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Design algorithm for the placement of identical segments in a large spherical mirror

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Abstract. We present a design algorithm to compute the positions of identical, hexagonal mirror segments on a spherical surface, which is shown to provide a small variation in gap width. A one-dimensional analog to the segmentation problem is developed in order to motivate the desired configuration of the tiling patterns and to emphasize the desire for minimizing segment gap widths to improve optical performance. Our azimuthal equidistant centroid tiling algorithm is applied to three telescope architectures and produces mirror segment arrangements that compare favorably with existing and alternative designs. © 2015 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.JATIS.1.2.024002](https://doi.org/10.1117/1.JATIS.1.2.024002)]

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1 Introduction

Since the invention of the telescope, astronomical observatories have implemented increasingly larger apertures. More recently, designs for both ground- and space-based astronomical telescopes have included optical surfaces that exceed the manufacturing limits for a single monolithic mirror. These designs have necessitated the development of segmented mirrors forming a single large optical surface out of smaller elements. Typically, these segmented mirrors use a hexagonal tiling pattern to achieve a high fill factor. However, the curvature of the optical surface prevents the hexagonal tiling from being perfectly uniform. As a result, telescope designs are forced to use mirror segments that are not identical in planform (the outlined shape of the segment), or to accept nonuniform gaps between segments, which increases the complexity of the mechanical design and degrades signal-to-noise performance through increased diffraction effects.

The first telescopes to use a segmented primary mirror were the Keck telescopes,¹ completed in 1993 and 1996, followed by the Hobby–Eberly Telescope (HET)² and South African Large Telescope.³ The James Webb Space Telescope (JWST) will be a space-based segmented telescope with 18 segments.⁴ A concept study for the next-generation space telescope is considering three designs ranging from an 8 m monolithic mirror to a 16.8 m segmented primary with 36 segments.⁵ Table 1 summarizes relevant parameters for these and other telescopes with segmented mirrors, including both existing and proposed designs. Most of these telescopes are designed with nonidentical segment geometries. The Thirty Meter Telescope (TMT), for example, uses six each of 82 unique mirror shapes to make up its 492 total segments.⁶ In order to accommodate maintenance and occasional resurfacing to restore reflectivity, the telescope will require the fabrication of seven of each segment type such that each shape has one spare.

The aspherical surface figure of the primary mirror in these telescopes requires the segment figures not to be identical. As a result, there is little benefit to requiring identical planforms for the segments. The design choice of using nonidentical segment planforms allows these telescopes to implement a uniform gap width between all segments. This maximizes the capture area of the primary aperture, but becomes impractical for telescope designs with too many segments to individually manufacture and calibrate.

The use of unique segments becomes infeasible as the aperture size continues to increase. For the proposed and eventually canceled Overwhelmingly Large Telescope (OWL), the inclusion of >3000 segments drove the design toward a spherical primary mirror composed of identical segments.¹³ The similarly sized In-Space Telescope Assembly Robotics (ISTAR) concept, with >5000 segments, is a design for a robotically assembled and serviceable space telescope.¹⁴ For such a design, segments could be replaced in orbit when damaged, but not resurfaced. Having a large number of uniquely shaped, spare segments in this case would be extremely inefficient. Therefore, these large telescope designs tend to use a spherical primary mirror with mass-produced identical segments in both planform and figure. However, in the future, advances in highly deformable, thin mirrors could enable such identical mirror segments to achieve an adequate optical figure even for an aspherical primary design.^{15,16}

This paper specifically addresses the problem of tiling identical hexagonal segments onto a spherical primary mirror. We present a design algorithm to compute the positions of these segments on the spherical surface, which is shown to provide a small variation in gap width. This design algorithm, which we will refer to in this paper as the azimuthal equidistant centroid tiling (AECT) algorithm based on the geometry developed in Sec. 4, is not computationally intensive and can be applied to a range of mirror geometries. We believe that it can be a valuable

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Table 1 Hexagonally segmented telescope designs and relevant parameters.

Name	Primary aperture dimension	Number of segments	Date constructed
Ground-based telescopes:			
Keck 1 ¹	10 m	36	1985 to 1993
Keck 2 ¹	10 m	36	1991 to 1996
Hobby–Eberly Telescope (HET) ²	11 × 10 m	91	1994–1997
South African Large Telescope (SALT) ³	11 × 10 m	91	2000 to 2005
Large Sky Area Multiobject Fiber Spectroscopic Telescope (LAMOST) ⁷	6.67 × 6.09 m	37	2001 to 2008
Gran Telescopio Canarias (GTC) ⁸	10.4 m	36	2002 to 2008
Euro50 ⁹	50 m	618	Merged with European Extremely Large Telescope (E-ELT)
California Extremely Large Telescope (CELT) ¹⁰	30 m	1080	Merged with Thirty Meter Telescope (TMT)
TMT ^{6,11}	30 m	492	2014 to 2022 (started)
E-ELT ¹²	39.3 m	798	2014 to 2022 (started)
Overwhelmingly Large Telescope (OWL) ¹³	100 m	3048	Canceled
Space-based telescopes:			
James Webb Space Telescope (JWST) ⁴	6.5 m	18	Expected 2018 launch
Advanced Technology Large-Aperture Space Telescope (ATLAST) ⁵	8 to 16.8 m	1 or 36	Preliminary
In-Space Telescope Assembly Robotics (ISTAR) ¹⁴	100 m	>5000	Preliminary

tool for rapid computation of optical configurations for early design studies, without requiring the time or computational power for an optimization-based technique. By focusing specifically on the problem of finding the ideal positions for mirror segments, the AECT algorithm would complement an overall system study that accounts for additional effects, including but not limited to manufacturing and positioning error, secondary optical elements, and active control systems.

In the following section, we provide an overview of the background and prior work on the design and analysis of the three-dimensional geometries of highly segmented telescope mirrors. Section 3 introduces a one-dimensional analog to the tiling problem in order to provide insight into the effect of different tiling options on the optical point spread function (PSF) using Fourier optics. Section 4 presents the details of our new AECT algorithm for tiling segments onto a spherical surface. Section 5 demonstrates that the proposed tiling strategy produces results that are competitive with alternative methods. We show this by applying the AECT algorithm to designs based on the HET, OWL, and ISTAR concepts. Finally, Sec. 6 concludes and summarizes.

2 Background

There has been much prior work on the design and analysis of large segmented telescopes. In this section, we briefly highlight

some specific work that is relevant or complementary to the problem of tiling identical segments on a sphere.

2.1 Telescope Design

The focus of this paper is on the effect of ideal positioning of segments on a primary mirror. However, the overall performance of a telescope is driven by many other factors. Many prior analyses have considered complete optical systems rather than just the primary mirror. For example, Jolissaint and Lavigne consider a 30 m design with adaptive optics,¹⁷ and Chanan et al. consider active control of the mirror segments¹⁸ in a full telescope system. The aberration introduced by a spherical primary mirror can be corrected through additional optical elements, such as the four-mirror, double Gregorian design used by HET.¹⁹

Additionally, operational techniques can be developed to mitigate performance limitations. Large ground-based telescopes are typically limited in angular resolution not by diffraction but by atmospheric distortion. This can be compensated by adaptive optics or by postprocessing. Speckle imaging techniques for postprocessing have been used to compensate for this effect and to achieve higher angular resolution. For example, the Keck telescope produced a 0.05 arc sec diffraction-limited near-infrared image.²⁰

The design of nominal mirror geometry in telescopes using identical segments is not well documented in the literature.

Published reports provide average segment gap width and ranges of gap width variation for the HET and OWL designs, but no details on the specific design algorithms used.^{2,13} The design of the TMT primary mirror segmentation, which uses variable mirror segments, was based on an optimization algorithm that started with a planar hexagonal grid and introduced a design variable that scales the tessellation as a function of distance from the optical axis to account for the mirror curvature.⁶ This grid was then cylindrically projected onto the mirror surface to define the individual segment planforms.

2.2 Optical Analysis

Techniques for predicting and characterizing the optical performance of a telescope can be partitioned into analytical and numerical methods. Fourier methods are commonly used in optical diffraction theory to compute the PSF of an aperture.^{21,22} Yaitskova et al. applied Fourier techniques to the study of large, highly segmented telescope mirrors.²³ The effects of errors from segment position and misfigure are described, while the variation in gap width is reported as having little effect on the PSF.

One approximation for the proportion of energy E_p outside the core PSF is

$$E_p = \frac{A_F - A_S}{A_F}, \quad (1)$$

where A_F is the surface area of the filled aperture and A_S is the total surface area of the segments not including the gaps.²⁴ This motivates the desire for a tightly packed arrangement of mirror segments to achieve the best image quality.

Numerical methods include ray-tracing techniques to model the optical behavior of a telescope and can be combined with numerical models of the telescope environment, structure, and other error sources to predict the overall system performance.²⁵

3 One-Dimensional Segmentation Analog

As discussed in the previous section, the use of Fourier optics to analytically determine the optical performance of a telescope has been described extensively in the literature.^{21–23} It is well known that segmentation of an optical aperture leads to a speckle pattern in the PSF. Variation in the width of gaps between mirror segments introduces intensity variations in the PSF, which increase the background intensity while diminishing the peak intensity of the speckles. This effect does not strongly influence the encircled energy of the PSF but does have a substantial effect on the ratio between the intensities of the central peak of the PSF and the nearest speckle peak. Reducing the average gap width between segments is an effective method for improving the contrast performance of a segmented telescope design by reducing the intensity of speckles near the central peak.

For readers who are less familiar with the conclusions presented in the previous paragraph, we provide, in this section, the following mathematical development demonstrating the effect of mirror segmentation on a PSF. In particular, we wish to characterize the effect of gap width variations between segments on the PSF of the aperture. This provides a direct connection between the geometry of the segments and a measure of the optical performance of the primary mirror. Considering Fraunhofer diffraction, the PSF $h(u, v)$ is computed from the two-dimensional Fourier transform of the aperture function $a(x, y)$ as

$$h(u, v) = |\mathcal{F}[a(x, y)]|^2. \quad (2)$$

In a segmented mirror, the full aperture function $a(x, y)$ can be expressed as the convolution of the segment aperture $s(x, y)$ with a grid factor $g(x, y)$, which is composed of an array of delta functions. The PSF can, therefore, be decomposed as the product of the individual Fourier transforms of the segment aperture and the grid factor:

$$h(u, v) = |\mathcal{F}[s(x, y)]\mathcal{F}[g(x, y)]|^2. \quad (3)$$

However, we find it worthwhile to consider first a simplified one-dimensional analog of the segmentation problem. This analog provides a more intuitive and easily visualized method to highlight the relevant features and constraints inherent in the two-dimensional transforms and three-dimensional geometries associated with real telescope systems. In the following sections, we adapt the two-dimensional notation introduced above for the one-dimensional cases of a fully filled aperture, a uniformly segmented aperture with gaps, and a segmented aperture with nonuniform spacing. For the last case, we quantitatively show that the effect of gap width variation is small relative to the effect of the average gap width.

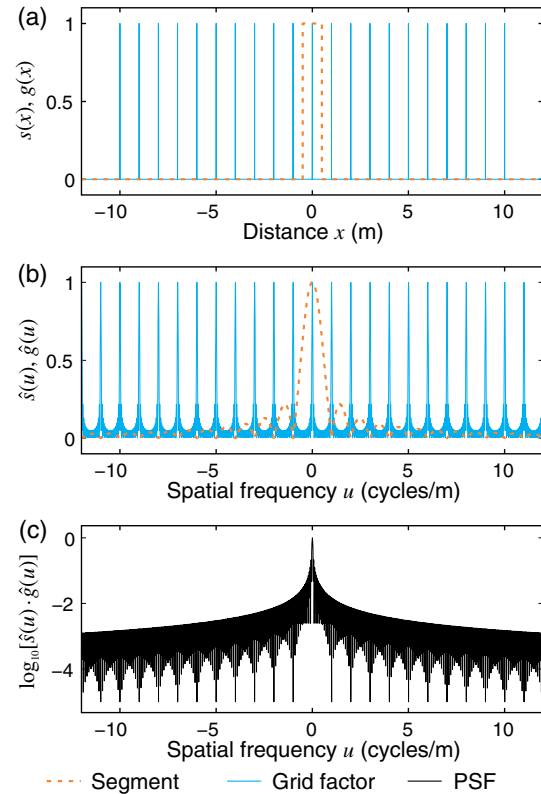


Fig. 1 Point spread function (PSF) of a fully filled linear aperture, composed of 21 segments each 1 m wide at a spacing of exactly 1 m. (a) Segment aperture (in dotted orange) and grid factor (in solid blue), plotted as a function of distance; (b) modulus squared Fourier transforms of the segment aperture and grid factor, plotted as a function of spatial frequency and normalized to a maximum of 1; (c) PSF computed as the squared product of the individual Fourier transforms, plotted logarithmically as a function of spatial frequency and normalized to a maximum of 1.

3.1 Fully Filled Aperture

For a one-dimensional, fully filled aperture, a segment of width d can be represented as

$$s(x) = \text{rect}(x/d), \quad (4)$$

and the corresponding grid function for an aperture size of D can be represented as

$$g(x) = \text{rect}(x/D)\text{III}(x/d), \quad (5)$$

where $\text{III}(x)$ is the Dirac comb function. The segment aperture and grid factor are plotted in Fig. 1(a) in orange and blue, respectively, for a numerical example with 21 segments each 1 m wide.

The Fourier transforms of these functions are

$$\hat{s}(u) = \mathcal{F}[s(x)](u) = \text{sinc}(\pi du), \quad (6)$$

$$\hat{g}(u) = \mathcal{F}[g(x)](u) = \text{sinc}(\pi Du)\text{III}(du), \quad (7)$$

with the sharp delta functions in the grid factor transforming into sinc functions because the total aperture does not extend to infinity. This is shown in Fig. 1(b), where the modulus squared of these two functions are plotted as functions of the spatial frequency f , aperture diameter D , and operating wavelength λ , 1 m^{-1} in the frequency coordinate u is equivalent to λ/D in the angle of observation or $\lambda f/D$ in spatial distance on the image plane.

When the two functions are multiplied together, the peaks of $\hat{g}(u)$ align exactly with the zeros in $\hat{s}(u)$. Accordingly, as shown in Fig. 1(c), the envelope of the PSF has a single peak and smoothly decays with increasing spatial frequency, though lobes in the PSF do occur, corresponding to the $\text{sinc}(\pi Du)$ component of the grid factor, exactly as if the PSF were computed for a single aperture $A(x) = \text{rect}(x/D)$.

3.2 Uniformly Segmented with Gaps

When gaps are introduced in the segmentation, the spatial frequency of the grid factor is scaled lower, and the peaks in the Fourier transform appear more closely spaced. Figure 2(a) shows the segment aperture and grid factor for a numerical example with the same 1 m segments as in the previous section, but spaced at a uniform interval of 1.1 m. As seen in Fig. 2(b), the grid factor peaks no longer align with the zeros in the transformed segment function. These result in speckles surrounding the central peak, which can be seen in the PSF in Fig. 2(c).

The effect of gap width on the optical performance of a telescope can be characterized by the encircled energy at a given radius and by the ratio of the central peak to the next largest peak in the PSF. The optical application determines the particular metric that is relevant for characterizing a given system. For example, exoplanet characterization applications using ultrahigh contrast imaging can require detection of light levels on the order of 10^{-6} to 10^{-10} times the central peak, using techniques such as coronagraphy.^{26,27} In Fig. 3, the encircled energy is plotted as a function of spatial frequency for a range of gap widths, and the ratio of the two highest peaks is plotted as a function of gap width, using the same one-dimensional example of a 21-segment array with 1 m segments.

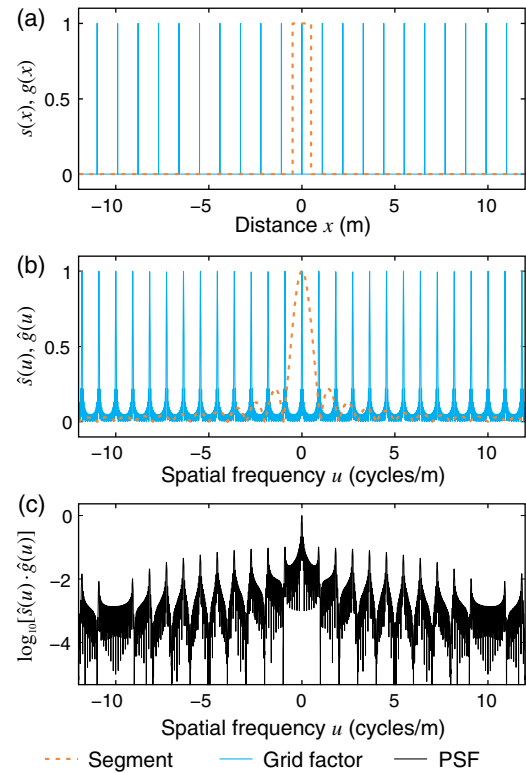


Fig. 2 PSF of a uniformly segmented aperture. The segments are 1 m wide but spaced at intervals of 1.1 m. The individual plots show (a) the segment aperture and grid factor, (b) the modulus squared Fourier transforms of the segment aperture and grid factor, and (c) the PSF computed as the squared product of the individual Fourier transforms. Here, the PSF shows a series of speckles, which are not present in the fully filled case.

For small gaps, the spatial frequency associated with 90% encircled energy is over an order of magnitude smaller than for larger gaps; this effect holds in general for encircled energy percentages of 50 through 99%, but the effect diminishes when considering encircled energies $> \sim 99.5\%$. The trend toward a higher percentage of encircled energy at a smaller spatial frequency implies that the cases with smaller gaps lead to better imaging resolution. The ratio of the peaks also shows a strong dependence on gap width, with the central peak being ~ 2000 times larger for a gap width of 10 mm, falling to a ratio of ~ 2.5 with a gap width of 1 m. The ability to distinguish a central spot from its speckle pattern is, therefore, improved at a smaller gap width. Both of these results support the conclusion that minimizing gap width is essential for limiting the effect of diffraction on optical performance.

3.3 Nonuniform Segmentation

If the grid factor is not uniformly spaced, the Fourier transform includes a slowly varying envelope attenuating the height of some peaks. This is shown in Fig. 4 for variable segment spacing uniformly sampled from a range of 1.075 to 1.125 m (gap widths of 75 to 125 mm), with the equivalent uniform segmentation shown lightly shaded for comparison.

However, from Parseval's theorem, the energy in the Fourier transform must remain the same as for the uniformly spaced configuration. The additional energy transferred from the

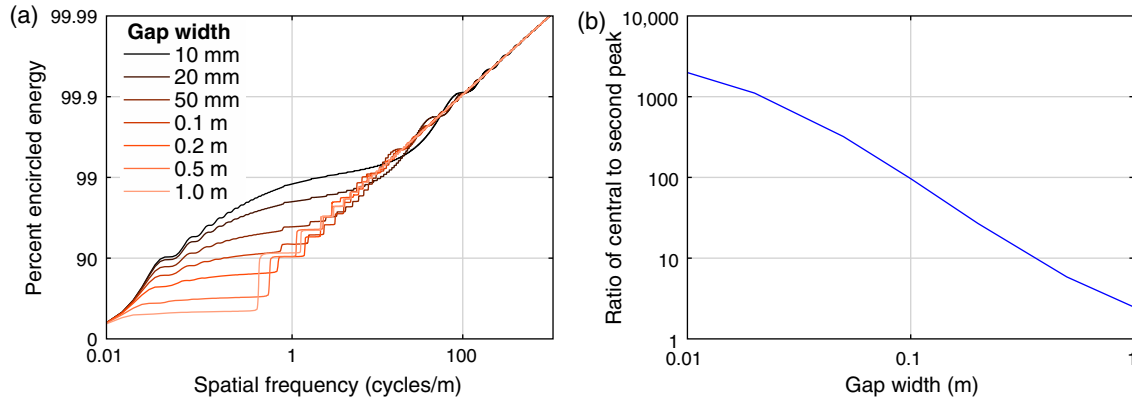


Fig. 3 Dependence of encircled energy and PSF peak ratio on gap width. (a) Percent encircled energy is plotted as a function of spatial frequency for gap widths ranging from 10 mm to 1 m. For a fixed value of encircled energy, smaller gaps tend toward lower spatial frequency, which is indicative of better imaging resolution. (b) The intensity ratio of the central peak to the next highest peak is plotted as a function of gap width, showing that larger gaps are associated with a speckle pattern that is less distinguishable from the central peak.

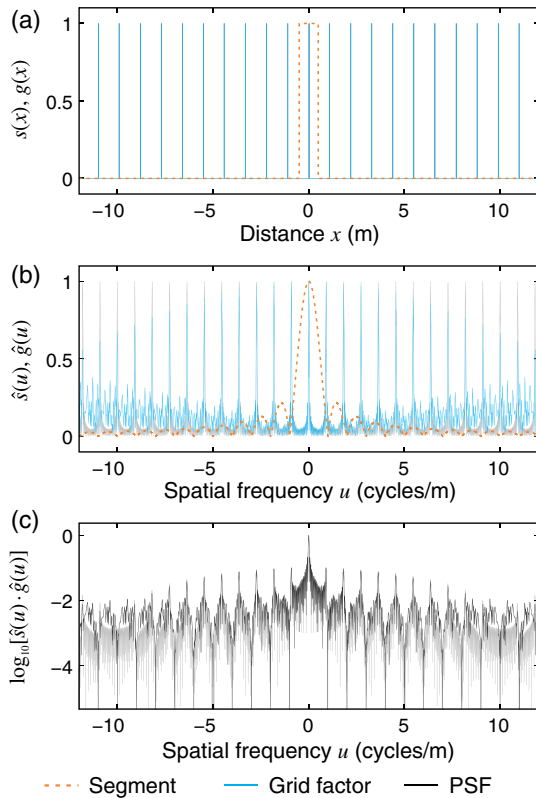


Fig. 4 PSF of an aperture with nonuniform segmentation, with the equivalent uniformly spaced segmentation shown lightly shaded for comparison. The segments are 1 m wide with random spacing sampled from a uniform distribution within a range of 1.075 to 1.125 m. The individual plots show (a) the segment aperture and grid factor, (b) the modulus squared Fourier transforms of the segment aperture and grid factor, and (c) the PSF computed as the squared product of the individual Fourier transforms. The speckles associated with the random segmentation fall off in amplitude with higher spatial frequency, but does not change much at low spatial frequency, while the energy between speckles seems to increase slightly at higher spatial frequency.

attenuated peaks is evident in the growing energy of the intermediate spatial frequencies between peaks.

In order to characterize the effect of random gap variation on the encircled energy and ratio of peak intensities, PSFs were considered using different ranges of gap width variation, from 1 to 100 mm for a mean gap width of 100 mm. For each range value, 200 sampled PSFs were computed. The standard deviation of the percent encircled energy over each population of 100 samples was computed as a function of spatial frequency, and the maximum standard deviation for each range is plotted in Fig. 5(a). The maximum standard deviation increases over the range of gap width variation, but even with the greatest variation considered, the encircled energy profiles do not deviate more than 0.2% from their mean values. The intensity ratio between the first and second peaks was also computed, and the maximum, minimum, and quartile values for each distribution are plotted in Fig. 5(b).

The maximum and minimum ratios remain relatively constant over the range of gap width variation, but the quartiles tend toward lower ratios for low variation, reach a maximum ratio for greater variation, and then decrease again. This is a relatively surprising result and indicates that a moderate level of randomness in the gap width may yield a greater likelihood of providing good contrast between the central and second peaks.

The conclusion that can be drawn from this one-dimensional exercise is that while the encircled energy profile is not influenced strongly by variations in gap width, the contrast performance can actually be enhanced by nonuniform spacing. However, the average gap width is a much stronger driver of both performance characteristics.

4 Segment Tiling Algorithm

The results presented in the previous section using the one-dimensional analog motivate a tiling strategy that minimizes the average gap width when positioning the mirror segments. However, most optical designs must also accommodate practical constraints accounting for factors including fabrication tolerances, mechanical clearances for motion of the segments under active control, or accessibility for construction and servicing. Typically, these impose a hard constraint on minimum gap width, but do not strongly penalize larger gap widths. With

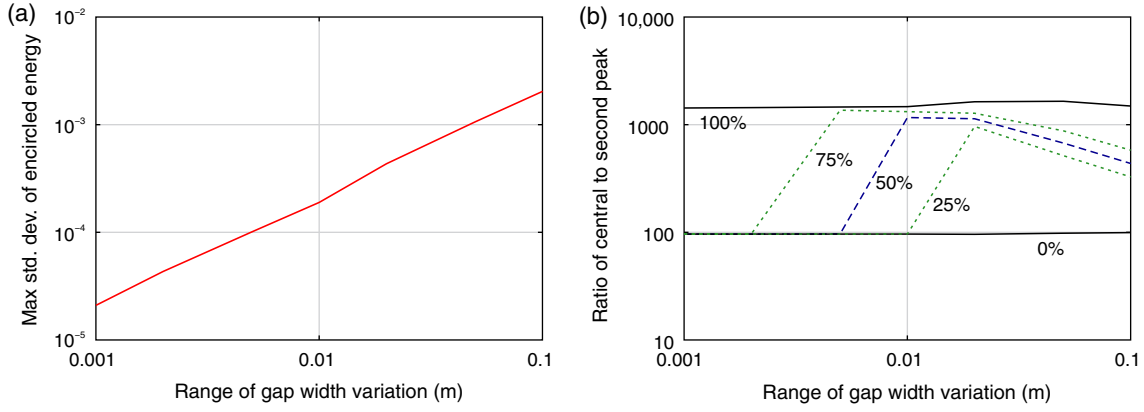


Fig. 5 Variation of encircled energy and peak ratio as functions of the gap width range over 200 sampled configurations. (a) For each value of total range, 200 random aperture functions are generated and the standard deviation of encircled energy, normalized by its mean value, computed as a function of spatial frequency. The maximum standard deviation over spatial frequency is plotted for each value of total range. The low value of maximum standard deviation over the full range of gap width variation shows that the encircled energy profiles do not change much with slightly perturbed segment positions. (b) The ratio of central to second peak amplitude was computed for each PSF, and the maximum, minimum, and quartiles of the distribution are plotted for each value of total range. While the maximum and minimum values do not vary much with gap width variation, the distributions are more heavily weighted toward a low ratio for small variation and tend toward higher ratios for greater variation.

these considerations in mind, we developed a tiling algorithm that demonstrates good performance in minimizing the range of gap width variation, which allows the mean gap width to be as close as possible to the minimum gap constraints. In this section, we start by specifying the geometry used in the tiling algorithm and then discuss the computation of mirror segment positions. Several performance metrics used to evaluate the algorithm are discussed, as well as alternative tiling methods used to determine segment position.

4.1 Mirror Geometry

The primary design parameters we used here for a segmented spherical mirror include the radius of curvature R , the side length of a hexagonal segment r , the nominal gap spacing w , and the number of rings of hexagons N around the central segment.

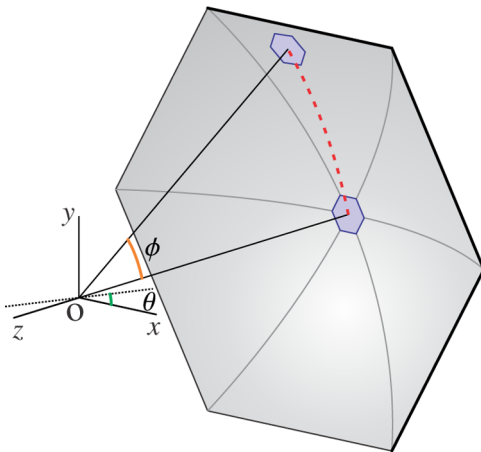


Fig. 6 Geometry used to describe mirror segment locations. A segment initially positioned at the center of the spherical optical surface is rotated about an axis passing through the origin O to reach its final position. The angle of the axis is defined by the spin angle θ and the rotation of the segment is defined by the pitch angle ϕ .

The geometry for describing mirror segment positions is shown in Fig. 6.

First, we define a fixed reference frame with origin O at the center of curvature of the spherical surface. Without loss of generality, we choose the z axis to be pointing away from the mirror surface and the x axis to be aligned with a vertex of the central segment. In this reference frame, the central segment has vertices located at points described in the fixed reference frame by the three-dimensional coordinates:

$$\mathbf{r}_v = \left[r \cos\left(\frac{\pi v}{6}\right) r \sin\left(\frac{\pi v}{6}\right) R \cos\left(\frac{r}{R}\right) \right], \quad (8)$$

$v = 1, 2, \dots, 6.$

All other segments can be fully defined as a sequence of three Euler angle rotations of this central segment about O , ensuring that the vertices remain on the spherical surface. While many rotation sequences are possible, the tiling geometry is most amenable to a 3-1-3 rotation sequence of a reference frame fixed to the segment geometry, which is defined as a spin angle θ about the fixed z axis, a pitch angle ϕ about the intermediate rotated x axis, and a clock angle ψ about the segment-fixed z axis. The rotated segment vertices are then located in the fixed reference frame at

$$\mathbf{r}'_v = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \times \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}_v, \quad (9)$$

where $\mathbf{r}_v$ as defined above expresses the coordinates of the segment vertices in a reference frame that is rotated with the mirror segment.

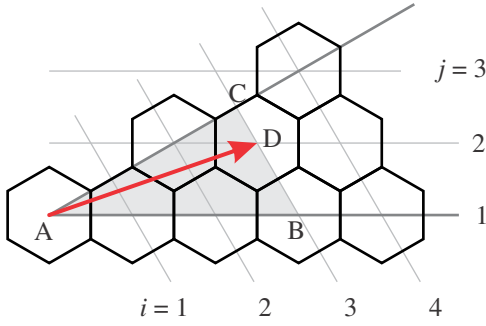


Fig. 7 One sector of the segmented mirror, indexed by ring number and position within each ring. The shaded triangle ABC is used to determine the location of a hexagon's centroid at D.

In order to maintain alignment of each segment with its neighbors, we choose the clock angle to be $\psi = -\theta$ and define each segment with only two angular parameters. This can mathematically be interpreted as a single rigid-body rotation of the pitch angle about the axis defined by the spin angle, as illustrated in Fig. 6.

The hexagonal tiling pattern inherently has 12-fold mirror symmetry. In the following section, we provide a solution for the rotation angles within a 30 deg sector, which can be replicated to fill the entire aperture.

4.2 Tiling Angles

We employ an approach to solve for spin and pitch angles based on a planar hexagonal tessellation. The distances between hexagon centroids in the radial direction on the plane are preserved in the azimuthal direction on the sphere, which is why we refer to this technique as the AECT algorithm. In contrast, the TMT algorithm applies a radial scaling to the planar tessellation and then projects cylindrically onto the sphere.⁶

The segments that are located with spin angles ranging from 0 to 30 deg are indexed by their ring number and position within the ring as shown in Fig. 7.

The first hexagon is the central hexagon, with angular coordinates $\theta = 0$ and $\phi = 0$. For each ring i from 1 to n , considering only a 30 deg sector, there are $j_{\max}$ new hexagons to be positioned, where $j_{\max} = \text{floor}(i/2 + 1)$. In order to accommodate the desired gap w , we define a tiling hexagonal side length

$$r_{\text{hex}} = r + \frac{w}{2 \cos(\pi/6)}, \quad (10)$$

such that a tightly packed planar tessellation of tiling hexagons will accommodate a gap of w between segment hexagons. We use this planar tessellation to compute the spin angle and the linear distance between the central and rotated segments, and then solve for the pitch rotation that has an arc length equal to this linear distance. For example, the segment labeled D in Fig. 7 has indices $i = 3$, $j = 2$. The angle CAD is computed based on known side lengths AC and CD with $\angle ACD = 90^\circ$, and the spin angle is equivalent to $\angle BAD = \pi/6 - \angle CAD$. With this method, the spin angles for each segment as a function of i and j are

$$\theta(i, j) = \frac{\pi}{6} - \tan^{-1} \left(\frac{i - 2j + 2}{i\sqrt{3}} \right), \quad (11)$$

for i from 1 to n and j from 1 to $j_{\max}$. The distance AD can be computed using the law of cosines from triangle ABD, with $\angle ABD = 60^\circ$ and known side lengths AB and BD. The corresponding pitch angles are

$$\phi(i, j) = \frac{\sqrt{3}r_{\text{hex}}}{R} \sqrt{i^2 + (j-1)^2 - (j-1)i}. \quad (12)$$

Each hexagon whose spin angle is not 0 or $\pi/6$ is then mirrored across the xz -plane to form a sector that spans 60 deg. Finally, all of the hexagons except the central one are patterned rotationally at intervals of 60 deg to fill the full field.

4.3 Performance Metrics

While the optical performance will ultimately depend on the full system configuration, we independently evaluate the performance of the AECT algorithm so that alternative methods can be compared. As with the one-dimensional example in Sec. 3, the PSF for the three-dimensional tiling can be computed. Additionally, a purely geometric evaluation can be made by considering the distribution of gap widths. In the following sections, we discuss the computation of both.

4.3.1 Gap width distribution

Mirror segments positioned on the spherical surface will be rotated in three dimensions and two adjacent edges will not necessarily be parallel or even coplanar. To avoid the ambiguity caused by this geometry, we approximate the continuously varying gap width by comparing distances between vertex pairs rather than distances between edges. Because these distances are more closely associated with the mechanical constraints of the telescope rather than the optical performance, we use the three-dimensional distances computed from the segments on the sphere rather than a projection onto a flat plane in front of the aperture.

For a tiling result where the segments are relatively well positioned and with a large enough gap, these vertex-to-vertex distances are adequate for characterizing the distribution of gap widths. However, if segments are positioned more closely, the vertex-to-vertex distances can be skewed and will result in an overestimate of the gap width. Additionally, this method will not discriminate between vertices that are spaced apart appropriately and those that are actually overlapping but spaced apart the same distance in the opposite direction. To address this

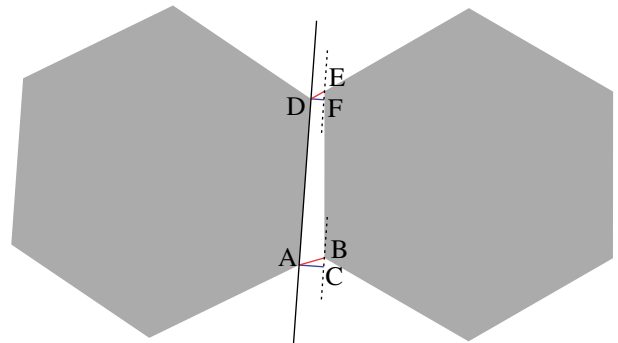


Fig. 8 Gap width represented by vertex-to-vertex distances between two hexagons. The actual vertex-to-vertex distance AB and DE are projected in the direction of the edge normal to obtain projected distances AC and DF.

limitation, each vertex-to-vertex distance can be projected onto a unit vector in the direction of the segment edge normal, within a plane tangent to the spherical surface, as depicted in Fig. 8. This provides a signed distance metric that is a more accurate representation of the gap widths.

In optical designs where the positions of the optical elements are globally fixed, the center of the primary element is often obscured. For such designs, the AECT algorithm can be applied without modification to compute only the positions of the mirror segments that are present. The resulting distribution of gap widths will have the same range but a lower average, since the segments in the central region have gap widths that are close to the maximum value on all sides while the segments near the edge are more closely spaced to adjacent segments in the same ring.

Once the distribution of gap widths is computed, it can be used to modify the parameters of the tiling angles in order to achieve a specific minimum, mean, or other metric. Specifically, the computed value for r_{hex} can be iterated to shift the gap width distribution without distorting the overall pattern.

4.3.2 Point spread function

The distribution of gap widths provides a verification that geometric design constraints are met, but does not directly provide a measure of optical performance. In order to provide a more direct optical measure of performance, the PSF of the aperture function is computed. However, in this paper, we consider only the geometry of the primary mirror, and as a result, there is no well-defined image plane on which to project the aperture function. In order to maintain consistency in comparisons between designs, we evaluate the PSFs based on an aperture function, which is the vertical projection of the mirror segments onto the xy -plane, in front of the primary aperture. In Sec. 5, we present PSFs for several aperture functions, computed using a gray-pixel technique to mitigate the effect of square pixels on the hexagonal structure.

4.4 Alternative Tiling Methods

Several alternative tiling methods were considered and are discussed in this section. One intuitive method is to use a 1-2-3 rotation sequence with constant angular increments. This places mirror segments along lines of constant latitude and longitude, but suffers from loss of 12-fold symmetry. With this method, the

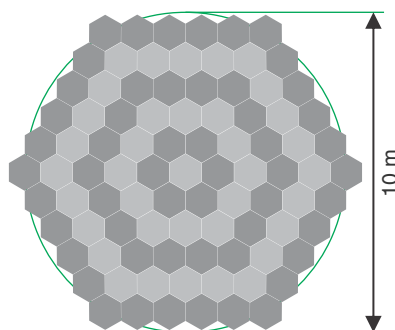


Fig. 9 Segment tiling configuration based on Hobby–Eberly Telescope (HET) parameters. This mirror configuration contains 91 segments, arranged with five rings (shown in alternating shades of gray) around the central segment.

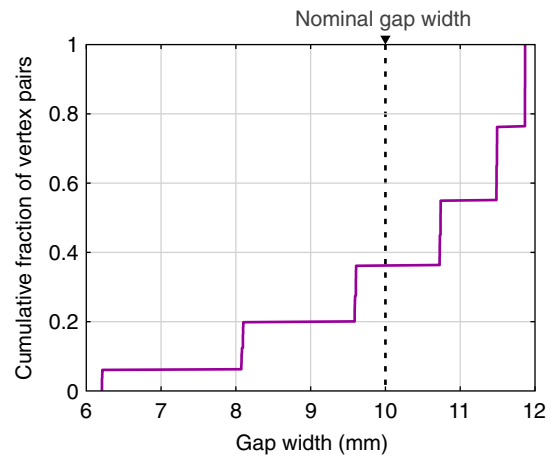


Fig. 10 Cumulative distribution of gap widths for the HET design. The stepped shape of the distribution shows that the gap widths are clustered around six values (the near-vertical segments) between 6.2 and 11.9 mm, with the median (cumulative fraction of 0.5) at 10.7 mm.

rows at extreme latitudes are compressed more closely together such that the segments on the equator must be spaced further apart to compensate. While this effect is also present in the AECT method described in Sec. 4.2, it is mitigated by the fact that 12-fold symmetry is enforced, so that this compression occurs more uniformly around the entire edge of the segmented mirror.

With the AECT method above, the triangle ABC in Fig. 7 is well defined on a plane with three known angles and three known side lengths. Another alternative is to work directly with spherical triangles defined on the optical surface. However, the planar relations between the side lengths and angles break down due to the angular defect of the curved surface. By imposing that triangle ABC is a spherical triangle (i.e., composed of great circle arcs), we can choose three known quantities and solve for the remaining three using spherical trigonometry. In order to maintain consistency with the 12-fold symmetry of the full mirror, we require that $\angle BAC$ remains 30 deg. We consider three alternative methods that can be applied, by using prescribed lengths AC and AB, one length AB and angle $\angle ABC$, or one length AB and angle $\angle ACB$. The spherical laws of sines and cosines provide the remaining three quantities. The constraint that this geometry imposes on rows of segments to lie on great circle arcs results in a tiling pattern that is less compact

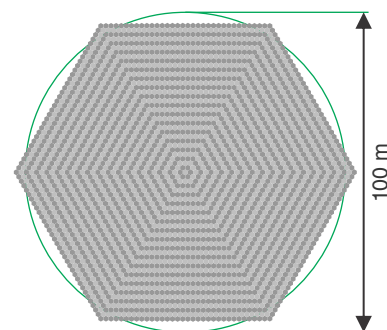


Fig. 11 Segment tiling configuration based on Overwhelmingly Large Telescope (OWL) parameters. This mirror configuration contains 3367 segments, arranged with 33 rings (shown in alternating shades of gray) around the central segment.

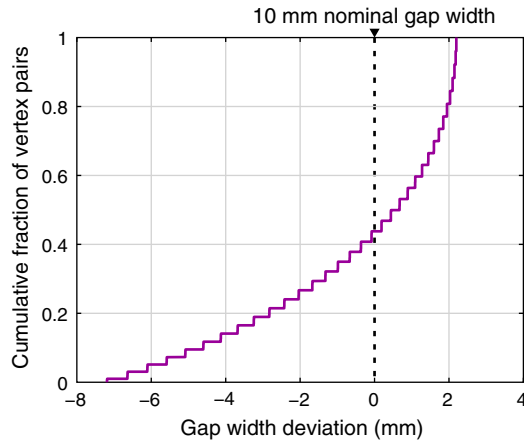


Fig. 12 Cumulative distribution of gap widths for the OWL design. Compared to the HET distribution in Fig. 10, the greater number of rings in the OWL design results in a more continuous distribution of gap widths. However, these widths are still clustered around specific values, as shown by the stepped shape of the plot.

than with the method above, resulting in a larger range of gap width variation. This is shown for a particular example in Sec. 5.2.

5 Application to Telescope Architectures

Using the AECT algorithm presented in Sec. 4, we generated possible mirror segmentation geometries for three telescope architectures. The first architecture is based on the optical parameters of the HET design in order to evaluate the tiling performance against an existing telescope. The second compares the tiling algorithm to the segmentation presented for the OWL phase A design, to exercise its performance with one

of the largest proposed telescope primary mirrors. We use this design study to provide comparisons between our AECT algorithm and some of the alternatives discussed in Sec. 4.4. Finally, we apply the algorithm to the ISTAR architecture, which is a less mature design. Because the optical design for ISTAR has not yet been frozen, parametric studies can be used to explore a range of geometries. With the ISTAR architecture, we present an extension to the AECT algorithm by leveraging the hierarchical structure described below.

For each of these architectures, mirror segment positions are computed based on the AECT algorithm, and the gap widths between segments are computed to evaluate the algorithm's performance. Using a MATLAB® script on a modern laptop computer, the mirror segment positions were computed in 0.6 to 3.9 ms depending on the number of segments, and the gap width distributions were computed in 3.9 to 130 ms. The principal computational benefit of this algorithm is that it produces satisfactory results through a single iteration or through optimization of a single parameter r_{hex} . In contrast, a general optimization of the segment positions would require potentially thousands or millions of evaluations of the gap width distribution.

5.1 Comparison with Hobby–Eberly Telescope Design

The HET is composed of 91 identical mirror segments, where each segment is 1 m flat-to-flat.² The spherical primary has a radius of curvature of 26.165 m, and the gaps between segments vary from 6.2 to 15.8 mm. By applying the AECT algorithm and targeting the same minimum gap width, we achieved a design, shown in Fig. 9, with gaps that ranged in width from 6.2 to 11.9 mm and a mean gap width of 10.3 mm. The distribution of gap widths is shown in Fig. 10.

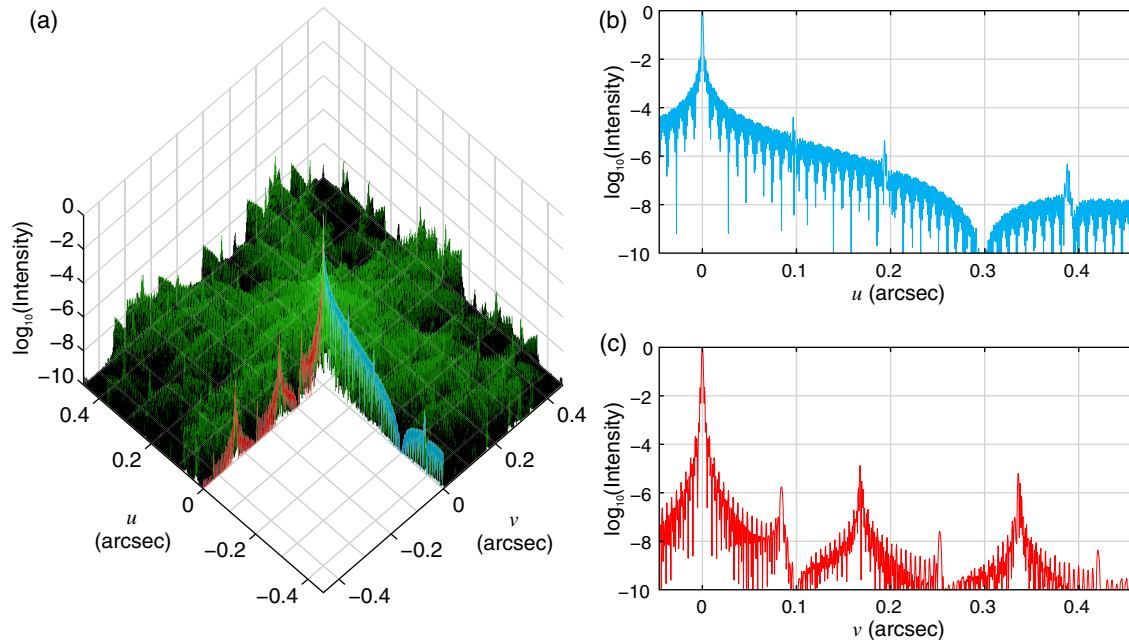


Fig. 13 PSF of a 100 m aperture based on the proposed OWL geometry. (a) Isometric view of the central portion of the PSF spanning a box size of 0.9 arc sec for $\lambda = 0.65 \mu\text{m}$. The color and height scale logarithmically with intensity. (b) and (c) Orthogonal cross-section profiles of the PSF, with directions selected to capture the neighboring peaks from the speckle pattern. The directions are indicated by the cutaway on the isometric plot.

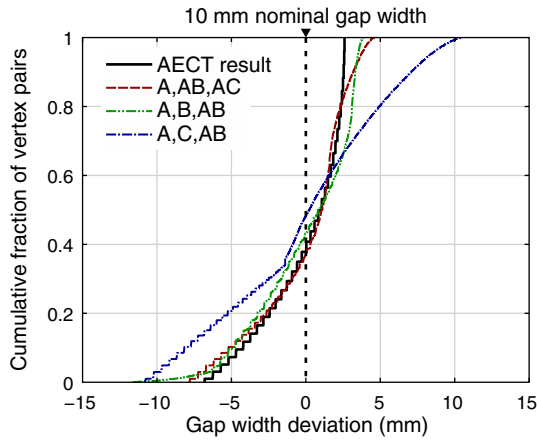


Fig. 14 Cumulative gap width distributions based on the baseline azimuthal equidistant centroid tiling (AECT) algorithm and the three alternative tiling methods using spherical trigonometry. The legend indicates the angles and side lengths that are prescribed, based on the notation in Fig. 7 and the spherical triangular tiling methods described in Sec. 4.4.

In this analysis and for the designs presented below, cumulative distributions of the gap widths are used rather than histograms in order to avoid sensitivity to binning. In each case, perpendicular projected distances between vertex pairs are computed as described in Sec. 4.3.1 and plotted in order of magnitude to obtain the cumulative distribution. The resulting gap width distribution using the AECT algorithm has a 40% reduction in the total range compared to the original HET design. Assuming a similar distribution of gap widths, the mean gap width can be reduced by $\sim 20\%$, leading to a reduction in diffraction effects as discussed earlier.

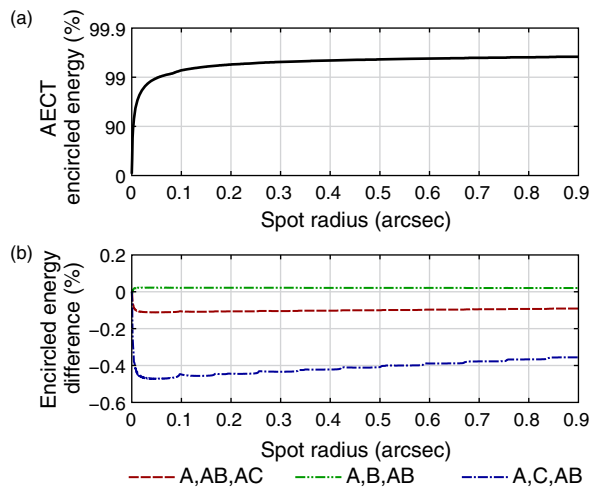


Fig. 15 Encircled energy profiles computed from the OWL PSFs. (a) Percent encircled energy of the baseline AECT design for the OWL configuration, plotted on a logarithmic vertical scale as a function of spot radius in arc seconds for $\lambda = 0.65 \mu\text{m}$. (b) Difference in encircled energy between the three spherical triangular tiling designs and the baseline AECT design, in percentage points. The legend indicates the angles and side lengths that are prescribed, based on the notation in Fig. 7. One alternative design shows slightly greater encircled energy than the AECT design, and the other two show much lower encircled energy in comparison.

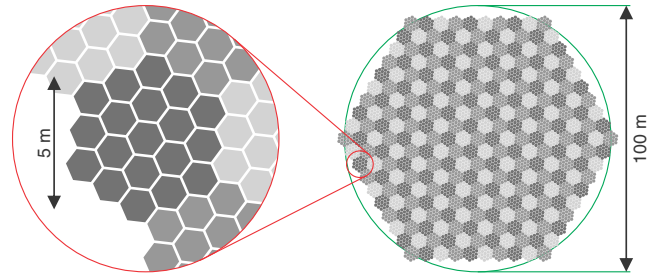


Fig. 16 Hierarchical segment tiling configuration for the In-Space Telescope Assembly Robotics (ISTAR) design, with 19 segments in a module (left) and 271 modules in the full primary (right). Individual modules are shaded in alternating shades of gray for clarity. All segments are identical to each other, and all modules are also identical to each other, which decreases the complexity of manufacturing and assembly.

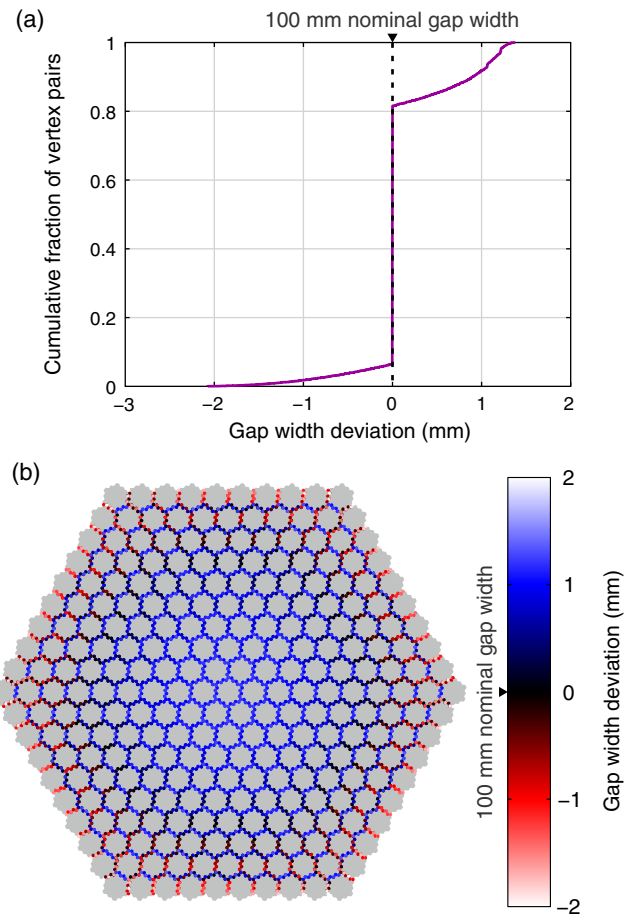


Fig. 17 Cumulative distribution of gap widths for the ISTAR design. (a) The total distribution of gap widths considering all adjacent segments regardless of their module. Here the intramodule gaps are seen as the large near-vertical segment that appears at zero deviation from the nominal width. The intermodule gaps, considering adjacent segments in different modules, comprise the remainder of the distribution and follows a trend similar to the OWL distribution. (b) The spatial configuration of intermodule gaps are plotted and color-coded based on deviation from the nominal 100 mm gap width. Smaller gaps are plotted in red and larger gaps in blue. Note that gaps between adjacent rings remain large from the center to the edge, while the gaps between modules in the same ring decrease in width from the center to the edge. Intramodule gaps are not plotted because of their small variation relative to intermodule gaps.

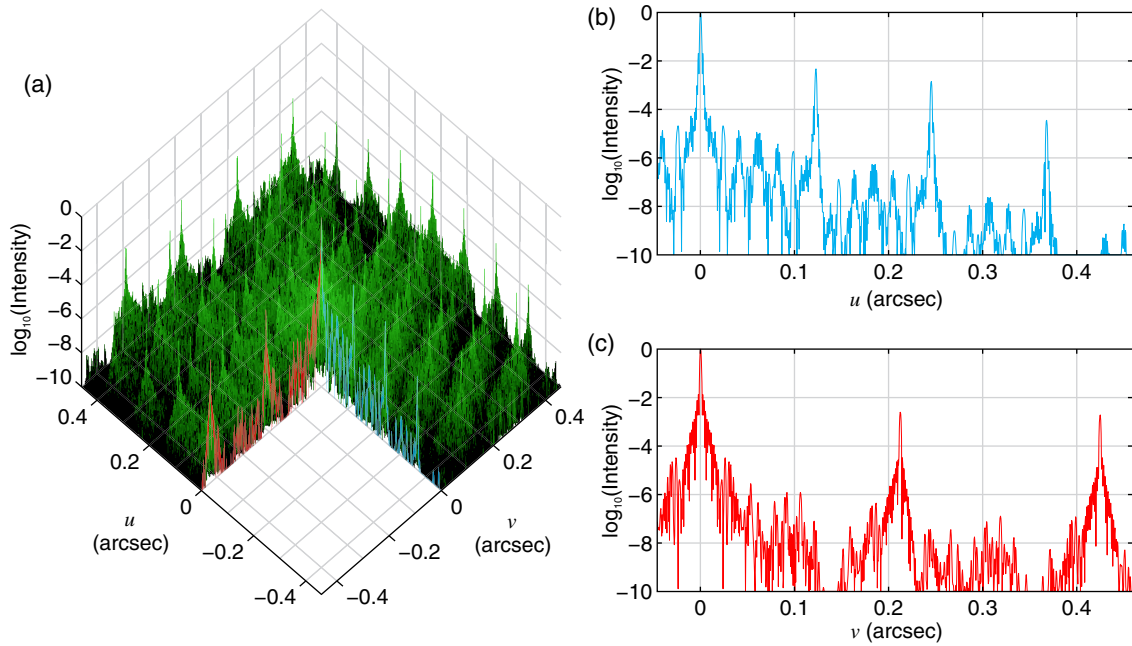


Fig. 18 PSF of a 100 m aperture based on the proposed ISTAR geometry. (a) Isometric view of the central portion of the PSF spanning a box size of 0.9 arc sec for $\lambda = 0.65 \mu\text{m}$. The color and height scale logarithmically with intensity. (b) and (c) Orthogonal cross-section profiles of the PSF, with directions selected to capture the neighboring peaks from the speckle pattern. The directions are indicated by the cutaway on the isometric plot.

5.2 Comparison with OWL Phase A Design

In the OWL phase A study, an optical configuration was presented for a 100 m spherical primary mirror with a 250 m radius of curvature, composed of 3048 mirror segments with a flat-to-flat dimension of 1.6 m (hexagonal side length of 0.92 m).¹³ The design of this primary mirror included variable gap widths ranging from 4 to 14 mm.

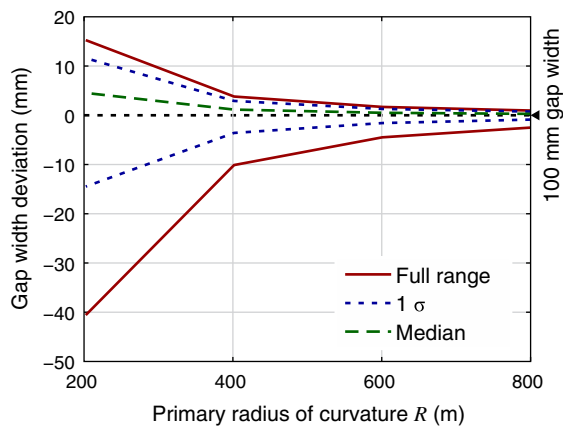


Fig. 19 Distribution of intermodule gap widths for ISTAR design as a function of primary mirror radius of curvature. Because of the large fraction of intramodule gaps and their small variation relative to the intermodule gaps, we show only the intermodule gaps here. As was evident in the upward curvature of the point design cumulative distributions above, the median and quartile values are biased toward the upper end of the distribution. The total range satisfies the project-imposed constraint of 10% maximum deviation from the nominal 100 mm gap only for mirror radius of curvature >400 m.

Using the proposed design algorithm with the OWL geometry, a fully filled configuration including 33 rings was generated, as shown in Fig. 11.

This design includes 3367 segments filling the entire hexagon, whereas the phase A design eliminated segments lying outside the circular aperture or behind an obscuration, leaving a total of 3048 segments. As a result, the distribution of gap widths presented here is conservative relative to a more complete design that accounts for removed segments. As discussed in Sec. 4.3.1, the elimination of the central segments behind the obscuration will result in a further reduction of the average gap width. The cumulative distribution of gap widths, computed as

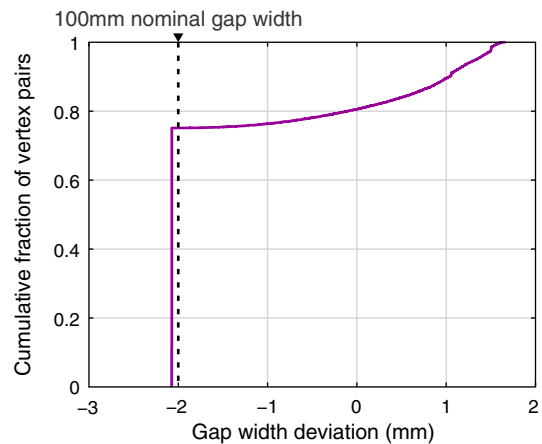


Fig. 20 Cumulative distribution of gap widths for alternative ISTAR design, minimizing the average gap width while keeping all gaps larger than the smallest in the original configuration. Here, the intramodule gaps appear at the bottom of the distribution, and the intermodule gaps comprise the upper curved portion of the distribution.

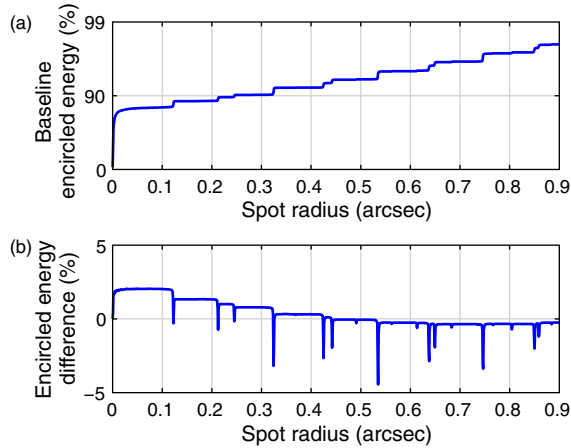


Fig. 21 Encircled energy profiles computed from the ISTAR PSFs. (a) Percent encircled energy of the baseline AECT design for the ISTAR configuration, plotted on a logarithmic vertical scale as a function of spot radius in arc seconds for $\lambda = 0.65 \mu\text{m}$. (b) Difference in encircled energy between the modified AECT design and the baseline design, in percentage points. The positive values at smaller spot radius indicate a higher percentage of encircled energy in the modified design.

the vertex-to-vertex distance projected onto the normal direction, is shown in Fig. 12.

An average gap width of 10 mm was targeted by iteratively modifying r_{hex} , and a total range of 9 mm in gap width variation was achieved, which is 10% better than the 10 mm range reported in the OWL phase A study.

The PSF for this aperture was computed using a gray-pixel bitmap image with 7.33 mm spatial resolution and 16-bit intensity resolution. This image was padded to a total of 40,000 pixels spanning a spatial dimension of 293 m. The central portion of the computed PSF is shown in Fig. 13 as an isometric view with two orthogonal cross-section profiles.

In order to compare the AECT design algorithm with alternatives using spherical trigonometry as described in Sec. 4.4, each of the three competing spherical triangular methods was used to compute a gap width distribution as well as PSFs. Figure 14 shows a comparison of the gap width distributions, with all three alternatives resulting in a wider range of gap widths than the baseline AECT design.

Figure 15 compares the difference between the encircled energy profiles of the baseline AECT design and the alternative spherical triangular designs.

The second alternative method shows a slightly greater encircled energy than the baseline design, despite the greater range in gap widths. However, the other two alternatives show much a lower encircled energy in comparison. Overall, the total percentage point difference in encircled energy is less than half a percentage point, which indicates a small effect on optical performance. The total range of gap widths is likely a more significant design driver, because the small range of the AECT design allows for a more aggressive reduction of the mean gap width.

5.3 Proposed Hierarchical Design for ISTAR

The robotically assembled concept proposed for the ISTAR project includes a 100 m spherical primary mirror assembled from modules of mirror segments.¹⁴ An objective of the design is to produce a mirror segment configuration that achieves an average gap width of 100 mm with no greater than 10%

variation and to simplify the manufacturing and assembly process by leveraging symmetry and modularity.

To achieve these goals, we developed a hierarchical extension to the AECT algorithm, where the positions of 19 segments were first computed to form a module with $n = 2$ rings, and then the module itself was treated as a larger hexagon and tiled to form the primary with $N = 9$ rings, totaling 5149 mirror segments. These are shown in Fig. 16. In order for the modules to tile such that the individual segments align properly, the module is rotated about its z axis by

$$\psi_{\text{mod}} = \tan^{-1} \left[\frac{1}{\sqrt{3}(2n+1)} \right] + \frac{\pi}{2}. \quad (13)$$

The effective size of the module tiling hexagon is defined by

$$r_{\text{mod}} = r_{\text{hex}} \sqrt{3n^2 + 3n + 1}, \quad (14)$$

and the spin and pitch angles are computed as described in Sec. 4.2.

Because the modules span a relatively small solid angle within the primary, there is very little variation in the intramodule gaps, spanning a range of $<2.5 \mu\text{m}$. This results in a total distribution, including both intermodule and intramodule gaps, where a large fraction of the gaps is very close to the desired gap width. In the ISTAR configuration, $\sim 75\%$ of all gaps are intramodule gaps, as can be seen in the distribution in Fig. 17(a). The total range of gap width variation, however, suffers slightly as a consequence, compared to a configuration where all mirror segments are tiled in a single large array as in the OWL design described in Sec. 5.2. Figure 17(b) shows how the gap widths are spatially distributed over the surface of the full primary. Because of the low variation of intramodule gaps, only the intermodule gaps are plotted, showing the larger gap widths near the center of the primary and the smaller gap widths near the rim.

The PSF for this aperture was computed using the same technique described above for the OWL PSF. The central portion of the computed ISTAR PSF is shown in Fig. 18 as an isometric view with two orthogonal cross-section profiles. Because of the imposed larger minimum gap width for robotic servicing, diffraction effects are much greater than in the OWL design, with speckle peaks approximately two orders of magnitude less intense than the central peak, compared to almost five orders of magnitude for OWL.

The design process was repeated for several values of the primary radius of curvature R , ranging from 200 m up to the baseline 800 m for the design described above. The range of intermodule gaps, plotted in Fig. 19, shows that the gap variations increase slowly as R decreases from 800 down to 400 m, but then grows more rapidly for smaller values of R . This parametric study demonstrated the value of using the AECT algorithm to quickly produce viable mirror geometries, without the need for intensive computation or optimization routines.

A feature of the hierarchical segmentation method is that an extra degree of freedom is introduced by separately tiling first a module and then the full mirror. This allows the gap width distribution to be manipulated. As discussed in Sec. 3.3, we would ideally like to produce a tiling geometry that has minimum average gap width subject to constraints. In the hierarchical method, we are able to more aggressively tile the segments within each module and achieve a distribution very close to the minimum

width constraint. The intermodule gap distribution would then target a larger gap width in order to avoid violating this constraint. An example of a possible gap width distribution is shown in Fig. 20. Here, the total range of gap widths has increased by 8.3% from 3.45 to 3.74 mm, but the average gap width has been reduced from the original configuration by 1.4 mm. While the original AECT scheme had an average gap width that was 2.07 mm (60% of the full range) above the minimum, this modification achieves an average gap width that is only 0.67 mm (18% of the full range) above the same minimum.

Figure 21 shows the encircled energy of the original design and the difference with the modified design. From this result, it appears that the modified design has up to two percentage points greater encircled energy at small radii, but that this effect does not persist beyond a radius of ~ 0.4 arc sec. Depending on the imaging application, one design may be preferable over the other, or a further parametric study using the AECT algorithm can be applied to characterize the range of possible designs.

6 Conclusions

In this paper, we presented the AECT algorithm to determine the positions of identical hexagonal mirror segments on a spherical surface. We used a one-dimensional analog to motivate the desire for a small range in gap widths and to bias the mean toward the low end of that range. We showed through comparison with HET and OWL designs that the algorithm compares favorably to prior techniques in terms of the total range of gap widths and presented a hierarchical concept based on the ISTAR geometry that is effective for biasing the gap width distribution and could potentially lead to improved designs for other large telescopes. The utility of the algorithm for use in a parametric design study was also demonstrated using the ISTAR concept.

From the one-dimensional analog, we verified that the PSF is not strongly influenced by variation in gap width and also found that the contrast performance could be enhanced by nonuniform spacing. Through the design studies with three telescope configurations, we showed that the AECT algorithm is capable of satisfying mirror segment placement constraints over a wide range of geometries and is well suited for use in early design studies. The HET study demonstrated that the AECT design could reduce the mean gap width by $\sim 20\%$ compared to the actual design. The OWL study demonstrated that the AECT algorithm can handle several thousand individual segments and showed a 10% reduction in total gap width range from the previously reported design. The ISTAR study demonstrated several approaches for using the AECT algorithm for preliminary design.

In future work, the AECT algorithm would also be a viable candidate to provide starting points for optimization of a final design. As future ground- and space-based telescopes target increasingly large apertures, the need for computationally efficient design algorithms will become more essential in order to manage the large number of mirror segments required.

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