Collapsible Rib-Tensioned Surface (CRTS) Technology Development
Final Report

S. Pellegrino

CUED/D-STRUCT/TR195

European Space Agency Contractor Report

ESA Study Manager: W.J. Rits

ESTEC Contract no. 11936/96/NL/FG

Release date: 2 April 2001

Study Summary

A new type of multipurpose deployable membrane reflector, known as the *Collapsible Rib Tensioned Surface* (CRTS) reflector, is being developed by the European Space Agency. Its underlying concept is the inherent geometrical stability of a doubly-curved membrane with high in-plane stiffness, maintained in a state of tension by a series of flexible, radial ribs. Its main components are an expandable hub, a series of collapsible ribs attached to the hub, and a precision-cut membrane. During deployment, the hub is contracted to minimise the loading on the ribs. After deployment, the hub is expanded, pushing the ribs outwards until the prestress of the membrane reaches a desired level.

The shape of a CRTS reflector is only an approximation to a perfect paraboloid, whose accuracy depends both on the number of ribs and the shape of the membrane between adjacent ribs. For prestressing the membrane biaxially, the surface would have to be saddle-shaped between the ribs, but a closer approximation to a paraboloid can be obtained by decreasing the radial prestress.

The first part of this report presents a methodology for the preliminary design of a CRTS reflector, given its aperture, focal length and required surface accuracy. A form-finding algorithm based on the force-density method, which was originally developed for cable net and membrane roofs, is adapted to membrane structures supported by flexible elastic elements. Different boundary conditions are implemented. An experimental study of the interaction between prestress distribution and shape in a one-sixth sector of a 1 m diameter symmetric reflector is presented.

This methodology has been extended to membranes of asymmetric shape, by introducing special prestress distributions which apply only in-plane forces on the ribs. Three different configurations of offset reflectors are considered, of which only two are found to be practically feasible.

For any chosen configuration of the reflector, the membrane surface is made out of flat pieces of Kevlar-reinforced Kapton foil. The cutting pattern for each piece and the initial rib profiles for the required prestress distribution are computed. Then, the shape error of the actual surface that corresponds to this cutting pattern and rib profiles is estimated, both by an approximate linear-elastic analysis and by a fully non-linear finite element analysis.

A study of the elastic folding and dynamic deployment of the collapsible ribs of CRTS reflectors has been carried out, showing that their behaviour is in many ways similar to that of straight tape springs — extensively studied in recent years and now well understood. Collapsible ribs, which are both longitudinally and transversally curved, can be folded by forming localised elastic folds. In analogy with straight tape springs, such folds have a characteristic longitudinal radius of curvature, zero transverse curvature, and carry an almost constant moment. However, when they are folded by inducing compressive stresses along the edges, they are bi-stable and hence this type of folds should be avoided, to guarantee the uniqueness of the deployed configuration.

A careful analysis of the natural frequencies of prestressed membrane structures has been carried out, starting from finite-element models of flat membranes with regular shapes, which could be compared to standard, analytical solutions. After showing that the finite element models are accurate for these simple structures, the same methodology is extended to CRTS reflectors. The first few natural frequencies of reflectors of different diameters and numbers of ribs are obtained, and are found to be relatively insensitive to prestress level and hub size. The fundamental natural frequency of the largest reflector that has been analysed, with a diameter of 10 m and focal length to diameter ratio of 0.78, is above 1 Hz.

The final part of the report presents the design, manufacture, and testing of a 1.5 m diameter offset CRTS demonstrator with twelve ribs, which fully validates the design methodology developed in this study. The expected root-mean-square surface error of the demonstrator was 2.3 mm and measurements of the actual shape obtained are in excellent agreement. Deployment tests carried out on this structure, with two different packaging schemes have confirmed that reliable deployment behaviour is achieved only if the ribs are folded in such a way that only tensile stresses are induced along the edges. Hence, a reflector packaged by wrapping the ribs around the hub has excellent deployment behaviour, but zig-zag folding of the ribs leads to erratic behaviour.

Contents of Final Report

This report consists of a brief Introduction, followed by six self-contained parts, each presenting the results of a separate work package. Each part was issued as a Technical Report during the course of the study and has different authors.

I. Introduction.

II. Shape and stress analysis of symmetric reflectors. Previously issued as Lai, You, and Pellegrino (1997).

III. Shape and stress analysis of offset reflectors. Previously issued as Lai and Pellegrino (1999).

IV. Non-linear finite element analysis. Previously issued as Lai and Pellegrino (2001).

V. Folding and deployment of curved ribs. Previously issued as Seffen, You, and Pellegrino (1997).

VI. Natural frequencies. Previously issued as Kukathasan and Pellegrino (2001).

VII. Design and testing of 1.5 m diameter offset demonstrator. Previously issued as Lai and Pellegrino (2001).

I. Introduction

S. Pellegrino

This part of the report provides an executive summary for the whole study and briefly introduces the six parts that follow.

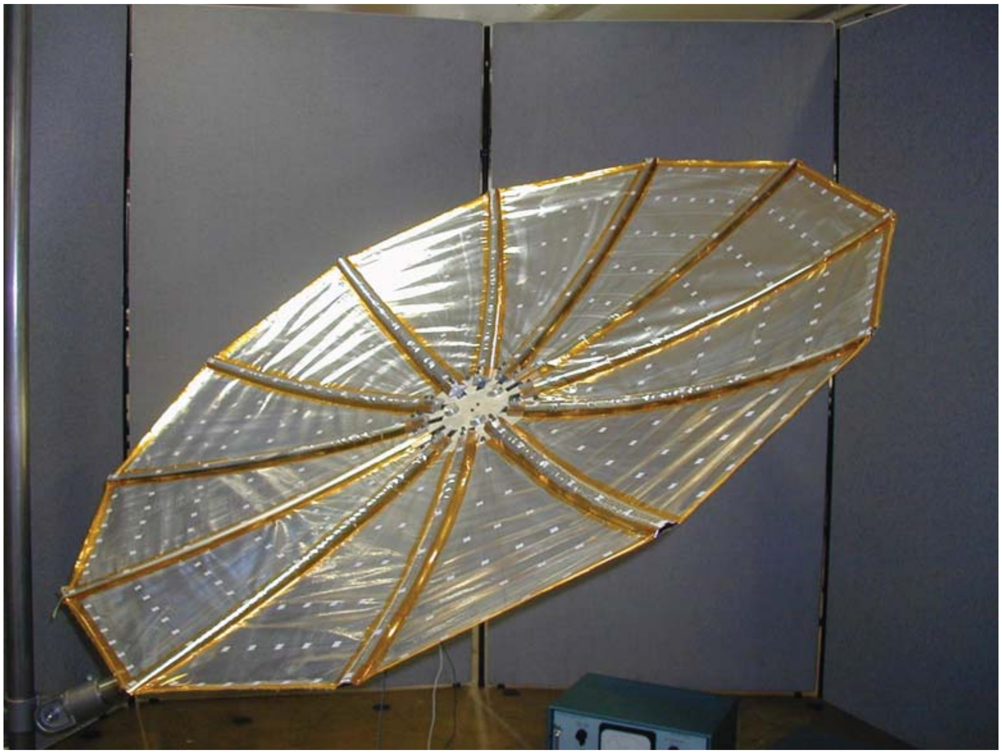

Figure 1: Demonstrator of CRTS concept.

1 Background

The concept of producing a radio frequency (RF) reflecting surface by tensioning an in-plane stiff membrane by means of collapsible ribs was put forward in 1991 by W.J. Rits of ESA-ESTEC.

A CRTS reflector consists of three main parts. A central expandable hub, a number of thin-walled foldable ribs connected radially to the hub, and a precision shaped membrane that is supported and tensioned by the ribs. During deployment the radius of the hub is at its minimum, so that the ribs can deploy the membrane without having to prestress it at the same time. Once the membrane has been fully deployed, the hub is expanded, thus pushing outwards the ribs. This has the effect of applying a state of prestress to the membrane, which sets it into its intended shape. A 1.5 m diameter demonstrator that was designed and constructed during the course of this study is shown in Figure 1.

Following a preliminary study of packaging and deployment issues related to CRTS reflectors, carried out in the Deployable Structures Laboratory at the University of Cambridge in 1993-94 (You and Pellegrino 1994), the European Space Agency commissioned a more extensive study whose principal aim was to extend the CRTS concept to offset configurations. This study, which has proved much more challenging than anticipated, has now been completed.

2 Aims and Purpose of Study

This report investigates the relationship between shape accuracy and prestress distribution in CRTS reflectors, in order to develop rational design criteria. Its main aims are to determine the surface accuracy of reflectors with different configurations, for different prestress distributions, and to develop feasible ways of designing and manufacturing actual structures. The packaging and deployment performance of CRTS reflectors are also briefly investigated, as well their deployed stiffness, which is linked to its natural frequencies of vibration.

A form-finding analysis procedure is developed to determine the shape of a membrane reflector surface in equilibrium under a given prestress distribution. A linearised method of analysis is developed to calculate the expected surface error of an actual structure formed by a series of flat membrane gores, whose results are verified by a non-linear finite element analysis.

A symmetric CRTS reflector is designed first, because of its simplicity. However, the reflecting surface of a symmetric antenna would be partly in the shadow of the subreflector and its supporting structure, as these lie in the direct path of the main beam. This results in a reduction of the effective aperture, and the blocked shadow can degrade the antenna efficiency. Offset configurations eliminate this effect by placing the subreflector and supports just past the edge of the aperture, hence they are the preferred choice for communication applications. A drawback is that the antenna structure is asymmetrical and therefore more complex to design and manufacture. A key difficulty is that the asymmetry of the prestress distribution normally results in high lateral loads on the ribs, which is not acceptable. Special offset configurations identified during the course of this study, though, make it possible to reduce and — in one case — practically eliminate the lateral loads.

A series of tools for the preliminary design of CRTS reflectors that can meet a set of specific requirements, i.e. aperture diameter, focal length, offset, surface accuracy, prestress level, etc. are developed and are then applied to the design of a demonstrator model. This structure was then manufactured and tested, thus obtaining much valuable information for the further development of CRTS reflectors.

3 Layout of Report

Following this Introduction, there are six main parts.

Part II investigates the shape and stress distribution of CRTS reflectors, both analytically and experimentally. The analytical part of the study establishes a methodology for the preliminary design of symmetric reflectors of given aperture, focal length, and target root-mean-square (rms) error. The concepts of a *reference surface* and of an *equilibrium surface* are introduced, and algorithms are developed to compute these surfaces and their associated rms error. Then, the cutting pattern for making the membrane is computed and the rms error of the *actual surface* is predicted. Estimates are made of the rms error of CRTS reflectors with apertures of 1, 3, 5, and 10 m, with 6, 12, and 24 ribs. Experimental measurements of prestress and shape of a one-sixth sector of a 1 m diameter reflector with 6 ribs, are compared with predictions obtained from the computational study.

Part III shows that CRTS reflectors with offset configuration are feasible, and presents extensive sets of results to aid the design of future reflectors. Prior to this study, it had been thought that setting up a prestressed membrane with asymmetric shape would require very stiff ribs. Although this remains an option, this part of the report presents two different ways of designing offset reflectors whose membranes are properly stressed and yet apply only in-plane loads on the ribs. Three different configurations of offset reflectors are introduced, whose rms errors are comparable but whose associated prestress distributions are different. It is found that one of these configurations can be used to produce reflectors with conventional offset shapes and prestress distributions that should be able to avoid the formation of wrinkles in the membrane. Another configuration produces prestress distributions as good as those previously obtained for symmetric reflectors, but requires a non-standard shape of the reflective surface. A simple and effective way of determining prestress distributions that are approximately in equilibrium is developed, which can be used in the design of reflectors of general shape.

Part IV presents a non-linear finite element simulation of the assembly process of a CRTS reflector, leading to estimates of its surface accuracy and prestress distribution. In Parts II and III preliminary estimates had been obtained by a linearised formulation; hence the main aim of this study is to assess the accuracy of estimates based on the earlier method.

Carrying out a fully non-linear simulation of an offset CRTS reflector is a formidable computational challenge, which required the development of special purpose software. Despite this, only for a limited set of test cases it was possible to achieve full convergence. A complete non-linear simulation of an 8-rib offset reflector with focal length of 0.9 m and diameter of 1 m is presented and the results are compared to the linear analysis. It turns out that the rms surface errors predicted by the two methods are practically identical but the stress magnitudes are less accurate, due to incomplete modelling of the sliding between the membrane and the cables and ribs of the reflector. It is concluded that the linearised method presented in Parts II and III is a very good way of determining the actual prestressed shape of a CRTS reflector.

Part V presents a study of the folding and deployment properties of the ribs of a CRTS reflector. These are a special type of tape springs, which are both longitudinally and transversely curved. It is shown that curved tape springs have much in common with straight tape springs and, in particular, localised elastic folds can be formed without difficulty. In analogy with straight springs, such folds have a characteristic longitudinal radius of curvature, zero transverse curvature, and carry a constant moment. The differences between the two types of springs are significant only for equal-sense folds (which induce compressive stresses along the edges) with small rotation angles, as curved springs folded this way are bi-stable. Hence, the uniqueness of the deployed configuration for a curved tape spring can be guaranteed only if opposite-sense folds are employed. Deployment tests presented in this part of the report show that relatively small gravity-induced effects can be sufficient to prevent the springs from deploying fully. This appears to be a problem for CRTS reflectors folded according to the zig-zag folding scheme, but not if the wrapping scheme is used.

Part VI presents a study of the natural frequencies of various prestressed membrane structures. A vibration analysis of flat membranes of arbitrary shape is done using the ABAQUS finite element package, and compared with analytical solutions; good agreement between the two solutions is obtained. The finite element model is then extended to CRTS reflectors. Natural frequency and mode shape estimates are obtained for various reflector diameters, hub dimensions, number of ribs and prestress levels. The analysis results indicate that the fundamental natural frequency of the reflector decreases with the increase of diameter of the reflector and does not change greatly with the increase of hub radius. Furthermore, the fundamental natural frequency increases with the increase of membrane prestress. A periodic variation in the results is observed.

Part VII presents the design, construction and testing of a 1.5 m diameter demonstrator with offset configuration, based on the circular configuration introduced in Part III. Key features of the design are: simplicity; ease of assembly, dismantling and adjustment; hub size capable of smooth and controlled expansion; and surface accuracy controlled by the membrane, not by the ribs. The rms surface error of the demonstrator was found to be 2.0 mm, practically coinciding with the value that had been predicted analytically. The demonstrator was packaged by wrapping the membrane and ribs around the hub, and the deployment sequence was recorded with a high-speed camera; deployment took about 1 s.

4 Main Conclusions

4.1 General Design Methodology

The shape error and prestress distribution of CRTS deployable membrane reflectors have been investigated in this report, both analytically and experimentally, and a methodology for the preliminary design of this type of reflectors has been established.

Given the aperture diameter, focal length, and target RMS error, as well as the maximum permissible ratio between the highest and lowest principal stress in the membrane, the key stages of the design process are:

1. Determination of the required number of ribs using the concept of a reference surface.
2. Computation of the equilibrium surface, i.e. a surface whose RMS error is better than the prescribed target, and which is in equilibrium under a state of prestress that satisfies everywhere the condition on the maximum ratio.
3. Computation of the cutting pattern.
4. Verification that the shape error and distribution of prestress if the membrane surface is made according to the computed pattern satisfies all the stated requirements.
5. Computation of the unstressed profile of the ribs.

The computations required to implement this design methodology are relatively straightforward and, apart from the search for the equilibrium surface — which in general cases may involve many iterations — require minimal or no iteration.

A non-linear finite element analysis has been developed to verify the results of the linear-elastic analysis which is used in Step 4, above. Although severe convergence problems were encountered in many cases when using this non-linear simulation, convergence was obtained in a few cases and for these cases it was found that linear and non-linear analyses results match well.

4.2 Offset Configurations

A key aim of this study was to investigate the applicability of the CRTS concept to offset reflectors. Soon after starting this study, it became clear that, because of the lower order of symmetry of offset reflectors, *gore by gore designs* would inevitably lead to large out-of-plane forces on the ribs, which would therefore have to be made very stiff. This would have posed no great problems for the analysis, but would have run against the CRTS concept, as the accuracy of the surface is supposed to come from the membrane, not from the support structure. Therefore, ways of producing *global designs* that reduce the out-of-plane loading on the ribs to zero were investigated.

Three different ways of designing reflectors with offset configuration have been identified. The first two configurations are obtained by intersecting the parent paraboloid with a circular cylinder parallel to the axis of the paraboloid; the third is obtained by intersecting the paraboloid with a circular cylinder whose axis is parallel to the normal to the paraboloid at the centre of the reflector. Only the second and third configurations are seriously worth considering.

The more conventional of these two solutions, called *central hub configuration*, puts the centre of the hub of the reflector at the point obtained by projecting the centre of the ellipse defined by the tips of the ribs onto the local tangent plane. To produce a successful design, one chooses a suitable initial prestress at the start of the form-finding process. This prestress needs to satisfy two-dimensional equilbrium, i.e. the prestress components in the tangent plane need to be in equilibrium at the gore-gore interfaces, as the ribs can carry only in-plane loading, and also at the gore-cord interfaces. Then, when three dimensional equilibrium conditions are imposed during the form-finding analysis, it is best to use boundary conditions that allow sliding in the direction perpendicular to the plane of the ribs, at connections between the membrane and the ribs.

The alternative solution is called *circular configuration*. It has the special feature that the tips of the ribs are not co-planar. Instead, their projections onto the local tangent plane lie on a circle. This greatly simplifies the choice of a suitable initial state of prestress because, as far as two-dimensional equilibrium is concerned, this configuration is axisymmetric and hence adjacent gores can be equally stressed. This solution produces a very even distribution of prestress but, because the shape of the reflecting surface in this configuration does not match the standard illumination pattern for antenna feeds, it is possible that a part of the surface may not be utilised.

4.3 Experimental Validation

Following preliminary experiments on a small one-gore model, a complete design of a 1.5 m diameter offset circular CRTS reflector with focal length of 0.9 m was developed and implemented as a complete structural demonstrator, which was then tested.

It is normally argued that to avoid the formation of wrinkles, thin membranes need to be subjected to biaxial states of prestress, but an alternative approach was followed. Recent research in the Deployable Structures Laboratory has shown that large wrinkles form only when either the membrane stretches (which Kevlar-reinforced Kapton will not do by any significant amount) or the boundary supports are non-smooth, hence the present design aimed for a distribution of prestress almost purely in the hoop direction. This approach had a number of advantages: (i) the reflector surface can be made from flat, in-plane stiff gores; (ii) the accuracy of the reflector can be easily predicted, as its shape coincides with a geometrically defined surface, i.e. the reference surface; (iii) the calculation of the cutting patterns for the gores and the analysis of the elastic deformation of the ribs can be done quite accurately by considering simple two-dimensional models.

A hub mechanism design was designed, in which each rib is mounted on a linear bearing and is pulled outwards by a single steel cable that applies a prestressing force. The ratios between the rib forces, related to the number of ribs, diameter, offset and focal length of the reflector, are controlled by varying the diameter of the pulleys supporting this cable. The whole system is driven by a single electric motor. This design has worked very well, but the present mechanism is too heavy with a mass of ≈ 6 kg and contains a large number of moving parts. Clearly, a different, although kinematically equivalent design will be needed for use in space. An obvious change would be to mount the ribs on springs perpendicular to the plane of the hub, thus removing all the linear guides. The idea of using a single cable to apply the prestressing forces to the base of each rib has also worked well. In future, the possibility of using a Nitinol wire —replacing both the steel cable and the electric motor, gears, and disk— is worth considering.

A general approach to the design of the curved ribs that support the membrane surface was presented. It takes into account the maximum bending moment in the rib and the yield limit associated with folding of the ribs. A rational approach to the design of the rib moulds was presented, taking into account the shape distortion of curved CuBe strips during heat treatment.

A procedure for accurately cutting the pieces of Kevlar-reinforced Kapton foil, bonding them along the edges to form the complete membrane of the reflector, attaching the Kevlar edge cord and the rib sleeves was developed. Of course, this is a key stage in the practical realisation of CRTS reflectors.

Two packaging techniques for CRTS reflectors, zig-zag folding and wrapping, that had been previously investigated by You and Pellegrino (1994) were tested. Of these two schemes, it was found that only wrapping works well for the collapsible ribs of CRTS reflectors. The problem with zig-zag folding — confirmed by the present study — is that downwards folds are bi-stable, i.e. they can be in equilibrium in a half-deployed configuration. On the other hand, it was confirmed that if the membrane and ribs are wrapped around the hub, they deploy very reliably. The deployment time of the 1.5 m demonstrator was about 1 s. Packaging the demonstrator was found to be far from straightforward, also due to its over-designed ribs, however it is believed that a folding machine could simplify this process considerably.

The surface accuracy of the demonstrator was carefully measured and its RMS error was found to be 2.0 mm, actually *lower* than the predicted value of 2.3 mm. This apparent anomaly is of no great significance.

5 Conclusions

The physical demonstrator designed, constructed and tested during the final part of this study has shown that it is possible to achieve the surface accuracy of the reference surface, which is the best possible shape that can be achieved by a CRTS reflector. Table 1 lists the RMS errors of 10 m diameter CRTS reflectors with 6, 12 and 24 ribs and F/D=0.78. For 24 ribs the error is about 3 mm which is suited to moderate precision reflector applications.

In conclusion, both the design methodology and the fabrication procedure were successfully validated. The CRTS concept is definitely feasible for future space applications.

Geometry	No. of ribs	RMS error [mm]
Symmetric Configuration	6	39.6
	12	9.6
	24	2.4
Standard Configuration	6	48.7
	12	13.2
	24	3.5
Central Hub Configuration	6	49.2
	12	13.4
	24	3.6
Circular Configuration	6	45.9
	12	12.5
	24	3.3

Table 1: Achievable RMS errors of 10 m CRTS reflectors with different configurations and F/D=0.78.

6 Recommendations for further work

Based on points raised in the various parts of this report, the following recommendations for further work are made. The following list is in order of significance, i.e. the most important first.

- Move ribs under the reflective surface. This will require considerable changes to the design presented in this dissertation; it is thought that a feasible approach would be to remove the rib sleeves, but instead connect the seam between adjacent gores directly to the upper edge of the ribs by means of "membrane webs."
- Validate predictions of natural frequencies against experimental measurements.
- Include overall buckling among the constraints considered in the design of the ribs.
- Design and make a folding machine, to wrap the demonstrator.
- Carry out further deployment tests and develop models of deployment dynamics.
- Modify the hub design to reduce the number of moving contacts. Explore the possibility of replacing the electric motor and steel cable with a Nitinol wire.
- Take more accurate measurements of the surface accuracy of the demonstrator, by increasing the number of target points.

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II. Shape and Stress Analysis of Symmetric Reflectors

C.Y. Lai, Z. You and S. Pellegrino

This part of the report investigates the shape and stress distribution of CRTS reflectors, both analytically and experimentally. The analytical part of the study establishes a methodology for the preliminary design of symmetric reflectors of given aperture, focal length, and target root-mean-square (rms) error. The concepts of a *reference surface* and of an *equilibrium surface* are introduced, and algorithms are developed to compute these surfaces and their associated rms error. Then, the cutting pattern for making the membrane is computed and the rms error of the *actual surface* is predicted. Estimates are made of the rms error of CRTS reflectors with apertures of 1, 3, 5, and 10 m, with 6, 12, and 24 ribs.

In the final, experimental part of the work presented herein, measurements of prestress and shape of a one-sixth sector of a 1 m diameter reflector with 6 ribs are compared with predictions obtained from the computational study.

III. Shape and Stress Analysis of Offset Reflectors

C.Y. Lai and S. Pellegrino

In this part of the report it is shown that CRTS reflectors with offset configuration are feasible, and extensive sets of results are presented, to aid the design of future reflectors. Prior to this study, it had been thought that setting up a prestressed membrane with asymmetric shape would require very stiff ribs. Although this remains an option, this part of the report presents two different ways of designing offset reflectors whose membranes are properly stressed and yet apply only in-plane loads on the ribs. Three different configurations of offset reflectors are introduced, whose root-mean-square errors are comparable but whose associated prestress distributions are different. It is found that one of these configurations can be used to produce reflectors with conventional offset shapes and prestress distributions that should be able to avoid the formation of wrinkles in the membrane. Another configuration produces prestress distributions as good as those previously obtained for symmetric reflectors, but requires a non-standard shape of the reflective surface.

A simple and effective way of determining prestress distributions that are approximately in equilibrium is developed, which can be used in the design of reflectors of general shape.

IV. Non-Linear Finite Element Analysis

C.Y. Lai and S. Pellegrino

A non-linear finite element simulation of the assembly process of a CRTS reflector is carried out, leading to estimates of its surface accuracy and prestress distribution. In Parts II and III of this report preliminary estimates had been obtained by a linearised formulation; hence the main aim of the study presented herein is to assess the accuracy of estimates based on the earlier method.

Carrying out a fully non-linear simulation of an offset CRTS reflector is a formidable computational challenge, which required the development of special purpose software. Despite this, only for a limited set of test cases it was possible to achieve full convergence.

A complete non-linear simulation of an 8-rib offset reflector with focal length of 0.9 m and diameter of 1 m is presented and the results are compared to the linear analysis. It turns out that the root-mean-square surface errors predicted by the two methods are practically identical but the stress magnitudes are less accurate, due to incomplete modelling of the sliding between the membrane and the cables and ribs of the reflector.

It is concluded that the linearised method presented in Parts II and III is a very good way of determining the actual prestressed shape of a CRTS reflector.

V. Folding and Deployment of Curved Ribs

K.A. Seffen, Z. You and S. Pellegrino

This part of the report presents a study of the ribs of CRTS reflectors. These are a special type of tape springs, which are both longitudinally and transversely curved. It is shown that curved tape springs have much in common with straight tape springs and, in particular, localised elastic folds can be formed without difficulty. In analogy with straight springs, such folds have a characteristic longitudinal radius of curvature, zero transverse curvature, and carry a constant moment. The differences between the two types of tape springs are significant only for equal-sense folds with small rotation angles, as curved tape springs folded in this way are bi-stable. Hence, the uniqueness of the deployed configuration for a curved tape spring can be guaranteed only if opposite-sense folds are used for packaging. Deployment tests presented in this report show that relatively small gravity-induced effects can be sufficient to prevent the springs from deploying fully. This appears to be a problem for CRTS reflectors folded according to the zig-zag folding scheme, but not if the wrapping scheme is used.

VI. Natural Frequencies

S. Kukathasan and S. Pellegrino

A study of the natural frequencies of various prestressed membrane structures is presented. A vibration analysis of flat membranes of arbitrary shape is done using the ABAQUS finite element package, and compared with analytical solutions; good agreement between the two solutions is obtained. The finite element model is then extended to CRTS reflectors. Natural frequency and mode shape estimates are obtained for various reflector diameters, hub dimensions, number of ribs and prestress levels. The analysis results indicate that the fundamental natural frequency of the reflector decreases with the increase of diameter of the reflector and does not change greatly with the increase of hub radius. Furthermore, the fundamental natural frequency increases with the increase of membrane prestress. A periodic variation in the results is observed.

VII. Design and Testing of 1.5 m Diameter Offset Demonstrator

C.Y. Lai and S. Pellegrino

The final part of this report presents the design, construction and testing of a 1.5 m diameter demonstrator with offset configuration, based on the circular configuration introduced in Part III. Key features of the design are: simplicity; ease of assembly, dismantling and adjustment; hub size capable of smooth and controlled expansion; and surface accuracy controlled by the membrane, not by the ribs. The root-mean-square surface error of the demonstrator is found to be 2.0 mm, practically coinciding with the value that had been predicted analytically. The demonstrator was packaged by wrapping the membrane and ribs around the hub. Deployment took about 1 s, and the deployment sequence was found to be very repeatable.