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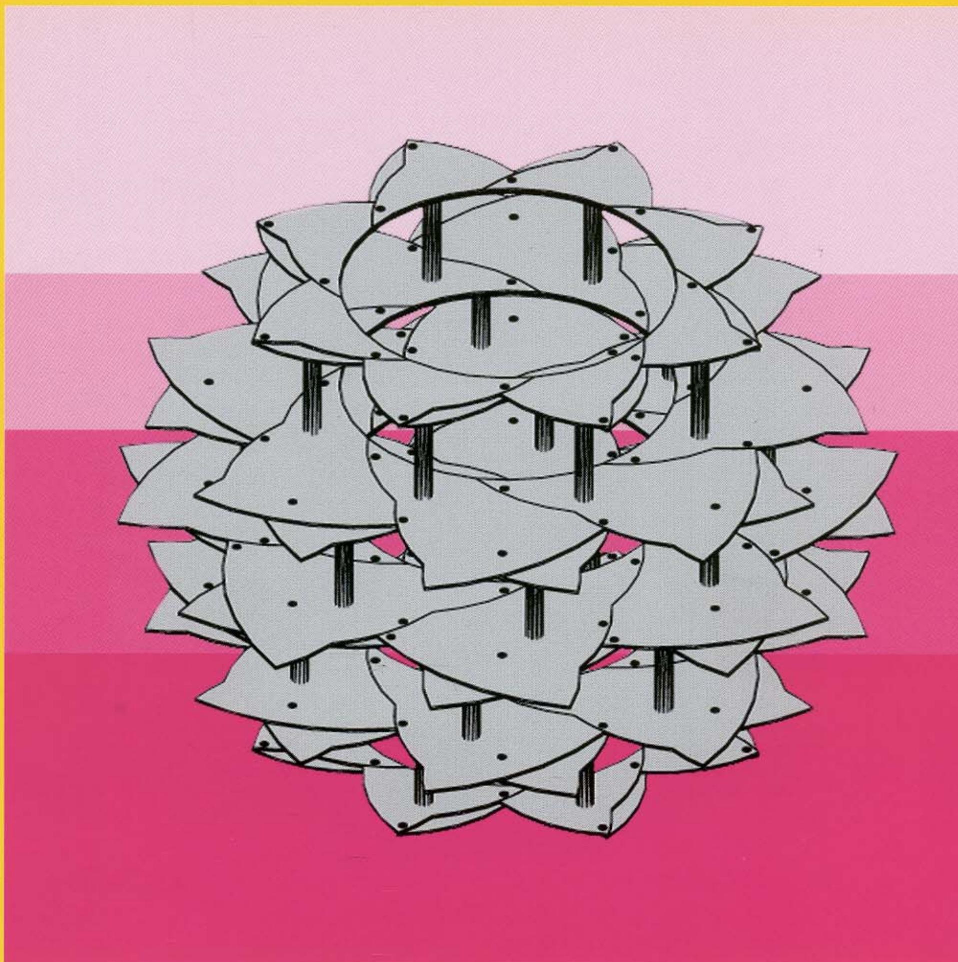

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EXPANDABLE "BLOB" STRUCTURES

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SUMMARY

This paper presents a methodology for designing single-degree-of-freedom expandable "free-form" structures composed of rigid blocks connected through simple cylindrical joints. The underlying idea is to interconnect two or more individually expandable plate structures. Using a two-dimensionally expanding sphere as a first example, the conditions that must be satisfied to preserve the internal mobility when connecting identical expandable plate structures are explained. These conditions are then extended to plate structures that are not identical and it is shown that a wide range of expandable free-form or "blob" structures can be designed through this approach.

Keywords: *Deployable structure, mechanism, pantograph.*

1. INTRODUCTION

This paper is concerned with the geometric design of expandable structures consisting of rigid elements connected through cylindrical hinges or *scissor joints* that only allow rotation about a single axis. The authors have been interested in developing stacked assemblies formed by rigidly interconnecting the *expandable plate structures* that they had previously developed [1]. The connections between individual plates can themselves be volume filling, and so the stacked structure can also become an expandable three-dimensional object. As the plate structures from which one starts can have any plan shape, and only simple kinematic constraints have to be satisfied in order for them to maintain their internal mobility in the stack configuration, nearly any shape can be generated, including so-called free-forms or *blobs*.

The approach presented in this paper starts by developing a method for rigidly connecting two identical and individually expandable plate structures, such that the assembled structure only possesses a single-degree-of-freedom. The kinematic constraints that must be satisfied by the connections and the connected plate structures are derived, and are shown to allow the stacking of non-identical plate structures as well, thereby allowing free-form profiles to be obtained for the assembled structure.

This paper is presented as follows. The next section briefly reviews the kinematic properties of the expandable plate structures that are to be connected, and their underlying bar structures. Then, Section 3 describes a method for connecting the plate structures such that they can form an expandable sphere. Section 4 is then concerned with the kinematic constraints that have been satisfied by these connections and the formulation of a general set of rules for such connections. These are then used in Section 5 to design an expandable free-form or blob structure demonstrating the possibility of designing vivid and exciting expandable structures using this method. A brief discussion concludes the paper.

2. BACKGROUND

Simple expandable structures based on the concept of *pantographic elements*, i.e. straight bars connected through scissor hinges have been known for a long time. One of the simplest forms of such pantographic structures is the well-known lazy-tong in which a series of pantographic elements are connected through scissor hinges at their ends to form two-dimensional linearly extendible structures.

More sophisticated expandable or deployable structures have been developed in recent years [2-4]. Many of these solutions are based on the so-called *angulated pantographic element* [5]. In its simplest form, shown in Figure 1, it consists of two identical angulated elements each composed of two bars rigidly connected with a kink angle α. Unlike pantographic elements composed from collinear bars, which have α = 0, such angulated elements can be used to form expandable closed loop structures if the conditions $\overline{AE} = \overline{CE}$ and $\alpha = 2\arctan(\overline{EF}/\overline{AF})$ are satisfied [6, 7]. These conditions guarantee that the following equation of geometric compatibility is satisfied for all deployment angles, γ

$$\tan(\alpha/2) = \frac{\overline{CE} - \overline{AE}}{\overline{AC}}\tan(\gamma/2) + 2\frac{\overline{EF}}{\overline{AC}} \qquad (1)$$

As the kink angle α is constant, i.e. independent of γ, the end joints of the pantographic element move along radial lines, and so an expandable closed loop structure can be formed from these elements.

You and Pellegrino [7] found that certain *multi-angulated elements*, i.e. pantographic elements with multiple kinks, have the same property. Consider the circular structure shown in Figure 2. It consists of two separate layers, shown with solid and dashed lines, each containing 12 identical multi-angulated elements, which here have three kinks. As the structure expands, from left to right in the figure, the hinges and hence both layers of elements expand radially. However, while moving radially each multi-angulated element in the solid-line layer also rotates *clockwise* as can be seen in the figure. The dashed-line layer, on the other hand rotates *counter-clockwise*. Note how the multi-angulated elements form three concentric rings of rhombus-shaped four-bar linkages all of which are sheared as the structure expands.

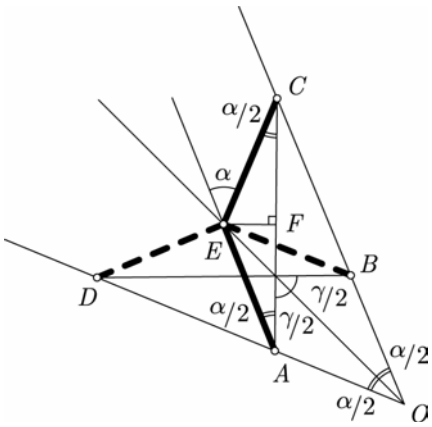

Figure 1. *Pantographic element consisting of two angulated elements, each formed by two bars.*

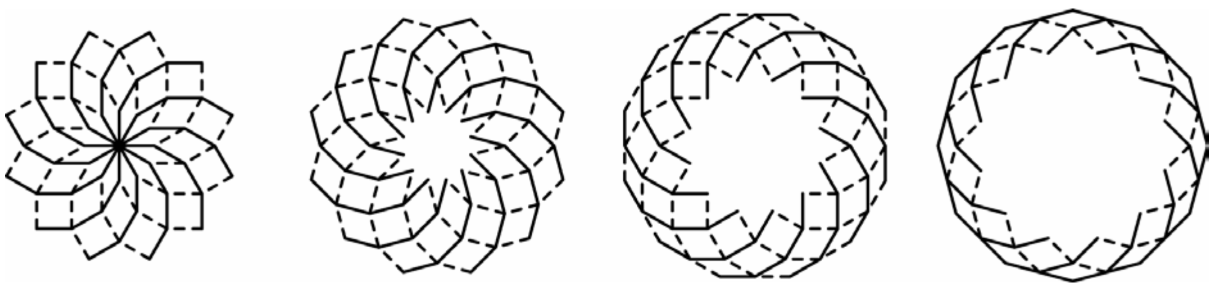

Figure 2. *Retractable structure formed from multi-angulated elements.*

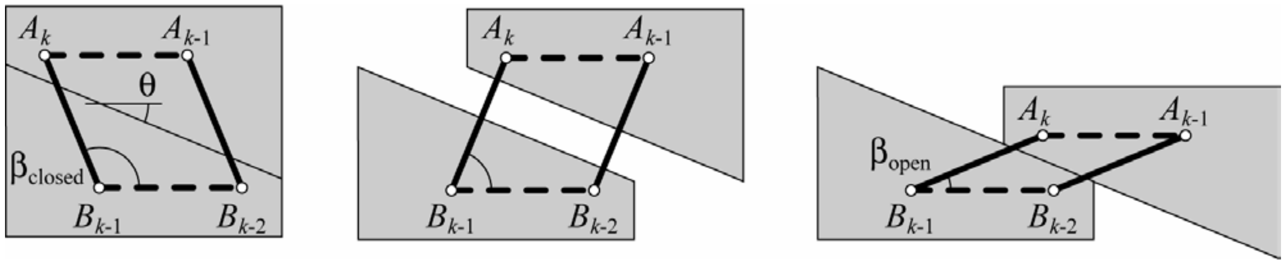

Figure 3. *Movement of four-bar linkage with two plates attached.*

Expandable structures composed from rigid plates and scissor or spherical joints have been proposed [8,9]. Reference [10] proposed covering the above basic circular bar structure with rigid panels to form a continuous, gap free covering. As an extension of his earlier work on bar structures, You [11] proposed to replace parts of the bar structure with rigid elements; this idea was further developed by Rodrigues and Chilton in their Swivel Diaphragm [12].

The present authors have proposed a method for covering any multi-angulated bar structure with plates [1]. By considering the shearing deformation of any four-bar linkage formed by the elements common to two pairs of consecutive angulated elements, the authors determined a general condition on the shape of the boundary between two covering elements, such that the plates would not restrict the motion of the structure while resulting in a gap and overlap free surface in either extreme position of the structure. The covering of a single rhombus is shown in Figure 3. The boundary angle θ for the plate elements is determined by:

$$\theta = \frac{\pi - \beta_{closed} - \beta_{open}}{2} \qquad (2)$$

In fact, instead of covering the bar structure with plates, it is possible to remove the angulated elements and connect the plates directly, by means of scissor hinges at exactly the same locations as in the original bar structure. Thus, the kinematic behaviour of the expandable structure remains unchanged. It was shown that, as long as the plate boundaries have a certain periodic shape, they need not be straight [1]. Figure 4 shows an example of such an expandable plate structure; this non-circular structure is formed by 26 plates of different shape.

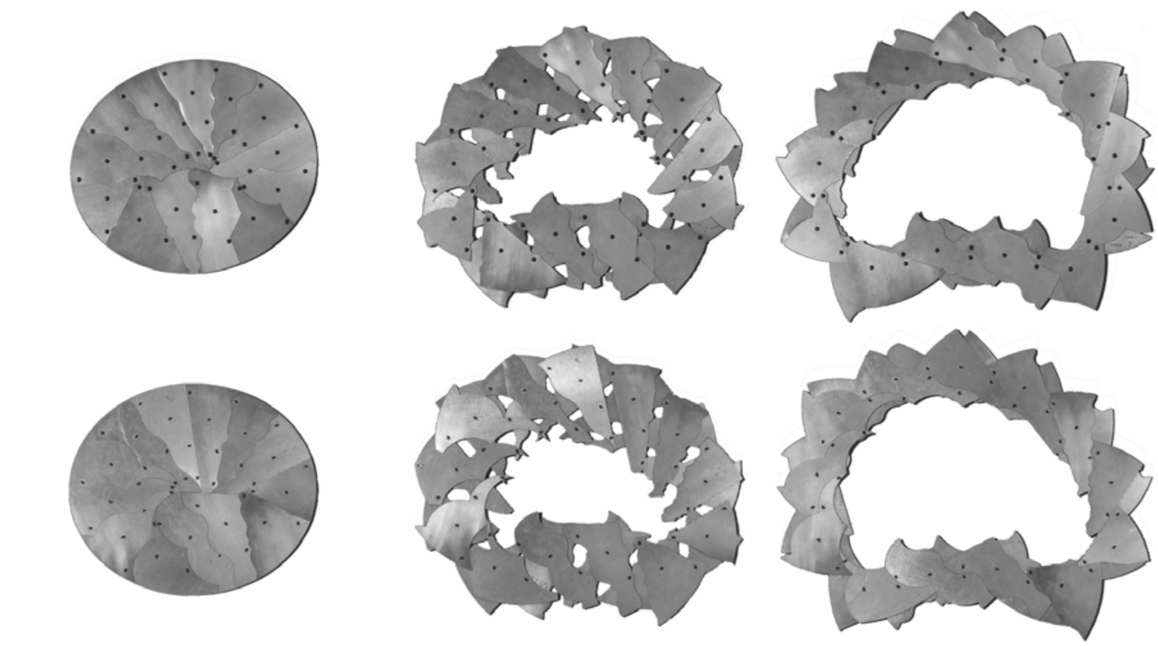

Figure 4. *Model of non-circular structure where all plate boundaries are different; both layers are shown.*

3. AN EXPANDABLE SPHERE

To investigate the possibility of creating three-dimensional expandable structures by stacking identical plate structures, a simple design for the plate structures was chosen. The design, shown in Figure 5, consists of 16 identical plate elements of which 8 form the bottom layer and 8 the top layer. As for the bar structure shown in Figure 2, the 8 plates forming the top layer move radially outwards while rotating *clockwise*, while the plates forming the bottom layer (of which only small parts can be seen in the figure) rotate *counter-clockwise*. From symmetry it can be concluded that all plates in the same layer rotate by identical amounts; the rotations of plates in different layers are equal and opposite.

Consider two such identical plate structures positioned above one another, which are to be rigidly connected. In order to preserve the internal mobility of the two plate structures, the rigid connections must be made from a single plate in one structure to a single plate in the other structure. However, noting the opposite rotations of the two layers in each structure, it is not possible to connect the plates in the top layer of the bottom structure to the plates in the bottom layer of the top structure while maintaining the internal mobility of these structures. Instead, the connections must be either between the top or the bottom layers, as these have identical motions. This can be achieved by swapping the top and bottom layers in one of the two structures so that connections can then be made between the facing and now identical layers of plates. Hence, by rearranging the order of the layers in one of the two structures it becomes possible to rigidly interconnect them, so they form a stacked and still expandable assembly.

A stacked assembly composed of identical plate structures would form a cylinder, but other profiles can be formed by varying the plan shape of the outer edge of the plate structure.

An expandable assembly that approximates to a spherical profile has been designed and built from 5 interconnected plate structures. From the model shown in Figure 6 it can be seen that the 5 structures conform to a spherical profile in the closed position. Note that the periodic boundaries of the plate elements on the upper face of each plate structure are convex in alternate directions, which confirms that the layers have been swapped in alternate structures. All five plate structures have been produced using the same plate template, only the outer boundaries of the plates have been made with different radii. Clearly, the allowable rotation between the extreme positions of neighbouring plates is identical throughout the structure, giving identical motions for the 5 plate structures.

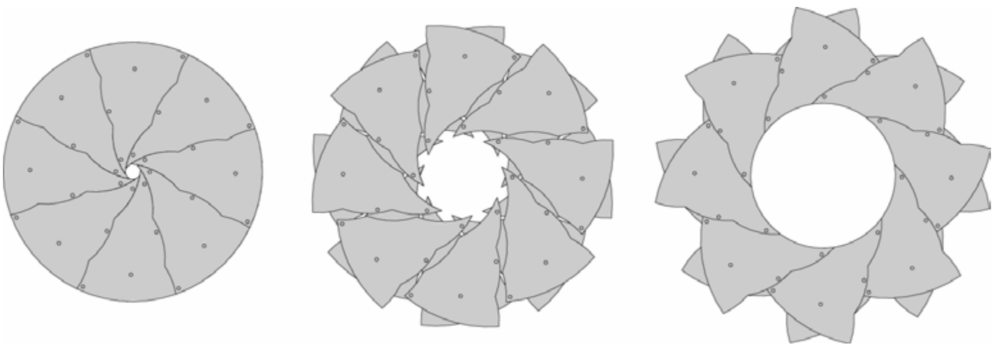

Figure 5. *Expandable circular plate structure.*

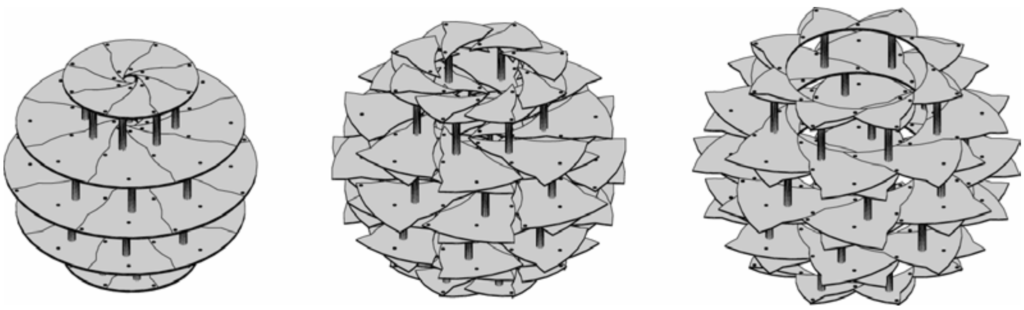

Figure 6. *Perspective views of expandable spherical structure.*

In Figure 6 the connections between the 5 structures have been shown as solid rods rigidly attached to the plate elements. However, as the motion of two connected plates is identical, the connectors themselves could be formed in a number of different ways. For example, to form an object with a smooth, continuous surface in the closed configuration, one can connect the plates using either solid or hollow blocks. By curving the outer wall of each block, and also the top/bottom surface of the blocks attached to the uppermost/lowermost plate structure, one can obtain a continuous spherical profile. The internal faces of the blocks could be made perpendicular to the original plate, and follow the periodic edge shape of the plates.

Figure 7 shows such a spherical model. It was constructed using identical plastic plate structures of which four were trimmed so that their outer boundaries form circles of different radii. The connections were made from identical blocks of light foam board, cut using abrasive water-jet cutting. The blocks were then glued to the back of the individual plastic plates that form the plate structures and the final spherical profile was obtained by removing the excess foam board material.

4. STACK STRUCTURES

To rigidly connect two expandable plate structures the motion of the individual plates being connected must be identical. Earlier, the motion of each plate was described as the combination of a radial motion, i.e. a translation and a rotation. Because the rotations in the two layers are equal and opposite, imposing an additional rigid body rotation to the whole structure, equal to the rotation undergone by one of the layers, the motions of the two layers become a pure rotation and a pure translation, respectively [10]. If, for example, the imposed rotation is such that the plates in the top layer of the bottom structure undergo a pure rotation, then each plate in this layer rotates about its own fixed centre.

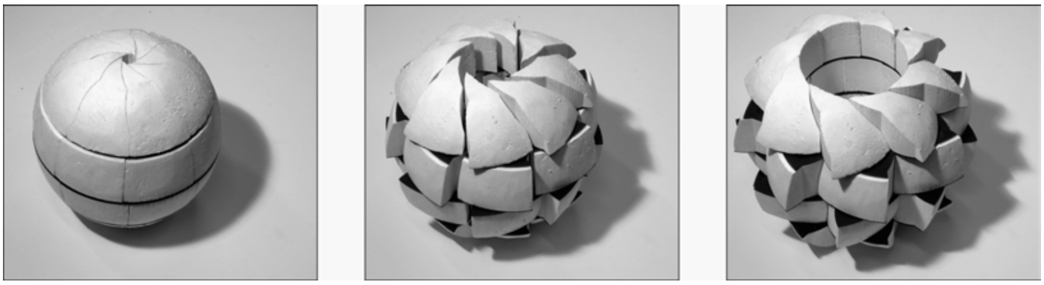

Figure 7. *Expandable sphere.*

Hence, consider two plate structures that are to be connected. Following the above approach, impose the same rigid body rotations on both structures; clearly *the plates in the two connected layers must rotate about the same axes of rotation and by the same amounts*. Since the rotations of two rigidly connected plates are automatically synchronised, the only kinematic constraint that must be satisfied is that the two plates have the same axes of rotation. This is the only condition that must be satisfied for any two plates belonging to different expandable plate structures to be connected without suppressing the internal mobility of the plate structures.

It was mentioned in Section 2 that an expandable plate structure is kinematically equivalent to a bar structure with identically positioned hinges. It is therefore possible to identify the centres of rotation for the equivalent bar structure, instead of the plate structures themselves. It is therefore proposed to start the design of an expandable stack assembly by considering its underlying bar structures; a particular embodiment as a plate structure will be determined later on.

Following Hoberman [5], an expandable bar structure of general shape is generated in its open configuration by considering an *n*-sided polygon. Each angulated element is then defined so that its central scissor hinge coincides with a vertex of the polygon and its end points coincide with the midpoint of the adjacent polygon sides. Thus, in Figure 8(a) we have defined two non-circular structures; structure I is drawn with a thinner line, structure II is drawn with a thicker line. Structures consisting of multi-angulated elements can be generated by adding any number of rhombuses to this base structure [7].

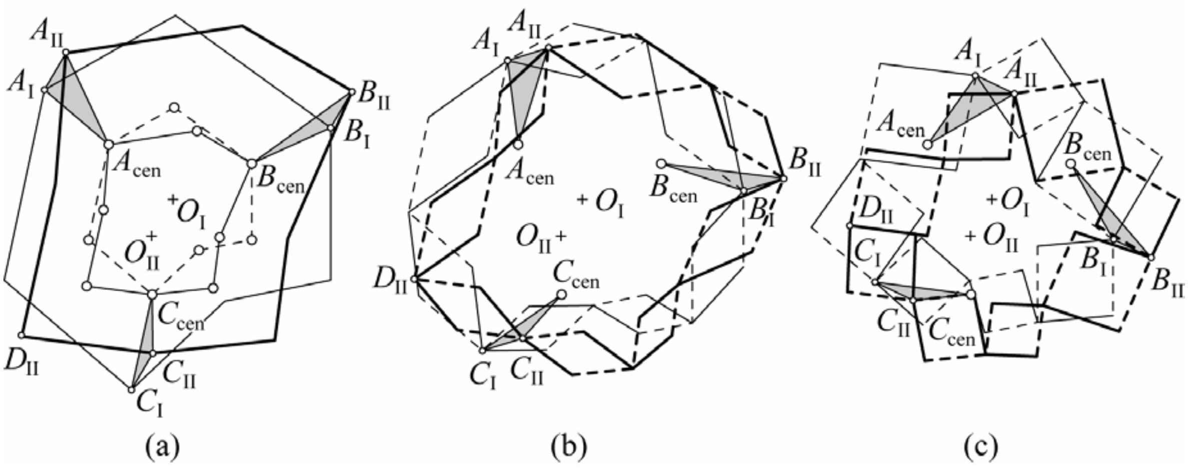

Figure 8. *Stacked bar assembly in three different configurations; (a) fully open; (b) intermediate; (c) closed.*

Kassabian, You and Pellegrino [10] showed that the axis of rotation for any particular angulated element is at the half-way point between the origin of the polygon, *O*, and the vertex defining the central hinge of the element. Hence, all the axes of rotation are at the vertices of a polygon that is half the size of that defining the elements themselves, as shown in Figure 8(a). Therefore, together with the location of the chosen origin, the initial polygon determines the location of the axes of rotation. Or, alternatively, a polygon defining the axes of rotation together with a chosen origin point define a bar structure, and hence also a plate structure that will rotate about these specific axes when it is expanded.

In Figure 8 note that the bar structures I and II have been defined such that three centres of rotation, A_{cen}, B_{cen}, and C_{cen}, are common to the two structures. Hence, the angulated elements with central scissor hinges *A*, *B* and *C* in the two structures can be rigidly connected to one another. For example, A_I and A_{II} can be connected by the rigid triangle $A_{cen}A_IA_{II}$. It can also be seen in the figure that the allowable rotation of the stacked structure is limited by contact at D_{II} in the extreme closed position.

For an assembly where *all* the plates in each layer are to be connected, the axes of rotation for all elements of the two layers must coincide. Hence, the polygons defining the axes must also be identical. As these polygons also define the open plan shape of the structures, when scaled to double size, the two layers must therefore be formed from identical polygons. However, the location of the origin for the two structures need not coincide, as was the case for the spherical assembly where they were both located on the central axis of the expandable sphere. In Figure 8 note the different origins O_I and O_{II}.

Note in Figure 8(a) that the axes of rotation could be chosen to be the same also for the elements in the dashed-line layers but this is only the case when the structures can be expanded fully, i.e. the dashed- and solid-line layers coincide when fully expanded. This is not normally the case as the physical size of hinges and plates will prevent this and thus different axes have to be identified if another structure has to be added to the stack, by connecting two facing dashed-line layers.

5. AN EXPANDABLE BLOB

Having determined the overall kinematic constraints that must be satisfied for two plate structures to be connected together, a more complex structure will be presented in this section. Three different configurations of this expandable structure are shown in Figure 9.

The first step in the design of this structure is to define its outer shape in the closed configuration. Next, one decides how many layers of blocks are to be used. Here, 6 layers have been used and hence there are 5 sets of connections to be designed. Then, one defines the underlying bar structures for any pair of plate structures that are to be connected. Here each bar structure consists of 6 identical angulated elements and hence has 6-fold symmetry.

These structures are shown in Figure 10 for a particular pair of plates, where the two structures have been drawn with thicker and thinner lines; note that the two layers to be connected are those shown with solid lines. Also note that individual elements of these bar structures rotate about the same axes of rotation (although they do not share a common origin) and hence they can be rigidly connected to each other, as shown in the previous section.

One of the complications of generating a structure with a complex three-dimensional shape is that the top and bottom faces of the blocks from which it is made may have different shapes. Hence, in addition to the underlying bar structures, the shapes of the top faces of the bottom layer of blocks and the bottom face of the top layer of blocks need to be defined.

In this particular case, see Figure 10, these shapes were chosen to be identical, though they are offset with respect to one another. In general, they need not be identical but, as the motion of the two solid-line underlying bar structures is identical, the boundaries determined using Equation (2) will always be parallel.

The shape chosen here is such that there is no central opening in the closed position, unlike the spherical model of the previous section where there is a small opening. The periodic shape of the boundaries was determined such that in the expanded configuration the central opening is perfectly circular, as before. But, unlike the previous model, this does not result in a smooth cylindrical opening through the whole structure as the individual layers have been offset. Another effect of the layers being offset is that the internal faces of the blocks need to be inclined. For the current model these faces have been created by extruding the lower face of the block along an inclined circular arc.

The periodic walls can hence be doubly curved, as shown in Figure 9, without impeding the motion of the structure.

Figure 9. *Perspective views of an expanding "blob" structure.*

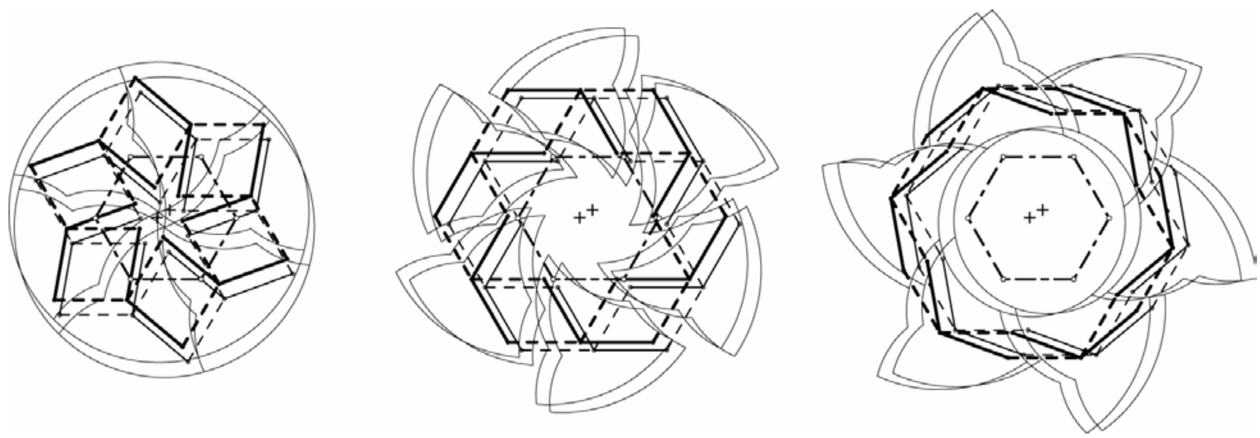

Figure 10. *Plate structures for generation of solid blocks.*

Note that, although the outer edges of the plates have been shown as circular arcs, their actual shape is determined by the three-dimensional profile of the structure. For the current "free-form" profile model the outer boundaries for the bottom and top faces of the blocks are not identical. This and the inclined, curved extrusion path result in each block being unique.

The model in Figure 9 has not yet been realised physically, though this would be possible using rapid prototyping techniques, for example. Its manufacture is complicated by the offset of the individual layers, which results in non-collinear hinges. Models without offsets are easier to manufacture and assemble; such models can be made from identical blocks connected using long thin rods for the connections. The required symmetric or free-form profile can be generated by machining the outer face of the assembled structure.

6. CONCLUSION

An investigation into three-dimensional expandable shapes made by rigidly interconnecting individually expandable plate structures has been presented. It has been shown that it is possible to create structures with highly irregular shapes; the internal mobility of the plate structures is preserved if simple kinematic constraints are satisfied. Several models have been designed and constructed to verify and demonstrate this finding, of which two have been shown in this paper.

The two models presented show that it is possible to create such expandable assemblies with almost any plan and profile shape. They can hence be visually pleasing and attractive for applications in architecture and design.

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