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This paper presents a novel kind of bi-stable structure: bi-stable space frames, based on a double-layer cylindrical architecture. An analytical method to carry out a preliminary assessment of the bi-stability of such space frames is introduced here and a series of bi-stable space frames found by this method are presented. Non-linear finite element simulations have been used to test one of the space frames and confirm the analytical predictions.

Introduction

Several bi-stable structures, which have two discrete stable configurations, have recently been found and the application of these structures to next-generation deployable and adaptive structures is currently being investigated at Cambridge. Many of the bi-stable structures that are known so far have the form of a metallic or composite cylindrical shells [1] [2] and assemblies of hinged bars and tape springs that are bi-stable are also known [3].

This paper will present a new kind of bi-stable structure –a bi-stable space frame– based on a double-layer cylindrical architecture. A method to carry out a preliminary assessment of the bi-stability of this kind of structure will be introduced and explained. A series of space frames having bi-stable properties have been found. Several key issues concerning the design of this kind of bi-stable structure have been revealed. Based on the geometrical estimation method, a cylindrical space frame that is potentially bi-stable has been selected and a series of detailed geometrically non-linear finite element simulations of this structure, varying several design parameters, have been done to investigated its bi-stability. Finally, a couple of physical models built by rapid prototyping technology will be presented.

Simple Geometrical Approach

Structural properties of space frames

A space frame is a structure system whose overall shape is two- or three-dimensional. It consists of linear elements that carry loads in a three dimensional way and these elements form a series of repeating and regular units [4]. For any space frame, the general structural properties can be usefully described in term of an equivalent continuum, which simulates the properties of basic repeating units in the space frame. Here we introduce the equivalent continuum modulus matrix for some simple examples.

Consider the two-dimensional lattice structure shown in Figure 1(a). It is assumed that all squares forming this structure are identical, and have the equal side length L and axial stiffness AE. A Cartesian coordinate system has been defined for the structure, whose axes are parallel to the sides of the space frame. As in continuum mechanics, we can introduce the equivalent modulus matrix $\mathbf{D}$ to describe its in-plane structural properties. The relationship between the in-plane axial and shearing force per unit length and the corresponding deformation variables is given by

$$\begin{bmatrix} N_x \\ N_{xy} \\ N_y \end{bmatrix} = \mathbf{D} \begin{bmatrix} \epsilon_x \\ \epsilon_{xy} \\ \epsilon_y \end{bmatrix} \quad (1)$$

in which the matrix on the right of the equation is the equivalent modulus matrix:

$$\mathbf{D} = \frac{AE}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

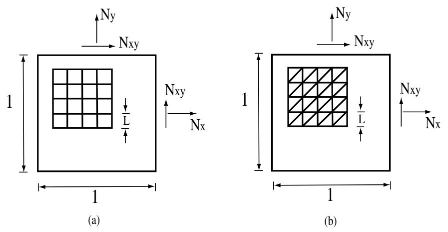

Fig. 1 Two dimensional repeating lattice based on (a) square and (b) right-angle triangle.

Similarly, for the two dimensional space frame based on right-angle triangles shown in Figure 1(b), the equivalent modulus matrix $\mathbf{D}$ can also be obtained by superposing the contribution of the additional diagonal bar to the modulus matrix in Equation (2):

$$\mathbf{D} = \frac{AE}{L} \begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} \end{bmatrix} \quad (3)$$

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Modulus matrices for lattices having several other in-plane forms have also also been derived [5].

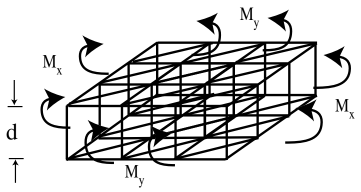

Fig. 2 Three dimensional space frame with identical top and bottom layer

For a three dimensional double-layer space frame as Figure 2 shows— in which the two layers are identical to the two-dimensional lattice considered previously (Figure 1(b)), but are now separated by a distance d, we can also develop an equivalent modulus matrix to describe its structural behavior under pure out-of-plane bending and twisting. If plane sections are assumed to remain plane and bars connecting the top and bottom layers provides a shear-rigid connection between layers, the relationship between bending and twisting moment per unit length and the corresponding curvatures and twist in the structure can be related as:

$$\begin{bmatrix} M_x \\ M_{xy} \\ M_y \end{bmatrix} = \mathbf{D}^* \begin{bmatrix} \kappa_x \\ \kappa_{xy} \\ \kappa_y \end{bmatrix} \tag{4}$$

where the modulus matrix $\mathbf{D}^*$ can be given by modifying the modulus matrix of the corresponding lattice in Equation (2):

$$\mathbf{D}^* = \frac{d^2}{2}\mathbf{D} \tag{5}$$

It can be shown that Equation (5) is valid for any double layer space frame with equal top and bottom layers. Equations (2) and (5) show that the elastic structural properties of any double-layer space frame can simply be derived from their corresponding two dimensional architecture.

Deformation of cylindrical space frame

We now consider the main object of our investigation—a double-layer space frame with cylindrical shape, whose mid-plane lies on the surface of a cylinder. To investigate its bi-stability, it is necessary to know the strain energy of the structure in all deformed configurations.

For simplicity, we initially assume that only a uniform and inextensional deformation of this space frame is allowed, and hence the mid-plane of the space frame has to remain on a cylindrical surface at all stages, as shown in Figure 3. A fixed global coordinate system $X-Y$ has been set in Figure 3 while a local coordinate system $x-y$ attached to the space frame will be allowed to rotate along with the structure itself. The initial configuration of the space frame is represented

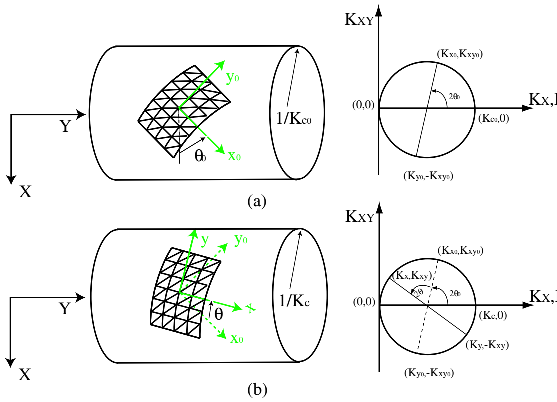

Fig. 3 Inextensional deformation of a cylindrical space frame: (a)Initial configuration (b) Deformed configuration)

by the curvature of the initial cylinder κ_{c_0} and the initial rotating angle θ_0. As shown in Figure 3(b), the deformed configuration will be denoted by curvature of current cylinder κ_c and the rotated angle θ.

The strain energy associated with any amount of bending and twisting of this structure (but note that the structure is not allowed to bend simultaneously in two different directions, because of the assumed inextensional deformation) is expressed by the equation

$$U = \frac{1}{2}\left[\Delta\kappa_x \ \Delta\kappa_{xy} \ \Delta\kappa_y\right] \mathbf{D}^* \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\kappa_x \\ \Delta\kappa_{xy} \\ \Delta\kappa_x \end{bmatrix} \tag{6}$$

where $\mathbf{D}^*$ is the equivalent modulus matrix of the space frame. $\Delta\kappa_x$,$\Delta\kappa_{xy}$ and $\Delta\kappa_y$ are the changes of curvature of the structure in the local coordinate system.

$$\Delta\kappa_x = \kappa_x - \kappa_{x_0} \tag{7}$$
$$\Delta\kappa_{xy} = \kappa_{xy} - \kappa_{xy_0} \tag{8}$$
$$\Delta\kappa_y = \kappa_x - \kappa_{y_0} \tag{9}$$

These curvatures can be obtained by Mohr's circle (Figure 3):

$$\kappa_{x_0} = \frac{\kappa_{c0}}{2}(1+\cos(2\theta_0)) \tag{10}$$
$$\kappa_{xy_0} = -\frac{\kappa_{c0}}{2}\sin(2\theta_0) \tag{11}$$
$$\kappa_{y_0} = \frac{\kappa_{c0}}{2}(1-\cos(2\theta_0)) \tag{12}$$

and

$$\kappa_x = \frac{\kappa_c}{2}(1+\cos 2(\theta_0+\theta)) \tag{13}$$
$$\kappa_{xy} = -\frac{\kappa_c}{2}\sin 2(\theta_0+\theta) \tag{14}$$
$$\kappa_y = \frac{\kappa_c}{2}(1-\cos 2(\theta_0+\theta)) \tag{15}$$

For any given cylindrical space frame, with specified initial configuration κ_{c_0} and θ_0, we can consider all possible inextensionally deformed configurations by varying the curvature of the cylinder κ_c and the twisting angle θ. Therefore, the strain energy of all configurations can be calculated and their stability can be examined from the contour plot of the energy.

Two examples of bi-stable space frame

We choose two example cases here to demonstrate this method of energy analysis. Two cylindrical double-layer space frames with the in-plane two dimensional architectures shown in Figure 1(a) and (b) have been selected and their structural properties are set as shown in Table 1. For these two space frames,

Table 1 Structural properties of space frames

Initial curvature of mid-plane	$\kappa_{c0} = 8.3\ \mathrm{m}^{-1}$
Initial rotating angle	$\theta_0 = 45°$
Side length of square	$L = 8$ mm
Depth of space frame	$d = 6$ mm
Cross section area (all bars)	$A = 0.196\ \mathrm{mm}^2$
Young's modulus of material	$E = 2.5$ GPa

the strain energy analysis method mentioned above has been performed to investigate the bi-stability. The contour plots of the strain energy of each space frame under inextensional deformation are shown in Figure 4 and Figure 5 respectively. It is clearly shown that there are two local minimum energy points in each diagram, which correspond to two stable configurations of each structure. Figure 4 shows the energy of a double-layer space frame with the same in-plane architecture as shown in Figure 1(a). One local minimum strain energy point is at $\kappa_{c_0} = 8.3\ \mathrm{m}^{-1}$ and $\theta_0 = 0$, corresponding to the initial stress-free configuration. The other local minimum energy point is at $\kappa_{c_1} = 8.3\ \mathrm{m}^{-1}$ and $\theta = 90°$. By referring to Figure 3, it is found that the mid-plane of the second configuration lies on the same cylinder as the initial one, but rotated by 90° in the $X - Y$ plane. Figure 5 shows the energy of a double-layer space frame with the same in-plane architecture in Figure 1(b). One local minimum strain energy point in this figure is also at initial configuration $\kappa_{c_0} = 8.3\ \mathrm{m}^{-1}$ and $\theta_0 = 0$. The second local minimum energy point is at $\kappa_{c_1} = 3.7\ \mathrm{m}^{-1}$ and $\theta = 90°$. Here, the mid-plane of the second stable configuration lies on a cylinder different to the initial state. It is also observed that in Figure 4, both local minimum energy points have the same energy value. This space frame is therefore a symmetrically bi-stable structure, meaning a structure that has two stable states with the same energy level. In Figure 5, the two stable configurations have two different energy levels. This space frame is therefore a asymmetrically bi-stable structure [6].

Design rules for bi-stable space frames

The fact that the above two space frames with different in-plane architectures have two varied second stable configurations indicates that the axial stiffness

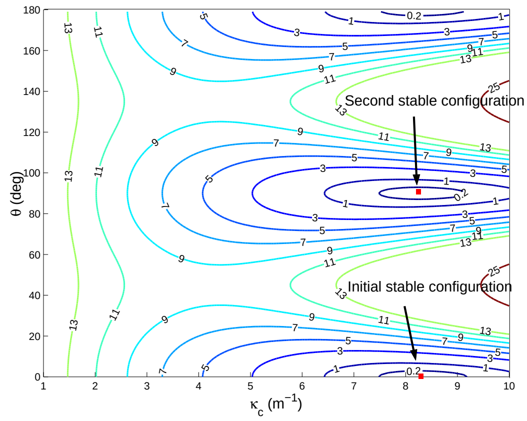

Fig. 4 Contour plot of strain energy of a space frame with lattice corresponding to Figure 1(a).

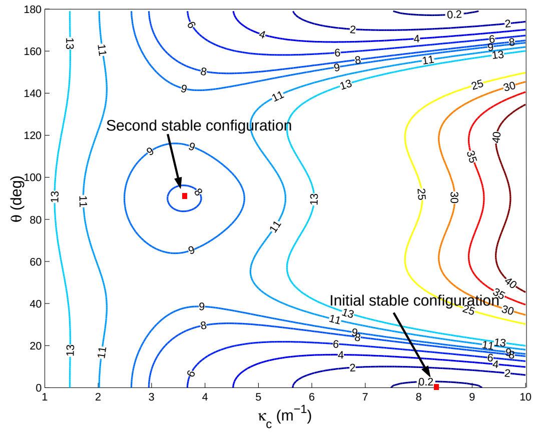

Fig. 5 Contour plot of strain energy of a space frame with lattice corresponding to Figure 1(b)

of diagonal bars (as shown in Figure 1) may have crucial effects on determining the second configuration. The energy analysis method has been applied on a series of space frames to investigate the effects of the axial stiffness of diagonal bars. The results are summarized in Table 2. Here the axial stiffness of diagonal bars is denoted as AE_1 while those of bars on the sides of squares are denoted as AE_2. The results clearly show that the second configuration is determined by the ratio of axial stiffness. With the increasing of the ratio, the curvature of second configuration increases up to the same value as the initial configuration. The rotating angle θ, however, remains the same.

Table 2 Effects of axial stiffness

AE_2/AE_1	0.5	1	2	3	5	10	∞
$\kappa_{c0}(m^{-1})$	8.3	8.3	8.3	8.3	8.3	8.3	8.3
$\theta_0(deg)$	45	45	45	45	45	45	45
$\kappa_{c_1}(m^{-1})$	2.7	3.7	4.7	5.5	6.3	7.2	8.3
$\theta(deg)$	90	90	90	90	90	90	90

Further research on additional space frame with other in-plane architectures has also been carried out.

The parameters of the models are set to the same as in Table 1. The analysis results show that for the space frames with the first two in-plane architectures (Figures 6(a) and (b)), the second stable configuration remains the same at $\kappa_{c_1} = 8.3$ m^{-1} and $\theta = 90°$. But for a model having the architecture shown in Figure 6(c), $\kappa_{c_1} = 5.4$ m^{-1} and $\theta = 79°$. For the model with architecture shown in Figure 6(d), $\kappa_{c_1} = 5.2$ m^{-1} and $\theta = 94°$. Combining the above results, it is demonstrated that the second configuration of this kind of bi-stable spaces frame is determined by the axial stiffness and the orientation of the diagonal bars. In other words, it is the diagonal bars that decide the shape of the second configuration.

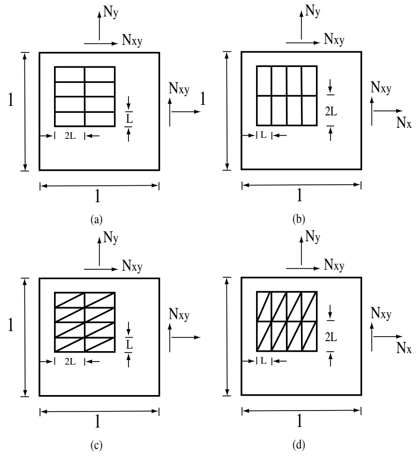

Fig. 6

Finite Element Modelling

Based on a series of assumptions, the above geometrical approach gives a preliminary assessment of the bi-stability of cylindrical space frames. A more realistic finite element model has been established here to confirm and test the bi-stability of space frame. The analysis is carried out with the ABAQUS [7] finite element package.

FE model establishment

We choose a space frame with an in-plane square lattice in Figure 1(a) as an example to run the FE analysis. The initial geometrical assessment has been carried out in the previous section and the contour plot of strain energy of the space frame is shown in Figure 4. In the FE model, the first thing to do is to determine the three dimensional architecture of the space frame because so far only the pattern in the top layer and bottom layer has been considered. In order to remove the internal mechanism in the structure, some triangular forms have been introduced to connect the top layer and bottom layer. A single three-dimensional lattice has been presented in Figure 7 to represent one cell unit of the space frame. A 12x12 double-layer cylindrical space frame model was then established with all the parameters specified as the same as those in the simple geometrical calculation presented in Table 1.

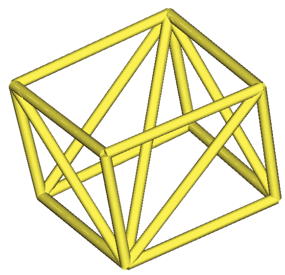

Fig. 7 Single units of a three-dimensional space frame

Simulation techniques

The coordinates of the nodes in the structure for each of the previously obtained configurations have been calculated. In the FE analysis, the structure has been switched from the initial configuration to the second one by applying temporary displacement constraints on 5 nodes in the structure. After the second configuration had been approached sufficiently closely, all temporary constraints are removed, and only six rigid body constraints are left, in order to test the stability of the structure in this new configuration. All the analysis is carried out as nonlinear geometrical simulations in ABAQUS.

Analysis results and discussion

Initially, truss element T3D2 was used in the analysis. The ABAQUS simulation shows that the space frame does gradually move to the second configuration under the displacement constraints of 5 nodes. Figure 8 shows the top view of the initial configuration of the space frame and Figure 9 shows the top view of the final configuration after the simulation is completed, corresponding to the second stable state. It is noted that, in the initial configuration, the axis of the cylinder on which the mid-plane of the space frame lies, is direction 2 , while in the second configuration it has rotated by 90° to direction 1. The diagram of the strain energy variation during the FE analysis is shown in Figure 10. The energy increases from initially zero energy to the intermediate high energy level and goes down to almost zero level at the end of STEP 12. After that, only 6 rigid body constraints are retained on the space frame and the energy level decreases to zero at the end of STEP 13. The ABAQUS results exactly match the results from the simple geometrical estimation.

Beam element B31 has also been used to model this space frame. The analysis procedure is the same as before. Figure 11 presents the energy change during

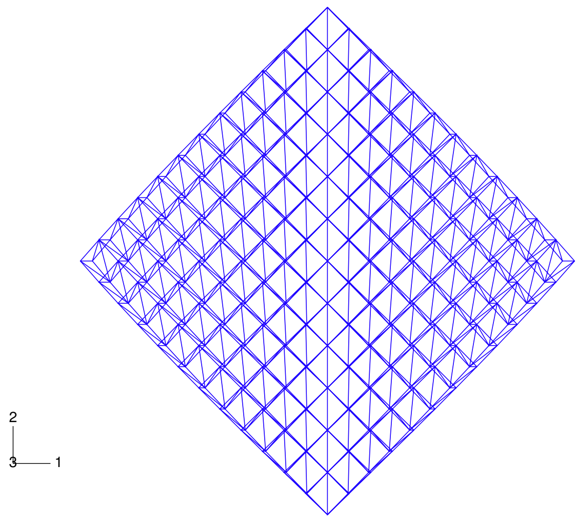

Fig. 8 Initial stable configuration.

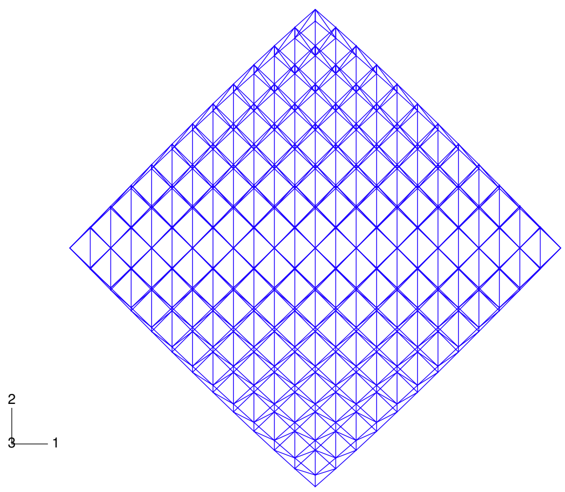

Fig. 9 Second stable configurations.

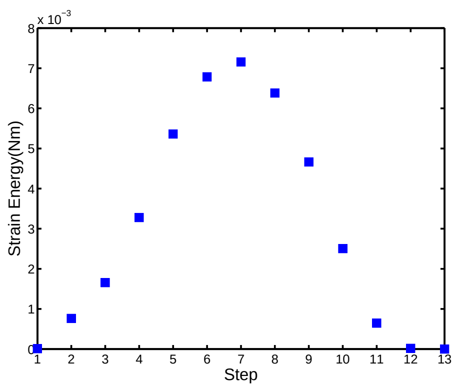

Fig. 10 Strain energy variation using truss elements.

the simulation. Because beam elements can endure bending deformation in addition to axial deformation, there is more energy stored in the structure when the displacement constraints have been applied. Therefore it can be observed in Figure 11 that the energy level at the second configuration is around half of the peak value. Also due to the beam elements, it is found that the ratio of diameter of bars to their length is crucial for the simulation to converge. Increasing this ratio may result in the analysis not converging. It is not reasonable to predict the stability of the second configuration in this case.

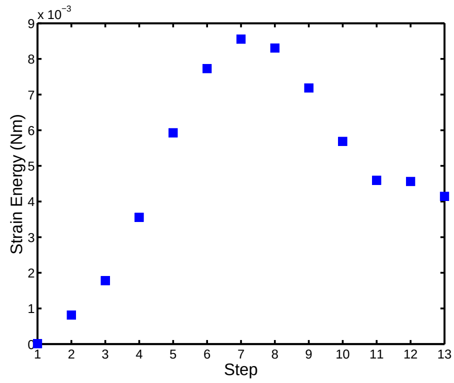

Fig. 11 Strain energy variation using beam elements.

Physical models

Research into the constructing of physical models of these bi-stable space frames has also been carried out in Cambridge. To achieve monolithic structures, rapid prototyping technology has been applied to build these models. Two sample models based on the information gained from ABAQUS analysis have been selected to be built. The structural parameters and material properties of model 1 are shown in Table 3. These parameters are selected to ensure the ABAQUS simulation can converge when both truss or beam elements are used. The analysis shows that during the change of the configuration, the maximum von Mises equivalent stress in the entire model is 20 MPa for truss elements and 11 MPa for beam elements. Both cases are lower than the maximum allowed stress 69 MPa.

Table 3 Parameters of model 1

Initial curvature of mid-plane	$\kappa_{c_0} = 8.3$ m^{-1}
Initial rotating angle	$\theta_0 = 45°$
Side length of square	$L = 8$ mm
Depth of space frame	$d = 6$ mm
Radius of cross section of bars	$r = 0.25$ mm
Name of mateial	SI50 (Resin)
Tensile modulus of material	$E = 2.48 - 2.69$ GPa
Tensile strength of material	48 50 MPa
Elongation @ break	$5.3 - 15.0\%$

Table 4 presents structural parameters and the material properties of model 2. During the simulation,

Table 4 Parameters of model 2

Initial curvature of mid-plane	$\kappa_{c_0} = 8.3\ \mathrm{m}^{-1}$
Initial rotating angle	$\theta_0 = 45°$
Side length of square	$L = 10$ mm
Depth of space frame	$d = 10$ mm
Radius of cross section of bars	$r = 0.5$ mm
name of mateial	Polyamide PA2200 (Nylon 12)
Tensile modulus of material	$E = 1.55 - 1.85$ Gpa
Tensile strength of material	$42 - 48$ Mpa
Elongation @ break	$15 - 25\%$

the maximum von Mises equivalent stress in the model is 42 MPa for truss elements and 48 MPa for beam elements. These also below the maximum allowable stress.

Conclusions and Future work

This paper has described double-layer cylindrical space frames which have two switchable stable configurations. The bi-stability of the structure was first analyzed by using a simple geometrical analysis. After that, a more refined Finite Element analysis was conducted, based on the initial estimation. This analysis has shown that the second configuration is indeed stable. Additionally, research on constructing physical models of this kind of bistable space frames has also been presented. Future work will involve investigation on space frames with other architectures. Further research must also be carried on to determine more suitable materials and manufacturing technologies for the construction of the bi-stable space frames.

Acknowledgments

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