{3,4} & r_{3,4} & s_{3,4} & \cdots \end{bmatrix} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\theta_3 \\ \vdots \\ \delta\theta_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$\mathbf{K}[\delta\theta_i] = \begin{bmatrix} 0 & 0 & \ldots & 0 \end{bmatrix}^T \tag{5}$$

where $\mathbf{K}$ is a tangent kinematic matrix ($6 \times n$ for an n-bar frame) and can be decomposed with the Singular Value Decomposition (Pellegrino [2]) into

$$\mathbf{K} = \mathbf{U}\mathbf{V}\mathbf{W}^T \tag{6}$$

Here, the (6×6) matrix $\mathbf{U}$ is orthogonal and the $(6 \times n)$ matrix $\mathbf{V}$ contains the r non-zero singular values along its leading diagonal. The $(n \times m)$ matrix $\mathbf{W}$, also orthogonal, comprises two submatrices, $\mathbf{W}_d$ and $\mathbf{W}_i$, of size $(n \times r)$ and $(n \times (n - r))$ respectively.

In general, Equation (5) can be solved numerically using a predictor-corrector scheme based on a standard Newton-Raphson iteration. If $r = 5$, then there is a single kinematic path for the structure to move along, and in this case the single column of the sub-matrix $\mathbf{W}_i$ provides a good initial approximation for the motion to be imposed. Hence, sets of finite rotations $d\theta_i$ and hence the full deployment path can be computed by defining

$$[d\theta_i] = [\Delta\theta_i] + [\Delta\theta_i'] \tag{7}$$

follows by, replacing the right-hand-side of Equation (5) by a vector $[e_i]$ of corrections arising from the predictor step,

$$\mathbf{K}[\Delta\theta_i] = [e_i] \tag{8}$$

Then, we employ the least squares solution of Equation (7) to determine the minimal correcting angles $\Delta\theta_i'$

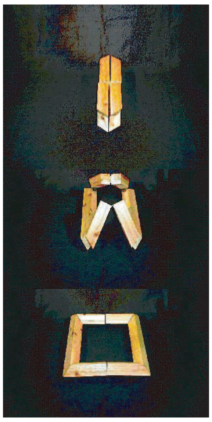
$$[\Delta\theta_i'] = -\sum_{j=1}^{r} \frac{w_j\ u_j^T}{v_{j,j}}\ [e_i] \tag{9}$$

The algorithm outlined above is repeated until either the joints or the members reach a physical boundary, at which point the structure is fully deployed.

If at some point $r \leq 4$ then a bifurcation point has been reached, and the structure is generally able to continue moving along several different paths.

3 Case study

We have analysed the deployment behaviour of several 6-rod linkages, and new insights into the kinematic behaviour of such structures have been obtained through careful study of the variation of the singular values of $\mathbf{K}$.



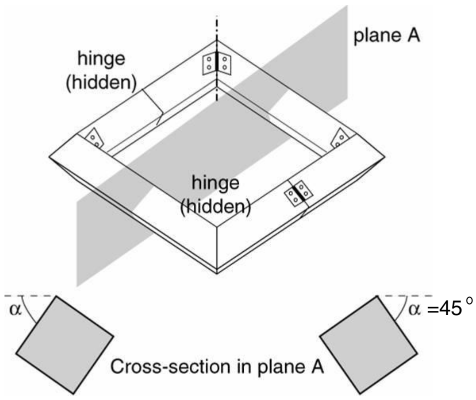


Figure 2: 6-rod closed-loop deployable frame

Consider the deployable frame shown in Figure 2, which consists of two short and four long rods, of lengths l_1 and l_2 respectively, connected by six revolute joints. The loop closure equation has the form

$$\mathbf{T}_1 \times \mathbf{T}_{\theta_1} \times \mathbf{T}_2 \times \mathbf{T}_{\theta_2} \times \mathbf{T}_3 \times \mathbf{T}_{\theta_3} \times \ldots \times \mathbf{T}_6 \times \mathbf{T}_{\theta_6} = \mathbf{I}$$

where

$$\mathbf{T}_1, \mathbf{T}_4 = \begin{bmatrix} -\frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} & l_1 \\ -\frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{6}}{3} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_2, \mathbf{T}_5 = \begin{bmatrix} -\frac{1}{3} & 0 & \frac{2\sqrt{2}}{3} & \frac{\sqrt{6}}{3}l_2 \\ 0 & -1 & 0 & 0 \\ \frac{2\sqrt{2}}{3} & 0 & \frac{1}{3} & -\frac{\sqrt{3}}{3}l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_3, \mathbf{T}_6 = \begin{bmatrix} -\frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{3}l_2 \\ 0 & 0 & 1 & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{3} & 0 & -\frac{\sqrt{3}}{3}l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_{\theta_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

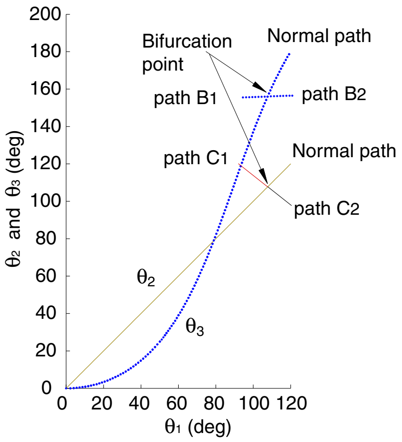


Figure 3: Variation of hinge angles during deployment

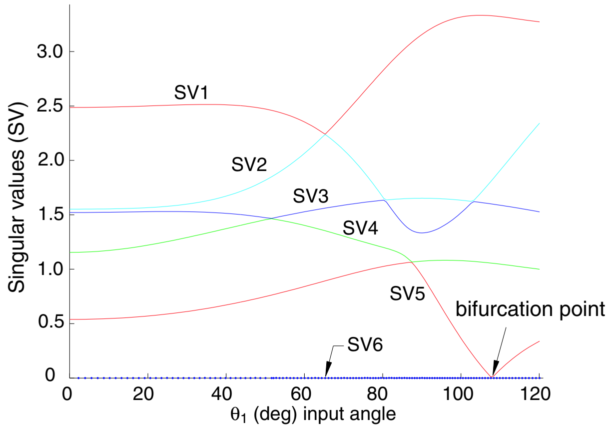


Figure 4: Singular values of kinematic matrix of frame, during deployment

Here, θ_i denotes the angle variable for joint i.

Figure 2 shows the deployment sequence of a 6-rod rectangular frame with $l_1/l_2 = 2.86$. Without assuming symmetric behaviour, Figure 3 shows a plot of the joint input-output angles. Figure 4 shows the corresponding variation of the singular values along the "normal" deployment path. Note that a bifurcation point is detected when $\theta_1 \approx 108°$; at this point there are two zero singular values, which indicates that the equation has a double infinity of solutions, and so there are two independent mechanisms for the structure, at this particular point.

Additional simulations, carried out for different values of the design variables, such as the ratio l_1/l_2, have shown that the singular values change accordingly, and in some cases the bifurcation point is lost. A sensitivity study of the structure can be conducted by assigning an objective function to each design variable, in order to optimize the structure overall performance. Progressive optimization is achieved by closely monitoring the fluctuation of singular values with respect to each individual design variable.

4 Remark

We have investigated the kinematic behaviour of several deployable frames, and considered the effects of manufacturing imperfections, in order to determine particular design configurations that are relatively insensitive to errors. Among these is the deployable frame shown in Figure 1, which has a uniquely defined deployment path.

References

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