

Experimenting with Measurement Error: Techniques with Applications to the Caltech Cohort Study*

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Abstract

Measurement error is ubiquitous in experimental work. It leads to imperfect statistical controls, attenuated estimated effects of elicited behaviors, and biased correlations between characteristics. We develop statistical techniques for handling experimental measurement error. These techniques are applied to data from the Caltech Cohort Study, which conducts repeated incentivized surveys of the Caltech student body. We replicate three classic experiments, demonstrating that results change substantially when measurement error is accounted for. Collectively, these results show that failing to properly account for measurement error may cause a field-wide bias leading scholars to identify “new” phenomena.

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1 Introduction

Measurement error is ubiquitous in experimental work. Lab elicitations of attitudes are subject to random variation in participants' attention and focus, as well as rounding due to finite choice menus. Despite the ubiquity of measurement error, fewer than 10% of experimental papers published in the last decade in leading economics journals mention measurement error as a concern (see Section 1.2 for details). Moreover, the tools for dealing with measurement error in experiments—most commonly improved elicitation techniques and multiple rounds—are relatively crude. This paper proposes a mix of statistical tools and design recommendations to handle measurement error in experimental and survey research.

At the heart of our approach is the combination of duplicate elicitations (usually two) of behavioral proxies and methods from the econometrics literature, particularly the instrumental variables approach to errors-in-variables (Reiersøl, 1941). While multiple elicitations would be impossible for a researcher using, say, the Current Population Survey, in experimental economics they are very easy to obtain.

The statistical tools discussed here deal with three types of inference breakdowns that arise from different uses of experimental proxies measured with error: as controls, as causal variables, or to estimate correlations between latent preference characteristics. We demonstrate the potential perils of measurement error, and the effectiveness of our techniques, using a unique new data set tracking behavioral proxies of the entire Caltech undergraduate student body, the Caltech Cohort Study (CCS), described in Section 2. We replicate within the CCS three classic and influential studies, and observe that 30–40% of variance in choices is attributable to measurement error. In all three of the studies we examine, accounting for measurement error substantially alters conclusions and implications.

First, we consider the most influential experimental study of the last decade, Niederle and Vesterlund (2007). That paper found that men are more likely to select into competition due to a preference for competitive situations that is distinct from risk attitudes and overconfidence. We replicate, as have many others, the fact that men choose to compete

more often than women. However, the gender gap in competition is well explained by risk attitudes and overconfidence once measurement error is properly accounted for. This is true both in our data, and that of the original study. Second, Friedman et al. (2014), summarizing their own research and that of many other scholars, find low correlations between different lab-based methods of measuring risk attitudes. As risk attitudes are fundamental to many economic theories, the failure to reliably measure them has troubling implications for lab experiments. In contrast, we find that many commonly used measures of risk attitudes are highly correlated once measurement error is taken into account. Third, we inspect the relationship between attitudes towards ambiguous and compound lotteries, following the setup of Halevy (2007). Ambiguity aversion is a rich area of theoretical exploration, and it is used to explain many behaviors: from equity trading to voting decisions. While Halevy finds a substantial correlation between attitudes towards compound risk and ambiguity, we find that accounting for measurement error leads to the conclusion that they are virtually identical.

As is well known, classical measurement error in a single variable biases estimates of effects towards zero. This attenuation bias is considered conservative, as it “goes against finding anything”—that is, it reduces the probability of false positives. However, as our results demonstrate, it may also lead to the identification of “new” effects and phenomena that are, in actuality, already reflected in existing research.

1.1 Simulated Examples

Here we present simulated examples to illustrate, for the unfamiliar reader, the problems created by measurement error, and summarize our approaches. In our first example, a researcher is interested in estimating the effects of a variable D —say, gambling—on some outcome variable Y —say, participation in dangerous sports—using an experimentally measured variable X —say, elicited risk attitudes—as a control. The model we use to simulate

data is

$$Y^* = X^* \text{ with } D = 0.5 \times X^* + \eta \text{ and } X = X^* + \nu, \quad (1)$$

where $\eta \sim \mathcal{N}[0, 0.9]$ (so the variance of D is ≈ 1), $X^* \sim \mathcal{N}[0, 1]$, and $\nu \sim \mathcal{N}[0, \sigma_\nu^2]$. That is, risk attitudes drive both gambling and participation in dangerous sports, but that attitude is measured through a lab-based elicitation technique that contains error. We assume the researcher only has access to $Y = Y^* - \varepsilon$, a noisy measure of Y^* , where $\varepsilon \sim \mathcal{N}[0, 1]$.

A diligent researcher would fit a regression model of the form

$$Y = \alpha D + \beta X + \epsilon, \quad (2)$$

hoping to control for the role of risk attitudes in the effect of gambling on participation in dangerous sports. Table 1 shows, from simulations, how the estimates, $\hat{\alpha}$ and $\hat{\beta}$, depend on how much measurement error there is in the variance of X , that is $\frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_{X^*}^2}$.

The estimated coefficients depend strongly on the amount of measurement error in X . With $N = 100$ —a typical sample size for an experiment—the coefficient on gambling $\hat{\alpha}$ becomes statistically significant in the average simulation when measurement error reaches approximately 1/3 of the variance in X . Statistically significant results are likely in such a setting, as we estimate that measurement error accounts for 30–40% of the variance of elicited proxies for risk attitudes—see (4) and surrounding text. Intuitively, false positives occur because the measurement error in X attenuates $\hat{\beta}$, allowing $\hat{\alpha}$ to pick up the variation in D related to X^* .

Depressingly, adding more observations does nothing to reduce the bias in the estimated coefficients. In fact, when $N = 1,000$, the approximate size of the CCS, $\hat{\alpha}$ appears statistically significant in the average simulation when measurement error accounts for only 10% of the variance in X . This emphasizes that issues with measurement error will not “wash out” once a study is large enough.

Correcting this is simple—elicit more controls. But questions remain: how many? and

Table 1: Simulated regressions of (2), with controls X measured with error. True model: $\alpha = 0, \beta = 1$.

Error as a percent of $\text{Var}[X]$:	0	10%	20%	30%	40%	50%
Panel A: $N = 100$						
$\hat{\alpha}$	0.00 (0.11)	0.06 (0.11)	0.11 (0.12)	0.16 (0.12)	0.21* (0.12)	0.26*** (0.12)
$\hat{\beta}$	1.00*** (0.12)	0.87*** (0.11)	0.75*** (0.11)	0.64*** (0.10)	0.54*** (0.10)	0.44*** (0.09)
Percent of time $\alpha = 0$ rejected at the 5% level with:						
1 noisy measure of X^*	5%	8%	15%	25%	37%	50%
5 noisy measures of X^*	5%	6%	6%	7%	9%	11%
10 noisy measures of X^*	5%	5%	5%	5%	6%	7%
20 noisy measures of X^*	5%	5%	5%	5%	5%	6%
Panel B: $N = 1,000$						
$\hat{\alpha}$	0.00 (0.03)	0.06* (0.04)	0.11*** (0.04)	0.16*** (0.04)	0.21*** (0.04)	0.26*** (0.04)
$\hat{\beta}$	1.00*** (0.04)	0.87*** (0.04)	0.75*** (0.03)	0.64*** (0.03)	0.54*** (0.03)	0.43*** (0.03)
Percent of time $\alpha = 0$ rejected at the 5% level with:						
1 noisy measure of X^*	5%	31%	81%	98%	100%	100%
5 noisy measures of X^*	5%	6%	11%	23%	42%	66%
10 noisy measures of X^*	5%	5%	7%	10%	16%	28%
20 noisy measures of X^*	5%	5%	5%	6%	8%	11%

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients and standard errors are averages from 10,000 simulated regressions.

how should these controls be used? If an experimenter is certain they can elicit the exact quantity they wish to control for (with measurement error), only two controls are necessary. One control should be entered linearly, and the other used as an instrument for the control. This will generate the proper coefficient on the control, and thus, on the variable of interest D . However, it is doubtful that this will ever be the case in practice. Even for a simple control like risk aversion, there are multiple, imperfectly correlated, elicitation methods, as we discuss in Section 4. Including multiple controls for multiple measures of multiple

Table 2: Simulated correlations when X and Y are measured with error ($N = 100$). True model: $\text{Corr}[X^*, Y^*] = 1$.

Error as a percent of $\text{Var}[X]$ and $\text{Var}[Y]$:	0	10%	20%	30%	40%	50%
$\widehat{\text{Corr}}[X, Y]$	1.00 (0.00)	0.90*** (0.02)	0.80*** (0.04)	0.70*** (0.05)	0.60*** (0.06)	0.50*** (0.08)
$\widehat{\text{Corr}}[\mathbb{E}[X], \mathbb{E}[Y]]$	1.00 (0.00)	0.95*** (0.01)	0.89*** (0.02)	0.82*** (0.03)	0.75*** (0.04)	0.66*** (0.06)
ORIV $\widehat{\text{Corr}}[X, Y]$	1.00 (0.00)	1.00 (0.01)	1.00 (0.02)	1.00 (0.04)	1.00 (0.06)	1.00 (0.10)

Notes: ***, **, * denote statistically significantly different from 1 at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients and standard errors are averages from 10,000 simulated regressions.

behaviors may not be practical.

Thus, in Section 3, we explore several different ways of including controls. First, we include them linearly, as suggested by Table 1. Then, we show that principal component analysis allows for a small, but informative, set of controls—preserving degrees of freedom. Finally, we elicit each control twice, and use the duplicate observation as an instrument. These different approaches all lead to the same conclusion—the gender gap in competitiveness can be explained by risk attitudes and overconfidence, although Niederle and Vesterlund (2007) concluded that it was a distinct phenomenon.

The problem of measurement error biasing coefficients is particularly acute when researchers estimate correlations between X and Y , as shown by the simulated results in Table 2. In this table, we vary the proportion of measurement error in both variables. Even a bit of measurement error causes significant deviations from the true correlation of 1. As measurement error accounts for 30–40% of the variation in our elicitations, it is extremely unlikely one would ever estimate a correlation close to 1, even if that were the true value. As in Table 1, increasing N does not affect point estimates, but shrinks standard errors.¹

To correct for measurement error, we expand on traditional instrumental variables ap-

¹Note that the standard errors are smaller in Table 2, as $\text{Var}[\varepsilon]$, set to 1 in all columns of Table 1, now varies across the columns: starting at 0 in the first column, and climbing up to 1 in the final column.

proaches to errors-in-variables. Our approach, which we call *Obviously Related Instrumental Variables* (ORIV) uses duplicate elicitations of X and Y as instruments, and produces an estimator which is more efficient than standard instrumental variable techniques. Specifically, we obtain duplicate measures of X , denoted X^a and X^b , which are both proxies for X^* that are measured with error. If measurement error in the two elicitations is orthogonal—as we assume—then the predicted values $\hat{X}^a(X^b)$ from a regression of X^a on X^b contain only information about X^* . We then use a stacked regression to combine the information from both $\hat{X}^a(X^b)$ and $\hat{X}^b(X^a)$, resulting in an efficient use of the data.² ORIV is easily extended to allow for multiple measures of the outcome Y . This is particularly useful in estimating correlations, where there is no clear distinction between outcome and explanatory variables, and measurement error in either can attenuate estimates.³

ORIV produces consistent coefficients, correlations, and standard errors. This is in contrast to one common way experimenters deal with multiple noisy elicitations: averaging. As can be seen from Table 2, while averaging reduces bias, it still leads to incorrect conclusions in the presence of small amounts of measurement error.

We apply ORIV, in Section 4, to show that various risk elicitation methods are more correlated than previously thought. We further use this technique to show, in Section 5, that ambiguity aversion and reaction to compound lotteries are very close to perfectly correlated—once we account for measurement error. This leads us to conclude, in Section 6, that failing to correct for measurement error has led the field to over-identify “new” phenomena.

1.2 Related Literature

Mis-measurement of data has been an important concern for statisticians and econometricians since the late 19th century (Adcock, 1878). Indeed, estimating the relationship between

²If measurement error is positively correlated across elicitations, then instrumented coefficients will still be biased downwards, although less so than without instrumenting. In our experimental design, we tried to weaken any possible correlation by varying the choice parameters, the grid of possible responses, and so on. See Section 2.1 and Section 4.4.2 for details and discussion.

³ORIV is equivalent to using all valid moment conditions in the generalized method of moments (GMM), however it is simpler and more transparent. See Appendix A.6 for a detailed comparison of ORIV and GMM.

two variables when both are measured with error is a foundational problem in the statistics literature (Koopmans, 1939; Wald, 1940). The use of instrumental variables to address the classical errors-in-variables problem was proposed by Reiersøl (1941) (see Hausman, 2001, for a review). This was first applied in economics by Friedman (1957) to estimate consumption functions. Since then, instrumental variables have been used to account for measurement error in an assortment of fields including medicine, psychology, and epidemiology.

The experimental literature has considered noise in lab data and its consequences, going back to at least Kahneman (1965). Nonetheless, in the decade from 2006–2015 only 9% of the 283 experimental (field and lab) papers in top 5 economics journals explicitly tried to deal with measurement error.⁴ One-fifth of these papers either used an experimental design aimed at reducing noise, or averaged multiple elicitations, a technique that may do little to reduce bias, as shown in Section 1.1. About one-half of these papers estimate structural models of participants’ “mistakes.” In particular, two-fifths estimate *Quantal Response Equilibrium* models.⁵ Most of the remaining papers use indirect methods to deal with noise, such as the elimination of outliers, or the informal derivation of additional hypotheses about the effects of noise, which are then tested.⁶ Our paper is, to our knowledge, the first to offer simple, yet general, experimental techniques for mitigating the effects of measurement error.

Many of the issues in experimental work due to measurement error are present in survey research as well. For example, Bertrand and Mullainathan (2001) and Bound, Brown, and Mathiowetz (2001) highlight the potential perils of measurement error in survey research

⁴We examined full, refereed papers published in *The American Economic Review*, *Econometrica*, *Journal of Political Economy*, *The Quarterly Journal of Economics*, and *The Review of Economic Studies*.

⁵These posit a structural model in which participant mistakes are inversely related to the payoff losses they generate: see Goeree, Holt, and Palfrey (2016) for details and a review.

⁶Some particularly innovative papers of this type are worth mentioning. Battalio et al. (1973) show that even small reporting errors can lead to a rejection of the generalized axiom of revealed preferences. Castillo, Jordan, and Petrie (2015) posit a structural model of measurement error in children’s risk elicitations. Coffman and Niehaus (2015) adjust for measurement error in self-interest and other-regard by projecting both on a common set of explanatory variables. Ambuehl and Li (2015) use instrumental variables to control for measurement error in one of their analyses. Like our paper, all of these only consider classical measurement error. However, by designing experiments to produce more specific forms of measurement error, experimenters may gain greater traction on their problem of interest. Standard texts on the statistics of measurement error, such as Buonaccorsi (2010), may be of use in future research.

in economics and social psychology. Bound, Brown, and Mathiowetz (2001) also note the potential usefulness of instrumental variable approaches. The closest paper to ours in that literature is Beauchamp, Cesarini, and Johannesson (2015), which considers measurement error in survey-based risk elicitations. It uses a latent variable model that allows inferences about the component of measured risk attitudes that is not due to measurement error. It emphasizes, as we do, the ubiquity of measurement error, and the paucity of concern about it.⁷

2 The Caltech Cohort Study

We administered an incentivized, online survey to the entire undergraduate student body of Caltech in the Fall of 2013 and 2014, and the Spring of 2015. The surveys included incentivized tasks designed to elicit an array of behavioral attributes. It also included a set of questions addressing students' lifestyle and social habits.⁸

The data used in this paper are from the Fall 2014 and Spring 2015 installments. In the Fall of 2014, 92% of the student body (893/972) responded to the survey. The average payment was \$24.34. In the Spring of 2015, 91% of the student body (819/899) responded to the survey. The average payment was \$29.08. The difference in average payments across years was due to the inclusion of several additional incentivized items in 2015.⁹ Of those who had taken the survey in 2015, 96% (786/819) also took the survey in 2014. As Section 4 requires data from both surveys, for consistency we use this subsample of 786 throughout.

There are several advantages to using the CCS to address questions of measurement error. The large size of the study allows us to document the non-existence of certain previously identified “distinct” behaviors with unusual precision. Furthermore, the inflation of standard

⁷In a different context, Aguiar and Kashaev (2017) suggest a nonparametric statistical notion of rationalizability of a random vector of prices and consumption streams when there is measurement error in consumption levels reported in surveys. They use this test to assess standard exponential time discounting.

⁸For screenshots of the 2015 survey, go to: leeatyariv.com/ScreenshotsSpring2015.pdf.

⁹The number of overall students was substantially lower in the Spring of 2015, as about 50 students departed the institute due to hardship or early graduation. Further, we did not approach students who had spent more than four years at Caltech, accounting for approximately 25 students.

errors that comes with using instrumental variables techniques does not threaten the validity of our inferences. Last, unlike most experimental settings, there is little concern about self-selection into our experiments from the participant population, due to our 90%+ response rates (see Snowberg and Yariv, 2018, and references therein). Thus, the issues we identify are due solely to measurement error, and not due to a small sample or self-selection.

Nonetheless, Caltech is highly selective, which may cause concern that the overall population is different from the pool used in most lab experiments. Three points should mitigate this concern. First, the raw results of the replications are virtually identical to those reported in the original papers. Second, responses from our survey to several standard elicitations—of risk, altruism in the dictator game, and so on—are similar to those reported in several other pools (see Appendix D for details). Third, while top-10 schools account for 0.32% of the college age population in the U.S., top-50 schools enroll only 3.77% of that population (using the *U.S. News and World Report* rankings). Thus, there seems to be little cause for concern that our participant pool is more “special” than that used in many other lab experiments. As the results reported in this paper are replications of other studies, these points suggest that our conclusions are likely due to our more sophisticated treatment of measurement error, rather than an artifact of the participant population.

2.1 Measures Used

Our results deal with a subset of the measured attributes, which we detail here. Question wordings can be found in Appendix E. Throughout, 100 survey tokens were valued at \$1.

2.1.1 Overconfidence

We break overconfidence into three categories, following Moore and Healy (2008). These measures are used in Section 3 as controls.

Overestimation and Overplacement: Participants complete two tasks: a five-question cognitive reflection test (CRT; see Frederick, 2005), and five Raven’s matrices (Raven, 1936). After each block of questions, each participant is asked how many they think they answered correctly. This, minus the participant’s true performance, gives two measures of overestimation. Each participant is also asked where they think they are in the performance distribution of all participants. This, minus the participant’s true percentile, gives a measure of overplacement. This gives three co-linear measures: performance, expected performance, and overconfidence; any two can be used to control for confidence and overconfidence.

Overprecision: Participants are shown a random picture of a jar of jellybeans, and asked to guess how many jellybeans the jar contains. They are then asked—on a six point qualitative scale from “Not confident at all” to “Certain”—how confident they are of their guess. This is repeated three times. Following Ortoleva and Snowberg (2015), each of these measures is interpreted as a measure of overprecision.

Perception of Academic Performance: A final measure of overconfidence asks participants to state where in the grade distribution of their entering cohort they believe they would fall over the next year. This is treated as a measure of confidence in placement.

2.1.2 Risk

Risk measures are used in Section 3 as controls, and in Section 4 as an outcome of interest. Further, the Risk MPL described below is used as an outcome of interest in Section 5.

Projects: Following Gneezy and Potters (1997), participants are asked to allocate 100 or 200 tokens between a safe option, and a project that returns some multiple of the tokens with probability p , otherwise nothing. In Fall 2014, two projects were used: the first returning 3 tokens per token invested with $p = 0.4$, and the second returning 2.5 tokens with $p = 0.5$. In the Spring of 2015, the first project was modified to return 3 tokens with $p = 0.35$.

Qualitative: Following Dohmen et al. (2011), participants are asked to rate themselves, on a scale of 0–10, in terms of their willingness to take risks. We use the Spring 2015 elicitation as a duplicate measure of the Fall 2014 elicitation.

Lottery Menu: Following Eckel and Grossman (2002), participants are asked to choose between six 50/50 lotteries with different stakes.¹⁰ The first lottery contains the same payoff in each state, and thus corresponds to a sure amount. The remaining lotteries contain increasing means and variances, allowing for an estimation of risk aversion.

Risk MPL: Participants respond to two Multiple Price Lists (MPLs) that ask them to choose between a lottery over a draw from an urn, and sure amounts. The lottery pays off if a ball of the color of the participant’s choosing was drawn. The first urn contains 20 balls—10 black and 10 red—and paid 100 tokens. The second contains 30 balls—15 black and 15 red—and paid 150 tokens.¹¹

2.1.3 Ambiguity and Compound Lotteries

Reactions to ambiguous and compound lotteries are considered in Section 5.

Compound MPL: This follows the same protocol as the Risk MPLs described above, except participants are told that the number of red balls would be uniformly drawn between 0 and 20 for the first urn, and between 0 and 30 for the second. As this is a measure of risk attitudes, it is also used as a control in Section 4.

Ambiguous MPL: This elicitation emulates the standard Ellsberg (1961) urn. It follows the same protocol as the two other MPLs. Participants are informed that the composition of the urn was chosen by the Dean of Undergraduate Students at Caltech.

¹⁰The variant we use comes from Dave et al. (2010).

¹¹In order to prevent multiple crossovers, the online form automatically selected the lottery over a 0 token certainty equivalent, and 100 tokens over the lottery. Additionally, participants needed only to make one choice and all other rows were automatically filled in to be consistent with that choice.

To reduce instructions, both of the MPLs for a given attitude (Risk, Compound, Ambiguity) are run sequentially, in random order. These three blocks are spread across the survey, and which block is given first, second, and third is randomly determined. As no order effects were observed, we aggregate results across the different possible orderings.

3 Mis-specified Controls and Measurement Error

To make the claim that an estimated effect is independent of other factors, many studies attempt to control for those other factors. If they are measured with error, one control, or even a few, may be insufficient to reliably assert the claim, as illustrated in Section 1.1. Here we show that properly dealing with controls measured with error has important substantive consequences. We do so by replicating the competitiveness and gender study of Niederle and Vesterlund (2007)—henceforth NV—within the CCS. Like NV, we find a robust difference in the rates at which men and women compete. However, NV conclude that:

Including these controls [for overconfidence, risk, and feedback aversion], gender differences are still significant and large. Hence, we conclude that, in addition to gender differences in overconfidence, a sizable part of the gender difference in tournament entry is explained by men and women having different preferences for performing in a competitive environment.

In contrast, we show that the gender gap is well explained by risk aversion and overconfidence.

Using the notation of Section 1.1, measurement error in X , in this case controls for risk aversion and overconfidence, can result in a biased estimate of the coefficient on D , in this case gender, on competition Y . To understand this intuitively, consider the model in (1) where $Y^* = X^*$, D and X^* are correlated, and $X = X^* + \nu$ is a noisy measure of X^* .¹² For illustration, consider an extreme case where the variance of ν is very large, so that X is almost entirely noise. Ignoring that noise in X , standard regression analysis could lead to

¹²Note that in many of our applications, these variables will be binary or ordered multinomials. In those cases, we still model the corresponding latent variable (X^* or Y^*) as continuous and the responses (X or Y) as stochastic. Measurement error then affects the probability that a given discrete choice is made.

the erroneous conclusion that Y and D are correlated, even when controlling for X .¹³

To put this in terms of our substantive example, it is well known that overconfidence is correlated with gender (see, for example, Moore and Healy, 2008), and, depending on the elicitation method, risk aversion may be correlated with gender as well (see Holt and Laury, 2014; Charness, Gneezy, and Imas, 2013, for surveys, as well as our discussion in Section 4.6). Thus, if competitiveness is driven by overconfidence or risk aversion, mis-measurement of these traits will lead to an overestimate of the effect of gender.

What can be done to mitigate this issue? There are several approaches. The first is to include multiple measures for each of the possible controls X . This approach reduces the effects of measurement error, while helping to ensure that elicited controls cover the potentially different aspects of the behavioral attribute being controlled for. However, this may have two shortcomings. First, it may come at the cost of too many degrees of freedom. Second, without a large number of controls for each aspect, this approach will not entirely eliminate the effects of measurement error. We therefore consider two further approaches: including principal components of the multiple controls, and instrumenting each control with a duplicate. This final approach—spanning the space of different aspects of the behavioral attribute, while instrumenting each of these aspects—is preferable whenever feasible.

3.1 Measuring Competitiveness in the Caltech Cohort Study

Part of the Spring 2015 survey mimicked the essential elements of NV’s design. First, participants had three minutes to complete as many sums of five two-digit numbers as they could. Participants were informed that they would be randomly grouped with three others at the end of the survey. If they completed the most sums in that group of four, they would receive 40 experimental tokens (or \$0.40) for each sum correctly solved, and would otherwise receive no payment for the task. Ties were broken randomly. As in NV, at the end of this task, participants were asked to guess their rank, from 1 to 4, within the group of four

¹³It is well known that measurement error in left-side variables may bias estimated coefficients in discrete choice models, see Hausman (2001). We use linear probability models as Y may also be measured with error.

participants. They were paid 50 tokens (or \$0.50) if their guess was correct.

Next, in parallel to the central task of NV’s design, participants were told they would have an additional three minutes to complete sums. But, before doing so, they chose whether to be paid according to a piece-rate scheme or a tournament. The piece-rate scheme paid 10 tokens for each correctly solved sum. The tournament had a similar payment scheme to the first three-minute task. However, a participant’s performance in the tournament would be compared to the performance of three randomly chosen participants in the *first* task.¹⁴ Otherwise, the payment structure was identical to that in the first task.

There are a few ways in which our implementation differs from NV’s:

Time and Payments: We gave participants three, rather than five, minutes to complete sums. Per-sum payments were scaled down by a factor of four. As with the rest of the CCS, participants were paid for all tasks, rather than a randomly selected one. This could have caused participants to hedge by choosing the piece-rate scheme.

Grouping of Participants: In NV, participants were assigned to groups of four where they could visibly see that there were two men and two women. This created an imbalance in the expected number of female competitors: $2/3$ of the group for men, $1/3$ for women. In the CCS, we randomly selected groups for each of the two tasks separately after the survey was administered. Thus, both genders faced the same expected profile of competitors.

Experimental Setting: Our tasks are on a survey, whereas NV use a lab setting. We ran our survey several months later in a lab environment to 98 Caltech students. We saw very similar results, but with larger standard errors due to the smaller sample.

Additional Tasks: NV include two additional parts: a preliminary task allowing participants to try out the piece-rate scheme, and a final choice that allows participants to select

¹⁴This ensured that the participant would not need to be concerned about the motivation, or other characteristics, that might drive someone to compete in the second task.

either an additional piece-rate or tournament payment scheme for their performance in the preliminary task. This final choice served as a control for risk aversion and overconfidence. As the CCS has multiple other controls for both of these traits, we omitted these two parts to reduce complexity and length.

The first three of these factors would only change interpretation if they affected men and women differently. However, as we replicate the gender gap in tournament entry found by NV, these do not seem to be of particular importance. The final factor implies that we cannot use an analogous control to that of NV's with the CCS data. Nonetheless, using NV's data and an analysis accounting for measurement error in their control, we find that the gender gap in tournament entry can be explained by risk aversion and overconfidence (see Section 3.3). More details about our, and NV's, implementation can be found in Appendix E.1.

3.2 Gender, Competition, and Controls

This subsection analyzes the extent to which risk aversion and overconfidence drive the gender gap in competitiveness. Table 3 summarizes specifications meant to illustrate different points. These are linear probability models, and hence, the coefficient on gender is directly interpretable as the percentage-point gap between men and women in choosing to compete.¹⁵

The first column shows the baseline difference in competition: Women choose tournament incentives 21.4% of the time, while men choose them 40.4% of the time, for a difference of 19.0 percentage points. This difference is highly statistically significant. While these numbers are somewhat lower than those reported in NV, their relative sizes are quite similar. The second column controls for participants' performance and estimates of their own rank linearly, as in NV's main specification. Similar to their results, the inclusion of these controls reduces the coefficient on gender by approximately 1/3.

There is, however, a non-linear relationship between expected rank and perceived prob-

¹⁵As noted in Footnote 13, discrete choice models may produce biased estimates of coefficients when the left-side variable is measured with error. Nonetheless, in our data, Probit and Logit specifications produce almost identical levels of statistical significance as in Table 3.

ability of victory in a competition.¹⁶ Therefore, the third column enters participants' subjective ranks non-parametrically, by including a dummy variable for each possible response (three categories). This estimation confirms that the effect of perceived rank in a competition is, indeed, non-linear, although the coefficient on gender remains unchanged.¹⁷ The third column also enters performance non-linearly (29 and 26 categories, respectively), as there is also a non-linear relationship between performance and competition. The coefficient on gender in the third column is lower than in the second.¹⁸

The fourth column introduces controls for risk aversion and overconfidence. Two (non-randomly) selected controls are entered, one for each attribute. This does not affect the gender coefficient, despite both controls having statistically significant coefficients. Two different (non-randomly) selected controls are entered in Column 5. Doing so cuts the coefficient on gender by more than half, and renders it statistically insignificant. Taken together, these columns show that the statistical significance of controls is not a good indicator of whether a trait is fully controlled for. Moreover, they suggest that measurement error in the controls themselves allows for the perception of competitiveness as a separate trait.

We note that these conclusions are not driven by our unusually large sample size. If anything, the size of our dataset helps reduce standard errors and identify weak effects. To see this, we draw a random sample of 40 women and 40 men (the size and gender composition of NV's experiment) from our data 10,000 times, and regress the competition decision on two overconfidence controls, two risk controls, and perceived rank in the first competition task. The coefficient on gender is significant at the 1% level 2.2% of the time, at the 5% level 7.6% of the time, and at the 10% level 13% of the time.

¹⁶If an individual believes a random participant is inferior to her with probability p , then her probability of winning is p^3 . Furthermore, her expected rank is given by $\sum_{i=0}^3 (i+1) \binom{3}{i} (1-p)^i p^{3-i} = 3(1-p) + 1$. Thus, with a reported rank of r (ignoring rounding), the probability of winning the competition is $(\frac{4-r}{3})^3$.

¹⁷Rates of competition are 65.6% for participants who predicted they would come in first (in a random group of 4), and 31.4%, 15.3%, and 5.0% for participants predicting they would come in second, third, and fourth (last), respectively. The distribution of guessed ranks differs from that reported in NV: our participants were better calibrated, and this likely resulted in the lower observed rates of tournament entry.

¹⁸This is entirely driven by including performance in the first task non-parametrically, as there are small differences in male and female performance in this task, as shown in Figure E.2 of Appendix E.1.

Table 3: Gender, competition, and controls

Dependent Variable	Chose to Compete ($N = 783$)							
Male	0.19*** (.034)	0.13*** (.030)	0.11*** (.031)	0.11*** (.031)	0.048 (.033)	0.050 (.034)	0.041 (.033)	0.0063 (.054)
Gussed Tournament Rank	-0.15*** (.017)	$F = 29$ $p = 0.00$	$F = 28$ $p = 0.00$	$F = 23$ $p = 0.00$	$F = 21$ $p = 0.00$	$F = 21$ $p = 0.00$	$F = 21$ $p = 0.00$	$\chi^2_3 = 8.8$ $p = 0.04$
Tournament Performance	0.086*** (.020)	$F = 1.6$ $p = 0.02$	$F = 1.6$ $p = 0.03$	$F = 1.6$ $p = 0.03$	$F = 1.6$ $p = 0.03$	$F = 1.5$ $p = 0.05$	$F = 1.5$ $p = 0.05$	$\chi^2_{30} = 36$ $p = 0.21$
Performance Difference	-0.021 (.017)	$F = 1.4$ $p = 0.09$	$F = 1.5$ $p = 0.07$	$F = 1.4$ $p = 0.11$	$F = 1.4$ $p = 0.11$	$F = 1.4$ $p = 0.11$	$F = 1.4$ $p = 0.11$	$\chi^2_{25} = 27$ $p = 0.34$
Risk Aversion: MPL #1		0.042*** (.015)						
Overplacement: CRT		0.026* (.015)						
Risk Aversion: Project #2				0.067*** (.016)				
Perceived Performance (pctile.): CRT				-0.042*** (.016)				
All Risk Aversion Controls						$F = 4.9$ $p = 0.00$		
All Overconfidence Controls						$F = 1.8$ $p = 0.05$		
First 5 Principal Components						$F = 37$ $p = 0.00$		
Instrumental Variables								$\chi^2_7 = 24$ $p = 0.00$
Adjusted R^2	0.038	0.23	0.26	0.27	0.28	0.29	0.22	

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients and standard errors on all non-dichotomous measures are standardized. F-statistics and p-values are presented when variables are entered categorically rather than linearly. There are 3 categories for Gussed Competition Rank, 29 categories for Tournament Performance, 26 categories for Performance Difference, 5 variables for Risk Aversion Controls, and 12 variables for overconfidence controls.

The sixth column enters all available controls for risk (6 controls) and overconfidence (an additional 12 controls). The coefficient on gender is relatively unchanged. If we enter these controls separately, we find that much of the decrease in this coefficient, compared with the third column, is due to risk controls. We revisit the relationship between gender and risk aversion in Section 4.6.

The number of controls in the sixth column (76, including categorical controls for performance) approaches the number of data points in a normally sized study—such as NV, which had 80 participants. Thus, we examine ways to preserve degrees of freedom. The simplest is to perform a principal components analysis of all 76 of the controls. In this case, entering just the first 5 principal components produces a very similar point estimate to entering all 76 controls. More on this technique can be found in Appendix B.1.

As discussed in Section 1.1, the potential bias in the gender coefficient comes from the fact that the coefficients on the noisy controls—assumed to be positively correlated with both gender and competitive behavior—are biased towards zero. Thus, in the final column, we instrument the risk aversion and overconfidence controls for which we have multiple elicitations. This approach combines consistent estimates of the coefficients on controls, while still ensuring we span the space of possible aspects of risk aversion and overconfidence in our data. While the point estimate of the gender coefficient is consistent, it is accompanied by higher standard errors that come with an instrumental variables specification.

When dealing with measurement error, a key advantage of the experimental approach is the ability to design one’s own controls. Using enough controls to span the space of aspects of a behavioral attribute, with a duplicate elicitation of each as an instrument, is preferable, if feasible. However, as mentioned, limits on participants’ time and attention may impose a constraint on the number of controls that can be elicited. When this constraint binds, an experimenter should think carefully about the trade-off between measuring an additional aspect and controlling for it perfectly via instrumentation. We return to multiple elicitations and instrumental variables strategies in Sections 4 and 5. Before doing so, we examine how

Table 4: Re-Analysis of Niederle and Vesterlund’s Data

Dependent Variable:	Choose To Compete (Y_i^a)			$Y_i^a - Y_i^b$	
Male	0.37*** (.11)	0.27*** (.11)	0.21** (.10)	0.075 (.12)	0.053 (.12)
Tournament Performance	0.016 (.019)	-0.003 (.019)	-0.012*** (.018)		-0.037 (.023)
Performance Difference	0.016 (.023)	-0.005 (.023)	0.012 (.023)		0.056** (.027)
Gussed Tournament Rank		-0.24*** (.066)	-0.20*** (.066)		-0.11 (.080)
Y_i^b			0.27** (.11)		
Adjusted R^2	0.13	0.24	0.29	0.00	0.054

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients and standard errors on all non-dichotomous measures are standardized. $N = 80$ for all regressions.

to correct for measurement error when using specially designed controls.

3.3 Using Designed Controls

NV control for risk aversion and overconfidence with another tournament entry choice. Namely, in the last stage of their experiment, participants are given a second opportunity to be paid for their performance in the piece-rate task from the beginning of the experiment. They can choose to be paid again as a piece rate, or to enter their performance into a tournament. The clever idea behind this additional choice is that it controls for all aspects determining tournament entry not directly related to a preference for competing—explicitly, risk aversion and overconfidence. Using their data, we show that accounting for measurement error in this control generates different conclusions than those of NV. Henceforth, we refer to the choice in the main task as Y_i^a , and that in the final task as Y_i^b .

NV regress Y_i^a on gender, performance, guessed tournament rank, and Y_i^b , as in the third column of Table 4. This reduces the coefficient on gender compared to their main specifi-

cations, which are displayed in the first two columns.¹⁹ However, suppose Y_i^b is positively correlated with both gender and competition, but contaminated with classical measurement error. This will bias the coefficient on Y_i^b downwards, and the coefficient on gender upwards.

As Y_i^b is designed to measure every part of the tournament entry choice *except* a preference for competition, it should enter the regression with a coefficient of 1. This can be implemented by regressing $Y_i^a - Y_i^b$ on gender, which produces an unbiased estimate of the effect of gender on tournament entry, controlling for Y_i^b . Intuitively, the only difference between these two variables is, by construction, a desire to compete. To see if this desire is correlated with gender, it should be regressed on gender. Doing so results in an insignificant coefficient on gender of 0.075. The inclusion of additional controls on the right side reduces the coefficient even further.²⁰

3.4 Substantive Interpretation

Our analysis shows, using both new data, and data from NV, that although men are more likely to select into competition, this is not due to a distinct *preference* for performing in a competitive environment. Rather, it is driven by differences in risk aversion and overconfidence. It is important to note that using multiple controls for risk aversion and overconfidence, or using principal components, does not allow us to say how important either of these factors is in explaining competition, only that together they explain much of the effect.²¹

Our results do not, by any means, imply that it is better to elicit risk attitudes and overconfidence instead of competitiveness. There is a tradeoff: competition is potentially more directly relevant for an array of economically important decisions, and is definitely a more parsimonious measure. Indeed, competition has been shown to predict several interesting

¹⁹We thank Muriel Niederle and Lise Vesterlund for generously sharing their data. The coefficients in Table 4 differ from NV due to our use of OLS—which is preferred to Probit for reasons described in Footnote 13. However, p-values are very similar.

²⁰The inclusion of additional controls should make the test more efficient in small samples. A formal exposition of this point, and other details of this sub-section, can be found in Appendix B.2.

²¹Van Veldhuizen (2016) uses clever experimental design to add refined versions of NV’s final task, and specifications following the previous subsection, to tease these effects apart.

behaviors, such as choice of college major (see, for example, Buser, Niederle, and Oosterbeek, 2014). However, risk aversion and overconfidence feature in many theories, and are therefore of potential use in bringing theory to bear on gender differences.

There are also practical considerations. NV report that their experiment had an average runtime of approximately 45 minutes. By using two tasks (rather than four) and allowing participants to solve sums for three minutes (rather than five), we reduced the average time participants spent on the competition task to around 8 minutes. Naturally, eliciting multiple measures of risk and overconfidence may be time consuming as well. Nevertheless, our entire survey had an average runtime of less than 30 minutes, including the competition task.

4 Measurement Error on Both Sides

It is well known that measurement error in outcome, or dependent, variables does not bias estimated relationships, although it increases standard errors. Measurement error in explanatory, or independent, variables is a much more serious problem, biasing estimated coefficients and distorting standard errors. This leads to an improper understanding of the relationship between explanatory variables and outcomes. These problems are compounded when estimating a correlation: the distinction between outcome and explanatory variables is blurred, and classical measurement error in either biases estimates towards zero. We introduce a simple method, *Obviously Related Instrumental Variables* (ORIV)—which is more efficient than standard instrumental variables techniques—to overcome these issues. We apply this technique to the estimation of the correlation between different measures of risk attitudes in this section, and between risk and ambiguity aversion in the next. The discussion in this section focuses on implementation, with the formal properties of the estimators developed in Appendix A.

4.1 Risk Elicitation Techniques

There is a substantial experimental literature assessing the validity of common experimental techniques for eliciting attitudes towards risk and uncertainty (see the literature review in Holt and Laury, 2014). These studies often elicit risk attitudes in the same set of participants using different techniques. By using a within-participant design, researchers attempt to understand technique-driven differences in elicited proxies for risk aversion. This type of work has generally found small correlations between different techniques, making it difficult to study the individual correlates of risk preferences. The literature concludes that risk elicitation is a “risky business”—the pun is not ours, see Friedman et al. (2014) for a survey.

However, none of the studies on which this conclusion is based account for measurement error. In what follows, we inspect several commonly used risk-elicitation techniques, and estimate their within-participant correlations using an instrumental variables strategy to account for measurement error. This generates much higher within-participant correlations than previously reported. Moreover, the corrected correlations suggest that elicitation techniques fall into one of two sets: those that elicit certainty equivalents for lotteries, and those that elicit allocations of assets between safe and risky options. The latter category exhibits greater correlations with other measures, and more stability over time. Further, elicitations based on allocation decisions display substantial gender effects—which are consistent with investment behavior in the field—while certainty equivalent elicitation do not.

This section uses four measures of risk as described in Section 2.1: Qualitative, Risk MPL, Project, and Lottery Menu. Before proceeding, we note how we transform variables for comparison. First, when using two measures from the same form of elicitation, we put them on a common scale. In particular, the certainty equivalents from the 30-ball urn Risk MPL (which go up to 150) are divided by 1.5 to be on the same scale as the certainty equivalents from the 20-ball urn Risk MPL (which go up to 100).²² Second, when comparing objects like estimated CRRA coefficients or derived certainty equivalents, these are also put on the same scale. For example, when examining the relationship between certainty

²²This implicitly assumes a CRRA utility function.

equivalents from the Risk MPLs and Projects—the former allowing for risk-loving answers and the latter not—those who give risk-loving answers in the Risk MPLs are re-coded to give a risk-neutral answer. Without this censoring, results are qualitatively similar.²³

4.2 A First Take on Measurement Error Correction

It is well known that measurement error attenuates estimated coefficients (see, for example, Buonaccorsi, 2010). Here we review that basic finding to set up a framework for our estimator.

To estimate the relationship between two variables measured with independent i.i.d. error, $Y = Y^* + \nu_Y$ and $X = X^* + \nu_X$ (with $\mathbb{E}[\nu_Y \nu_X] = 0$ and $\text{Var}[\nu_k] = \sigma_{\nu_k}^2$), the ideal regression model would be $Y^* = \alpha^* + \beta^* X^* + \varepsilon^*$. Instead, we can only estimate $Y = \alpha + \beta X + \varepsilon$, where α is a constant and ε is a mean-zero random noise. Annotating finite-sample estimates with hats and population moments without hats, this results in an estimated relationship of

$$\begin{aligned} \hat{\beta} &= \frac{\widehat{\text{Cov}}[Y, X]}{\widehat{\text{Var}}[X]} = \frac{\widehat{\text{Cov}}[\alpha + \beta^* X^* + \varepsilon + \nu_Y, X^* + \nu_X]}{\widehat{\text{Var}}[X^* + \nu_X]} \\ \mathbb{E}[\hat{\beta}] &= \text{plim}_{n \rightarrow \infty} \hat{\beta} = \beta^* \frac{\sigma_{X^*}^2}{\sigma_{X^*}^2 + \sigma_{\nu_X}^2} < \beta^*. \end{aligned} \quad (3)$$

The estimated coefficient $\hat{\beta}$ is thus biased towards zero. Importantly, the bias in (3) depends on the amount of information about the true explanatory variable X^* in X .

In a lab experiment, it is relatively easy to elicit two replicated measures of the same underlying parameter X^* . That is, suppose we have $X^a = X^* + \nu_X^a$ and $X^b = X^* + \nu_X^b$, with ν_X^a, ν_X^b i.i.d. random variables, and $\mathbb{E}[\nu_X^a \nu_X^b] = 0$. With the additional assumption that $\frac{\text{Var}[\nu_X^a]}{\text{Var}[X^a]} = \frac{\text{Var}[\nu_X^b]}{\text{Var}[X^b]} \equiv \frac{\text{Var}[\nu_X]}{\text{Var}[X]}$, we have

$$\widehat{\text{Corr}}[X^a, X^b] \rightarrow_p \text{Corr}[X^a, X^b] = \frac{\sigma_{X^*}^2}{\sigma_{X^*}^2 + \sigma_{\nu_X}^2}, \quad (4)$$

²³This censoring affects 22% of the responses in the 20-ball urn, and 32% of responses in the 30-ball urn.

which allows us to ballpark the degree of bias in estimated coefficients. The modal correlation between two elicitations of the same risk measure is approximately 0.6, suggesting that the variance of measurement error is of the order of 2/3 of the variance of X^* .²⁴

Using instrumental variables, the second noisy measure of X^* can be used to recover a consistent estimate of the true coefficient β^* . As a corollary of (4), note that $\widehat{\text{Cov}}[X^a, X^b] \approx \widehat{\text{Var}}[X^*]$ consistently estimates $\text{Var}[X^*]$ in the population. We apply two-stage-least-squares (2SLS) to instrument X^a with X^b

$$X^a = \pi_0 + \pi_1 X^b + \varepsilon_X \quad \Rightarrow \quad \hat{\pi}_1 = \frac{\widehat{\text{Cov}}[X^a, X^b]}{\widehat{\text{Var}}[X^b]} \approx \frac{\widehat{\text{Var}}[X^*]}{\widehat{\text{Var}}[X^b]}, \quad (5)$$

and then condition on this instrumented relationship to estimate $Y = \alpha + \beta(\hat{\pi}_0 + \hat{\pi}_1 X^b) + \varepsilon_Y$.

This second stage regression provides

$$\hat{\beta} = \frac{\widehat{\text{Cov}}[\alpha^* + \beta^* X^* + \varepsilon^* + \nu_Y, \hat{\pi}_0 + \hat{\pi}_1 X^b]}{\widehat{\text{Var}}[\hat{\pi}_0 + \hat{\pi}_1 X^b]} \approx \frac{\beta^* \hat{\pi}_1 \widehat{\text{Var}}[X^*]}{\hat{\pi}_1^2 \widehat{\text{Var}}[X^b]} \rightarrow_p \beta^*,$$

a consistent estimate of β^* , the true relationship between Y^* and X^* .

4.3 Two Instrumentation Strategies

Multiple measures for X^* admit multiple instrumentation strategies that will only produce the same estimate with infinite data. The ORIV estimator consolidates the information from these different formulations, producing a more efficient estimator.²⁵ In our working example, we have two equally valid elicitations and two possible instrumentation strategies: one may

²⁴In practice, if one is not certain that the variation in X^a and X^b due to measurement error is identical, one can measure the proportion of measurement error in each elicitation j separately as $\text{Cov}[X^a, X^b]/\text{Var}[X^j]$. In general, it is advisable to standardize variables, at which point the formulation here and in (4) coincide.

This formulation also allows a correction factor for the attenuation bias in the regression estimates from (3) dating back to Spearman (1904). Define the “disattenuated” estimator of β as $\tilde{\beta} = \frac{\hat{\beta}}{\widehat{\text{Corr}}[X^a, X^b]}$. The continuous mapping theorem implies that $\tilde{\beta}$ is a consistent estimator for β . However, this approach is less efficient than ORIV.

²⁵Combining these estimators is the same as using both valid moment conditions in GMM, see Appendix A. ORIV offers a simpler and more transparent correction technique than GMM. In our settings, GMM and ORIV are equally efficient.

Table 5: Correlation between different risk measures is understated due to measurement error.

Dependent Variable	Qualitative Assessment	Lottery Menu
Project #1	0.30*** (.034)	0.24*** (.034)
Project #2	0.29*** (.034)	0.29*** (.034)
Project #1 (Instrumented)	0.55*** (.067)	0.60*** (.073)
Project #2 (Instrumented)	0.58*** (.069)	0.50*** (.070)
Risk MPL #1	0.16*** (.036)	0.19*** (.035)
Risk MPL #2	0.17*** (.035)	0.23*** (.035)
Risk MPL #1 (Instrumented)	0.22*** (.048)	0.44*** (.069)
Risk MPL #2 (Instrumented)	0.21*** (.047)	0.37*** (.067)

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients are from regressions where both the right and left-side variables are standardized, and thus are correlations. $N = 775$.

instrument X^a with X^b , or X^b with X^a . In this subsection, we illustrate the divergent results these two strategies may produce. The next subsections show how to combine these sources of information into a single estimated relationship.

Table 5 shows estimated correlations between different elicitation techniques.²⁶ These are first estimated using a standard regression, and then the two different instrumental variables strategies discussed above. Although different instrumentation strategies may produce similar results—as in the third and fourth columns of Table 5—they may also produce different results—as in the seventh and eighth columns. Moreover, given that estimated standard deviations—inflated by measurement error—are used to standardize the variables in Table 5, neither strategy produces an accurate correlation. The next subsection deals with both of these issues.

4.4 Obviously Related Instrumental Variables

We construct ORIV estimates and corrected correlations in three steps. First, we consider the case where only explanatory variables are measured with error. We then extend the analysis to the case where both the outcome and explanatory variables are measured with error. Finally, we show how to derive consistent correlations from the consistent and asymptotically efficient ORIV estimates of the regression coefficient β . Throughout, we focus on designs in which there are at most two replications for each measure.²⁷ This is done for simplicity, and because it fits precisely the implementation carried out using the Caltech Cohort Study.

4.4.1 Errors in Explanatory Variables

The ORIV regression estimates a stacked model to consolidate the information from the two available instrumentation strategies. In the model of Section 4.2 this can be written as:

²⁶The coefficients are from regressions where both the left- and right-side variables are standardized, which removes scale effects and provides for easy comparison.

²⁷When there are two replications of each, the ORIV estimator is twice as efficient as instrumental variables—that is, the variance of the ORIV estimator is one-half that of instrumental variables estimators. Appendix A extends the ORIV estimator to settings where more than one replicate is available.

$$\begin{pmatrix} Y \\ Y \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \beta \begin{pmatrix} X^a \\ X^b \end{pmatrix} + \varepsilon, \text{ instrumenting } \begin{pmatrix} X^a \\ X^b \end{pmatrix} \text{ with } W = \begin{pmatrix} X^b & 0_N \\ 0_N & X^a \end{pmatrix}, \quad (6)$$

where N is the number of participants and 0_N is an $N \times N$ zero matrix. To implement this, one should create a stacked dataset and run a 2SLS regression. This is equivalent to estimating a first stage, as in (5), for both instrumentation strategies, then estimating

$$\begin{pmatrix} Y \\ Y \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \beta \begin{pmatrix} \hat{X}^a \\ \hat{X}^b \end{pmatrix} + \varepsilon, \quad (7)$$

where \hat{X}^a and \hat{X}^b are the predicted values derived from the two first-stage regressions. Sample Stata code illustrating this estimation procedure appears in Appendix C.2.

With a single replication, the stacked regression will produce an estimate of β^* that is the average of the estimates from the two instrumentation approaches in prior subsections. Intuitively, with no theoretical reason to favor one estimate or the other, it is equally likely that the smaller is too small as it is that the larger is too large.²⁸ The estimator splits the difference, leading to a consistent estimate of β^* , the true relationship between Y^* and X^* .

Proposition 1. *ORIV produces consistent estimates of β^* .*

This technique uses each individual twice, which results in standard errors that are too small, as the regression appears to have twice as much data as it really does. Many practitioners understand intuitively the idea that one should use clustered standard errors to treat multiple observations as having the same source.²⁹

²⁸Over-identification can be addressed using GMM, which allows testable restrictions that can be evaluated using a standard Sargan test.

²⁹Mathematically, clustering is needed as $\text{Cov}[\varepsilon_i, \varepsilon_{N+i}] = \text{Var}[\varepsilon_i^*]$ for $i \in \{1, 2, 3, \dots, N\}$. This implies that the variance-covariance matrix of residuals is given by

$$\begin{pmatrix} (\text{Var}[\varepsilon^*] + \beta^2 \text{Var}[\nu^a])I_N & \text{Var}[\varepsilon^*]I_N \\ \text{Var}[\varepsilon^*]I_N & (\text{Var}[\varepsilon^*] + \beta^2 \text{Var}[\nu^b])I_N \end{pmatrix}, \text{ where } I_N \text{ is an } N \times N \text{ identity matrix.}$$

Clustering takes care of the common ε_i^* for participant i on- and off-diagonal.

Proposition 2. *The ORIV estimator satisfies asymptotic normality under standard conditions. The estimated standard errors, when clustered by participant, are consistent estimates of the asymptotic standard errors.*

Simulations using t-Tests show that these asymptotic standard errors reject the null slightly too often at the 1%, 5%, and 10% levels. Bootstrapped standard errors, in contrast, produce effective rejection rates equal to the nominal rate. Thus, asymptotic standard errors slightly understate the true variability of estimates.

4.4.2 Designing Experiments for ORIV

Our analysis assumes that measurement error is independent across elicitations. To increase the chances this is true, we recommend experimental designers follow a few standard practices. First, duplicate elicitations should use different numerical values. Second, if using an MPL, the response grid should be constructed so that implied values (usually the midpoint between two choices) are not the same. Third, duplicate items should be placed in different parts of the survey or study to alleviate any tendency for consistency of responses.

Even after adopting these practices, a common component of measurement error may still remain. Assume that this common component is independent across measures (but not elicitations of that measure).³⁰ In this case, when estimating the correlation between the two measures using ORIV—or any other technique to deal with measurement error—there is a residual error in each measurement and assessed correlations would still be attenuated. Even in such settings, ORIV would be advised as it corrects for one component of measurement error. However, resulting estimates would be conservative. This illustrates a general point:

As the diagonal terms of the residual covariance matrix differ in whether $\text{Var}[\nu^a]$ or $\text{Var}[\nu^b]$ remain, a different weighting of X^a and X^b is optimal for efficiency. Such a condition can be most easily identified by comparing the unconditional variances of X^a and X^b . If they differ substantially, a feasibly-efficient GMM or weighted Feasible-Generalized Least Squares (FGLS) approach would yield asymptotically efficient (and equivalent) estimators. In our dataset, which is an order of magnitude larger than most, FGLS-weighting does not produce different results, suggesting the assumption of homogenous errors underlying ORIV is reasonable in our application. We present the asymptotic equivalence between feasible-efficient GMM and FGLS in Appendix A.6. This appendix concludes with a discussion of the numerical challenges that may arise in implementing a naive FGLS ORIV estimator due to rank-deficiency in the residual covariance matrix.

³⁰In particular: $X^i = X^* + \eta_X + \nu_X^i$ and $Y^i = Y^* + \eta_Y + \nu_Y^i$, with $\mathbb{E}[\eta_X \eta_Y] = \mathbb{E}[\nu_X^a \nu_X^b] = \mathbb{E}[\nu_Y^a \nu_Y^b] = 0$.

survey- or session-based measurement error is difficult to estimate or interpret. In particular, if we see different patterns of responses across surveys or sessions, it would be challenging to disentangle correlated measurement error from a change in preferences.

4.4.3 Errors in Outcome and Explanatory Variables

When estimating the relationship between two elicited variables, there is no reason to believe that one is measured with error (X), but the other is not (Y). The existence of measurement error in Y does not change Propositions 1 and 2, although estimated standard errors will, of course, increase, reflecting the degree of uncertainty in the estimated coefficient. Still, if one has access to two estimates of Y there is no reason not to use them, as they will increase efficiency. Moreover, when estimating *correlations* between two variables, classical measurement error in either will attenuate the estimated correlation.

To incorporate two measures of Y^* with measurement error ($Y^a = Y^* + \nu_Y^a$, $Y^b = Y^* + \nu_Y^b$, $\mathbb{E}[\nu_Y^a] = \mathbb{E}[\nu_Y^b] = 0$) in the ORIV estimation procedure, one would simply estimate

$$\begin{pmatrix} Y^a \\ Y^a \\ Y^b \\ Y^b \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} + \beta \begin{pmatrix} X^a \\ X^b \\ X^a \\ X^b \end{pmatrix} + \varepsilon \quad \text{with instruments } W = \begin{pmatrix} X^b & 0_N & 0_N & 0_N \\ 0_N & X^a & 0_N & 0_N \\ 0_N & 0_N & X^b & 0_N \\ 0_N & 0_N & 0_N & X^a \end{pmatrix}.$$

4.4.4 Estimating Correlations from Consistent Coefficients

ORIV produces $\hat{\beta}^*$, a consistent estimate of β^* . Notice that

$$\hat{\beta} = \frac{\widehat{\text{Cov}}[X, Y]}{\widehat{\text{Var}}[X]} \quad \text{implying} \quad \hat{\rho}_{XY} = \hat{\beta} \sqrt{\frac{\widehat{\text{Var}}[X]}{\widehat{\text{Var}}[Y]}}$$

where ρ_{XY} is the correlation. Thus, we need consistent estimates of $\text{Var}[X^*]$ and $\text{Var}[Y^*]$ to recover $\hat{\rho}_{XY}^*$. The problem is that $\text{Var}[X] = \text{Var}[X^*] + \text{Var}[\nu_X]$. As $\text{Var}[Y]$ is biased as

well, it is not clear if transforming the regression coefficient into a correlation will generate an estimate that is biased up or down. Nonetheless, we have

$$\text{Cov}[X^a, X^b] = \text{Cov}[X^* + \nu_X^a, X^* + \nu_X^b] = \text{Var}[X^*] \quad \text{so} \quad \hat{\rho}_{XY}^* = \hat{\beta}^* \sqrt{\frac{\widehat{\text{Cov}}[X^a, X^b]}{\widehat{\text{Cov}}[Y^a, Y^b]}}$$

Proposition 3. *$\hat{\rho}_{XY}^*$ is consistent with an asymptotically normal distribution, where standard errors can be derived using the delta method. These standard errors can be consistently estimated using a bootstrap to construct confidence intervals.*

An example of how to estimate correlations and bootstrapped standard errors using ORIV in STATA can be found in Appendix C. A STATA program to estimate correlations and bootstrapped standard errors is available from the authors upon request.

4.5 Corrected Correlations between Risk Elicitation Techniques

We now use ORIV estimators to examine the correlations between different risk measures. Table 6 contains both the raw and corrected correlations. Both the Risk MPL and the Project measures were elicited twice. The qualitative risk assessment was elicited once in the Fall of 2014 and again in the Spring of 2015, the latter serving as the duplicate elicitation. The Lottery Menu measure was elicited only once.³¹

Previous work comparing different risk-elicitation techniques often transforms them into a common scale (see Deck et al., 2010, for an example). For comparability, we do the same in Table 6. This is not theoretically advisable as it introduces a non-linear change in the structure of measurement error, which may lead to inconsistent estimates. However, for the question at hand, this makes little difference in the results.

³¹For correlations involving the Lottery Menu task, we multiply by $\sqrt{\widehat{\text{Var}}[((X^a)')' (X^b)'] / \widehat{\text{Var}}[Y]}$. This is valid so long as the measurement error in the measures of X and Y are equal—that is, $\text{Corr}[X^a, X^b] = \text{Var}[Y^*] / \text{Var}[Y]$, as shown in (4). Having only one measure of Y , we have no measure of $\text{Var}[Y^*]$, and cannot test this. However, the correlation between elicitations for the three that we do have are 0.67 (s.e. 0.027, projects), 0.62 (s.e. 0.028, qualitative), and 0.59 (s.e. 0.29, Risk MPLs), so this seems reasonable.

In the top panel, we consider correlations between the unaltered measures. That is, we use units given by the elicitation techniques. The second panel translates these various measures into CRRA coefficients, except for the qualitative assessment, which does not lend itself to transformation. The third panel uses the imputed CRRA coefficients to calculate the implied certainty equivalent of a lottery with a 50% probability of 100 tokens and a 50% probability of 0 tokens. Note that in the case of the Risk MPLs, this is the same as the questions' natural units, as these are elicitation of certainty equivalents over 50/50 lotteries.

All three panels suggest similar conclusions. First, the corrected correlations are substantially higher. While the raw correlations are arguably low, never exceeding 0.5 (and uniformly below 0.27 when considering imputed CRRA coefficients), corrected correlations are dramatically higher, reaching levels as high as 0.73. Whether this correlation is “high” or “low” is largely a judgement call. However, the literature seems to consistently suggest that correlations above 0.7 are very high (see, for example, Cohen, 1988; Evans, 1996). Moreover, many perceived strong links correspond to correlations that are 0.7 or less. For example, the correlation between parents' and their childrens' heights hovers around 0.5 (Wright and Cheetham, 1999); the correlation between average parents' education and their children's education ranges from around 0.30 in Denmark to 0.54 in Italy, with most western countries falling somewhere in-between (Hertz et al., 2007). Second, some measures are noticeably more correlated. Namely, the Project measure is most correlated with our other elicitation techniques. It is most highly correlated with the Lottery Menu measure, with corrected correlations of 0.55–0.73, depending on measurement units. The Lottery Menu also exhibits relatively high correlations with other measures. The lowest correlations are observed between the Risk MPL and the Qualitative measure.

4.6 Substantive Implications

There are good reasons to suspect that the Risk MPL measure captures risk attitudes over a different domain than the other measures: its smaller correlation with other measures, and

Table 6: Correlation matrices before and after accounting for measurement error

In Units Given by the Questions

	Raw Correlations			Corrected for Measurement Error		
	Project	Qualitative	Lottery Menu	Project	Qualitative	Lottery Menu
Qualitative	0.26*** (.029)			0.40*** (.043)		
Lottery Menu	0.47*** (.029)	0.25*** (.032)		0.71*** (.046)	0.40*** (.052)	
Risk MPL	0.19*** (.032)	0.13*** (.033)	0.22*** (.030)	0.30*** (.048)	0.19*** (.047)	0.38*** (.053)

Measured in CRRA coefficients

	Raw Correlations			Corrected for Measurement Error		
	Project	Qualitative	Lottery Menu	Project	Qualitative	Lottery Menu
Qualitative	0.20*** (.032)			0.36*** (.047)		
Lottery Menu	0.27*** (.040)	0.24*** (.033)		0.55*** (.078)	0.38*** (.053)	
Risk MPL	0.18*** (.037)	0.070** (.033)	0.22*** (.037)	0.37*** (.078)	0.10** (.047)	0.42*** (.071)

Measured in certainty equivalent of a 50/50 lottery over 0/100 tokens

	Raw Correlations			Corrected for Measurement Error		
	Project	Qualitative	Lottery Menu	Project	Qualitative	Lottery Menu
Qualitative	0.25*** (.029)			0.44*** (.046)		
Lottery Menu	0.38*** (.026)	0.23*** (.032)		0.73*** (.077)	0.37*** (.051)	
Risk MPL	0.24*** (.043)	0.13*** (.033)	0.20*** (.027)	0.43*** (.067)	0.19*** (.047)	0.34*** (.047)

Notes: ***, **, * denote statistical significance at the 1%, 5% and 10% level, with bootstrapped standard errors from 10,000 simulations in parentheses. $N = 775$.

the fact that, unlike other risk measures, it is uncorrelated with gender, as shown in Figure 1.³² These differences account for differences in the explanatory power of risk controls on the gender gap in competitiveness in Section 3.2.

The fact that different measures yield different conclusions about the relationship between gender and risk attitudes is reflected in the behavioral literature, which reaches mixed conclusions about the general relationship between gender and risk (see, for example, Byrnes, Miller, and Schafer, 1999, for a review of the relevant experimental work in psychology, and Croson and Gneezy, 2009 and Eckel and Grossman, 2008b for reviews of related experimental work in economics). On the other hand, the finance literature has found a more consistent difference between men and women when considering risky financial investments (see, for example, Barber and Odean, 2013; Embrey and Fox, 1997; Farrell, 2011). The Project measure intentionally mimics a stock/bond portfolio choice (or risky/safe assets), and the gender-based behavior in this task is similar to that seen in real financial investments: men invest more aggressively than women.³³

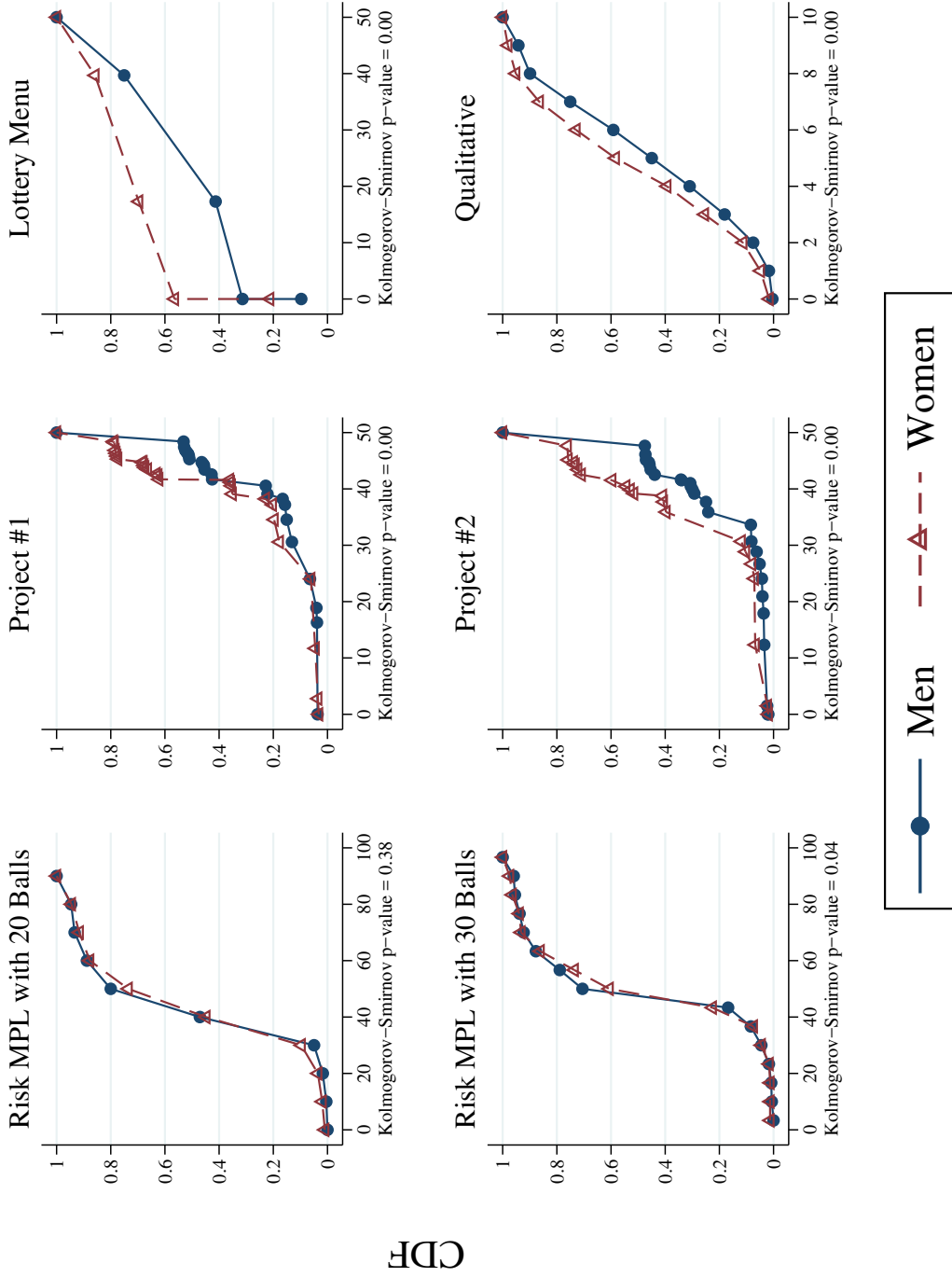
There is also variation in the consistency of responses to the different risk elicitation techniques across time. The Project task, Risk MPL, and Qualitative assessment were all elicited in both the Fall 2014 and Spring 2015 installments of the survey. The risk attitudes elicited by the Project task exhibit more stability than the Risk MPLs—a correlation of 0.65 (0.038) for the Project measure(s), compared with 0.42 (0.063) for the Risk MPL (both corrected for measurement error). The Qualitative elicitation was performed only once per survey, and the uncorrected correlation between these elicitations was 0.62.

Taken together, these different measures may be representative of risk attitudes in different settings. This would not be surprising, as psychologists have found that risk attitudes do

³²In addition to the usual concerns with the Kolmogorov-Smirnov test, it is not valid for discrete distributions. In such cases, the p-value is only approximate, and may lead to extreme implications, such as the p-values of 1.000 found in Figure 2. As such, we provide p-values only to aid visual inspection.

³³The lotteries in the Lottery Menu task can be viewed as corresponding to different investment allocations between a safe option and a risky one that pays three times the amount invested with 50% probability, see Eckel and Grossman (2002). Eckel and Grossman (2008a) observed that results are not sensitive to whether or not lotteries are described as risky investments to participants, which is consistent with the Lottery Menu task exhibiting similar patterns to the Project measure.

Figure 1: Risk aversion differs by gender, except when elicited using Risk MPLs.



Notes: All panels except for the qualitative assessment are put on the same scale: a certainty equivalent of a 50/50 lottery paying 0/100 tokens, via a CRRA utility function.

differ across contexts (Slovic, 1964; see also Kruger, Wang, and Wilke, 2007 and references therein for more recent work). While there are many criteria on which one might evaluate such measures, the Project-based measure seems particularly attractive due to its stability, correlation with other popular measures, and the fact that the literal interpretation of the measure is consistent with evidence from the field in finance.³⁴

5 New Traits

Measurement error may lead researchers to believe that an observed behavior is not well explained by current theory. We have already shown one example of this in Section 3, due to controls measured with error. It is natural to think that bias in correlations, explored in the last section, may similarly cause researchers to underestimate the relationship between two variables, and thus declare them distinct when they are, in fact, not. In this section we provide a potential example of this phenomenon by examining attitudes towards ambiguity.

Ambiguity aversion refers to a preference for known risks over unknown risks. First introduced by Ellsberg (1961), this preference implies that an ambiguity averse individual would prefer a lottery with a known probability distribution of rewards over a similar lottery in which the probability distribution of rewards is unknown. This behavior is expressed in the *Ellsberg Paradox*, where participants prefer a bet on the draw of a black ball from an urn with, say, 10 red and 10 black balls than on an urn with 20 balls of unknown composition. Ambiguity aversion is widely studied, and used to explain incomplete contracts, volatility in stock markets, selective abstention in elections, and so on (Mukerji, 2000).

Segal (1987, 1990) suggests that choices under ambiguity may come from improperly compounding a sequence of lotteries. For instance, in the Ellsberg Paradox scenario above, a participant might view a draw from the ambiguous urn as having two stages: First, the number of red balls is randomly determined, according to some subjective probability; sec-

³⁴Recent evidence suggests a global risk component that explains individuals' choices across investment domains. This component exhibits some of the features of the Project measure (Einav et al., 2012).

ond, a ball is drawn from the urn. If an individual fails to properly reduce these two lotteries into one, a bias will result. Halevy (2007) experimentally tests this proposition. In his study, participants face both an Ellsberg urn, and an urn where the number of red balls is uniformly determined. In his results, Halevy reports correlations of around 0.5 between behaviors in both treatments. Nonetheless, his results suggest that half the variation in the responses to ambiguous and compound lotteries is independent. This implies a strong, but imperfect, link between ambiguity aversion and (negative) reactions to compound lotteries.

In this section, we replicate Halevy’s exercise, adding duplicate measures of certainty equivalents of both ambiguous and compound lotteries. Correcting for measurement error using ORIV, ambiguity aversion and reaction to compound lotteries appear almost identical.

5.1 Ambiguity Aversion and Reaction to Compound Lotteries

As described in Section 3.1, the Risk MPL, Compound MPL, and Ambiguous MPL are all implemented similarly. All ask for a participant’s certainty equivalent value of a draw from an urn if a certain color ball is drawn. All allow the participant to select the color of the ball associated with positive payment. All have the same number of balls, and the same payoff. The only difference is how the distribution of balls in the urn is specified: half black and half red for Risk; drawn from a uniform distribution for Compound; or unknown for Ambiguous. Each measure is replicated twice: once with a 20-ball urn and a payoff of 100 tokens if the correct color ball is drawn, and once with a 30-ball urn and a 150-token payoff.

Our data show evidence of ambiguity aversion, as well as a negative reaction to compound lotteries. In particular, the certainty equivalents of the ambiguous urns are 2.5 and percentage points lower than those of the risky urns (s.e. of 0.48 for 20-ball urn, and 0.46 for 30-ball urn); and the certainty equivalents of the compound lotteries are 2.9 (0.51) and 2.8 (0.51) percentage points lower than those of the risky urns. Note that these differences are statistically significant, but not significantly different from *each other*. On average, ambiguity aversion and reaction to compound lotteries are identical.

Table 7: The correlation between certainty equivalents is substantial.

	Raw Correlations			Corrected for Measurement Error		
	Risk CE	Compound CE	Compound Reaction	Risk CE	Compound CE	Compound Reaction
Compound CE	0.55*** (.035)			0.74*** (.043)		
Ambiguous CE	0.60*** (.033)	0.65*** (.027)		0.78*** (.037)	0.85*** (.029)	
Ambiguity Aversion			0.44*** (.039)			0.86*** (.059)

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with bootstrapped standard errors from 10,000 simulations in parentheses. $N = 774$.

Table 7 reports the raw and corrected correlations between the three measures. The raw correlation between ambiguous and compound certainty equivalents are 0.65. This is in line with Halevy (2007), who reports a correlation of 0.45 in the first round of his experiment, and a correlation of 0.71 in his robustness round. However, once measurement error is corrected for in our data, the correlation is much higher: 0.85.

Corrected correlations between certainty equivalents of risky and compound or ambiguous urns are substantial as well—0.74 and 0.78, respectively. This leads to an important point: the high correlation between ambiguity aversion and reaction to compound lotteries may be because the certainty equivalents of both reflect risk attitudes as well. Thus, we subtract the risk certainty equivalents from each of the compound and ambiguous certainty equivalents, leaving measures of ambiguity aversion, and (negative) reaction to compound lotteries. This results in a smaller raw correlation of 0.44, but the same correlation of 0.86 when measurement error is taken into account. Moreover, the 95% confidence interval for this correlation is (0.74, 0.98). Thus, while we can reject the null hypothesis that the correlation is 1, we cannot reject the null that the correlation is very close to 1.³⁵

³⁵The 99% confidence interval is (0.71, 1.01). Halevy’s data has a correlation of 0.47 in the main experiment, and 0.75 in the robustness round.

Here, unlike in Section 3, our large sample size is likely the reason we find any difference between these behaviors. Drawing a random sample of 104 observations (the size of Halevy’s experiment) 10,000 times, the correlation between ambiguity aversion and reaction to compound lotteries differs from 1 only 1.2% of the time at the 1% level, 4.8% of the time at the 5% level, and 9.0% of the time at the 10% level. It should be noted that these are the results from standard confidence intervals that are well-calibrated. While correlations have an upper bound of 1, and thus suffer from the Andrews’s (2001) problem, our estimator can take on values greater than 1, and is normally distributed around 1 when that is the true correlation. Thus, any time a correlation greater than 1 is calculated using ORIV, it should be interpreted as strong evidence that the correlation is actually 1.

Finally, we plot the CDFs of the various measures in Figure 2. The left-most panels show certainty equivalents for risky urns and ambiguous urns. The fact that these diverge below 50 is evidence of ambiguity aversion. The center panels show the certainty equivalents for ambiguous urns and compound urns: the distributions appear identical. The final panels show the distributions of ambiguity aversion and reaction to compound lotteries. Once again, these appear identical.

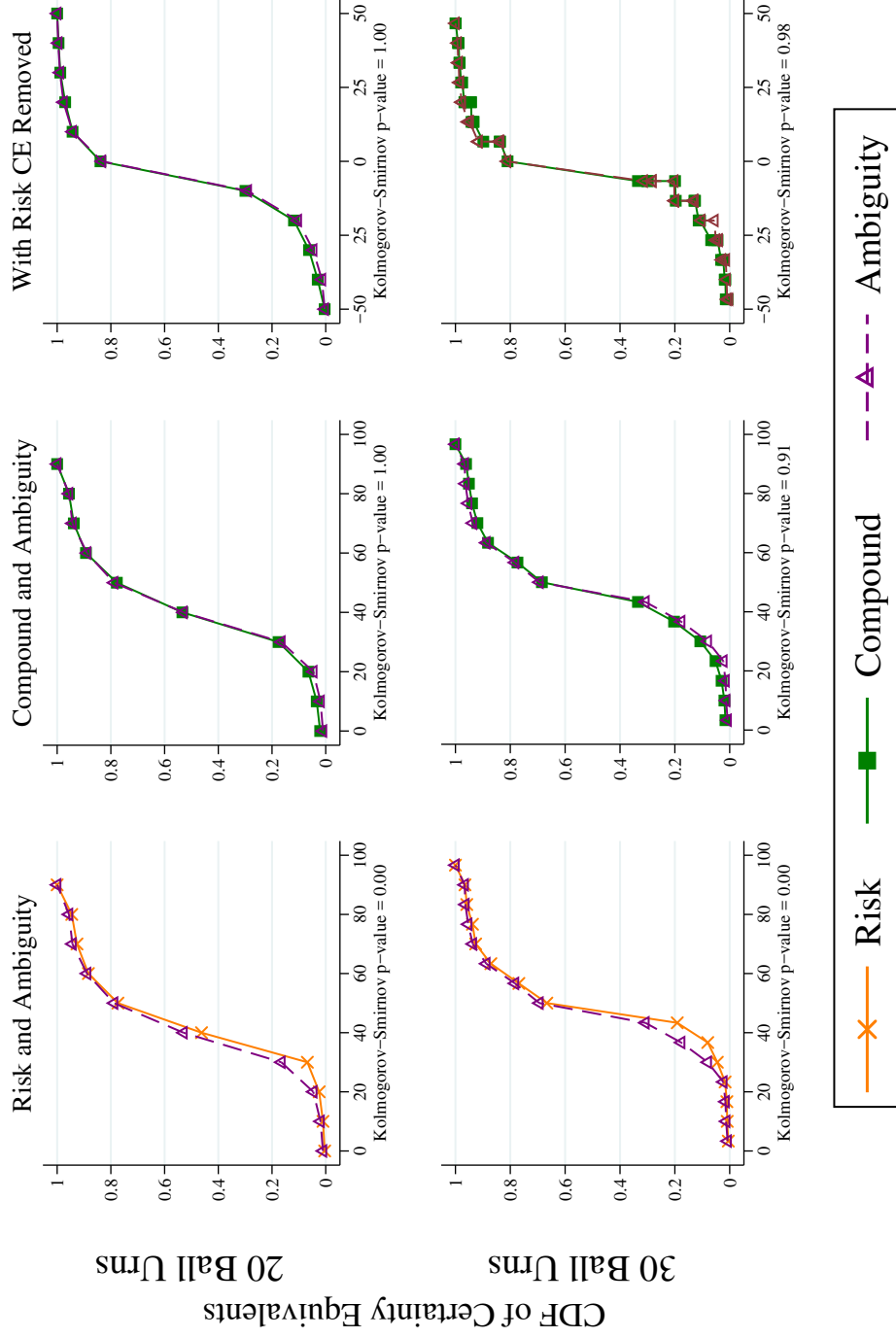
5.2 Substantive Implications

Ambiguity aversion and reaction to compound lotteries appear remarkably similar once measurement error is accounted for. It is worth noting that this is compatible with the original description of Knightian (1921) Uncertainty,

[T]he essential fact is that “risk” means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomena depending on which of the two is really present and operating.

That is, compound lotteries may be no more “susceptible to measurement” for individuals—even Caltech students, who are mathematically inclined—than those that are ambiguous.

Figure 2: Population responses to risky and ambiguous lotteries differ, but are identical for ambiguous and compound lotteries.



Notes: The first two columns are certainty equivalents for a lottery paying 0/100 points. This is the natural scale for the 20-ball urns; for the 30-ball urns the expressed certainty equivalent is divided by 1.5, implying a CRRA utility function. The final column subtracts the certainty equivalent of the risky lottery from those of the compound and ambiguous lotteries.

This suggests to us that the defining characteristic, in addition to risk attitudes, in determining valuations of these lotteries is some notion of complexity. Regardless of the philosophical interpretation of these results, it is clear that any successful model of ambiguity aversion should also predict behavior in complex, but fully specified, risky environments.

6 Conclusion

If measurement error is such a ubiquitous, but easily correctable, issue, why has the experimental literature paid so little attention to it? The answer likely lies in the fact that attenuation bias driven by measurement error is a conservative bias. That is, it biases the researcher against false positives. So when asked about measurement error, a researcher can confidently answer that it would “go against finding anything.”

However, measurement error creates another, field-wide, issue that has been little appreciated. It leads to the over-identification of “new” phenomena. Indeed, our paper shows two examples of this: previously, competitiveness was thought to have a component unconnected to risk aversion and overconfidence, and ambiguity aversion and reaction to compound lotteries (the latter a form of risk aversion) were thought to be distinct. Our results show that both of these beliefs are unlikely to be true. Moreover, our finding that elicitation methods are more highly correlated than previously appreciated suggests that fewer elicitation methods are needed than previously thought. Given that these are the only three results we examined in trying to understand the influence of measurement error in experiments, it seems likely that the over-identification of new phenomena is a substantial problem.

That measurement error may lead to the identification of new phenomena where none exist may feed into the recent mushrooming of methodological work suggesting the high rates of non-replicability of research discoveries (see Ioannidis, 2005; Simonsohn, 2015, and references therein). Using the techniques developed here to account for measurement error may help researchers discover, in a more robust fashion, the deep connections between different

attitudes and effects.

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Online Appendix—Not Intended for Publication

A Sampling Properties of the ORIV Estimator

There are several textbooks and survey articles that present methods for inference in linear and non-linear models with measurement error, a topic that is also addressed in almost any statistics and econometrics textbook. A common approach to this problem exploits repeated measurements (or replicates) to characterize the severity of measurement error. After estimating these distributional features of the measurement error, researchers can compute correction factors that dis-attenuate sample estimates. Our ORIV method provides a simple unifying method for consistent estimation that integrates all information available for relating two variables to one another. To our knowledge, it is the first to do so. In this technical appendix, we present a standard suite of results establishing consistency and asymptotic normality, along with consistent standard errors, for the ORIV estimator. These properties are proved using textbook-standard arguments presented primarily for completeness.

A.1 Assumptions for the ORIV Model

To establish the Sampling Properties of ORIV, we first detail the assumptions underlying the linear regression model in latent variables (Assumption 1) and then present the classical measurement error model for our contaminated observations (Assumption 2). The assumptions here are standard, with the usual exogeneity restrictions and bounds on moments to admit asymptotic analysis, and all results can be readily extended to allow for heteroskedasticity, autocorrelation, or other relevant features of the observed data.

Assumption 1 (Linear Regression Model in Latent Variables). *The relationship between the latent variables Y^* and X^* satisfy the usual assumptions for linear regression:*

1. *Linear model: $Y^* = \alpha + X^*\beta^* + \varepsilon^*$*

2. *Exogeneity*: $\mathbb{E}[\varepsilon^*|X^*] = 0$

3. *Variability in treatment*: $0 < \text{Var}(X^*)$, and, $\mathbb{E}[(X^*)^4] < \infty$

4. *Variability in residuals*: $\text{Var}[\varepsilon^*|X^*] = \sigma_{\varepsilon^*}^2$, and, $\mathbb{E}[(\varepsilon^*)^4|X^*] < \infty$

5. *Independent sampling of individuals*: $(Y_t^*, X_t^*, \varepsilon_t^*) \perp (Y_s^*, X_s^*, \varepsilon_s^*) \forall t \neq s$.

Assumption 2 (Replicated Classical Measurement Error). *We observe multiple replicates for noisy measurements of Y^* and X^* that are tainted by classical measurement error.*

1. $X^k = X^* + \nu_X^k$, $k = 1, \dots, K_X$

2. $Y^k = Y^* + \nu_Y^k$, $k = 1, \dots, K_Y$

(a) $\mathbb{E}[\nu_X^k|X^*, Y^*] = 0$

(a) $\mathbb{E}[\nu_Y^k|X^*, Y^*] = 0$

(b) $\mathbb{E}[(\nu_X^k)^2|X^*, Y^*] = \sigma_{\nu_X^k}^2$

(b) $\mathbb{E}[(\nu_Y^k)^2|X^*, Y^*] = \sigma_{\nu_Y^k}^2$

(c) $\mathbb{E}[(\nu_X^k)^4|X^*, Y^*] < \infty$

(c) $\mathbb{E}[(\nu_Y^k)^4|X^*, Y^*] < \infty$

(d) $\nu_X^k \perp \nu_X^l$, $\forall k \neq l$

(d) $\nu_Y^k \perp \nu_Y^l$, $\forall k \neq l$

3. *Independence across measures*: $\nu_Y^k \perp \nu_X^l$, $\forall k, l$

4. *Independent sampling of individuals*: $\forall s, t \in \{1, \dots, N\}$, *s.t.* $t \neq s$:

$$\nu_{X,t}^k \perp \nu_{X,s}^l, \forall k, l \in \{1, \dots, K_X\}, \text{ and, } \nu_{Y,t}^k \perp \nu_{Y,s}^l, \forall k, l \in \{1, \dots, K_Y\}$$

Our objective is to perform consistent and (potentially) efficient inference on β , for which we propose the ORIV regression.¹ Defining 1_N as a $(N \times 1)$ vector of 1's and 0_N as a $(N \times 1)$

¹The conditional independence restrictions imposed in Assumptions 1 and 2 are stronger than necessary for our purposes and can be weakened to mean independence conditions using standard arguments. We make these assumptions for ease of interpretation. In principle, the conditional independence assumption provides meaningful additional restrictions that could enhance the identification power, estimator efficiency, and testable restrictions of the model. This is an interesting direction for future research.

vector of 0's, the independent and dependent variables are:

$$\begin{aligned} \tilde{Y} &\equiv [Y^{1'}, \dots, Y^{K_Y'}]' & \tilde{X}^k &\equiv 1_{K_Y} \otimes X^k \\ Y_{OR} &\equiv 1_{K_X} \otimes Y & X_{OR} &\equiv [\tilde{X}^{1'}, \dots, \tilde{X}^{K_X'}]' \end{aligned}$$

Now construct the instruments for each model:

$$\begin{aligned} W^k &\equiv [X^1, \dots, X^{k-1}, 0_N, X^{k+1}, \dots, X^{K_X}] \\ \tilde{W}^k &\equiv 1_{K_Y} \otimes W^k \end{aligned} \quad W_{OR} \equiv \begin{bmatrix} \tilde{W}^1 & 0 & \dots & 0 \\ 0 & \tilde{W}^2 & \ddots & 0 \\ 0 & \dots & \ddots & \vdots \\ 0 & \dots & 0 & \tilde{W}^{K_X} \end{bmatrix}$$

Then, as we soon show, the ORIV estimator fits the linear model with the exogeneity restriction:

$$Y_{OR} = X_{OR}\beta^* + \varepsilon_{OR}, \quad \mathbb{E}[\varepsilon_{OR}|W_{OR}] = 0.$$

Letting $P_{W_{OR}} \equiv W_{OR}(W'_{OR}W_{OR})^{-1}W'_{OR}$ denote the projection matrix onto the column space of W_{OR} , we can write the ORIV estimator for β using the usual formula:

$$\hat{\beta}^* = (X'_{OR}P_{W_{OR}}X_{OR})^{-1}X'_{OR}P_{W_{OR}}Y_{OR}. \quad (\text{A.1})$$

A.2 ORIV Estimator Consistency

We now present the standard arguments for IV estimator consistency in the ORIV setting, establishing the result stated in the text's Proposition 1:

Proposition 1. *ORIV produces consistent estimates of β^* .*

Proof. We begin by verifying the exogeneity condition $\mathbb{E}[\varepsilon_{OR,n}|W_{OR,n}] = 0$. This requires breaking ε_{OR} down equation-by-equation to verify the condition for each IV specification

included in the model.

$$\begin{aligned}
\varepsilon^{i,j} &= Y^i - X^j \beta^* \\
&= Y^* + \nu_Y^i - X^* \beta^* - \nu_X^j \beta^* \\
&= X^* \beta^* + \varepsilon^* + \nu_Y^i - X^* \beta^* - \nu_X^j \beta^* \\
&= \varepsilon^* + \nu_Y^i - \nu_X^j \beta^*.
\end{aligned}$$

Note the ORIV estimator interacts $\varepsilon^{(i,j)}$ solely with $W^{(j)}$, so that:

$$\begin{aligned}
\mathbb{E}[\varepsilon^{i,j}|W^j] &= \mathbb{E}[\varepsilon^* + \nu_Y^i - \nu_X^j \beta^* | W^j] \\
&= \mathbb{E}[\varepsilon^* | W^j] + \mathbb{E}[\nu_Y^i | W^j] + \mathbb{E}[\nu_X^j | W^j] \beta_{OR} \\
&= 0.
\end{aligned}$$

The first term above cancels by the exogeneity condition in assumption 1.2, the second by the classical measurement error assumption 2.3, and the last by assumptions 2.1(a) and 2.1(d) because the k^{th} column of W^k is zeroed out.

We now recall the formula for $\hat{\beta}^*$ from equation A.1:

$$\begin{aligned}
\hat{\beta}^* &= (X'_{OR} P_{W_{OR}} X_{OR})^{-1} X'_{OR} P_{W_{OR}} Y_{OR} \\
&= (X'_{OR} P_{W_{OR}} X_{OR})^{-1} X'_{OR} P_{W_{OR}} X_{OR} \beta^* + (X'_{OR} P_{W_{OR}} X_{OR})^{-1} X'_{OR} P_{W_{OR}} \varepsilon_{OR} \\
&= \beta^* + (X'_{OR} P_{W_{OR}} X_{OR})^{-1} X'_{OR} P_{W_{OR}} \varepsilon_{OR}.
\end{aligned}$$

To establish consistency of the estimator, notice that:

$$\begin{aligned}\hat{\beta}^* - \beta^* &= (X'_{OR} P_{W_{OR}} X_{OR})^{-1} X'_{OR} P_{W_{OR}} \varepsilon_{OR} \\ &= \left(\frac{1}{N} X'_{OR} W_{OR} \left(\frac{1}{N} W'_{OR} W_{OR} \right)^{-1} \frac{1}{N} W'_{OR} X_{OR} \right)^{-1} \\ &\quad * \frac{1}{N} X'_{OR} W_{OR} \left(\frac{1}{N} W'_{OR} W_{OR} \right)^{-1} \frac{1}{N} W'_{OR} \varepsilon_{OR}.\end{aligned}$$

The bounded fourth moments in assumptions 1.4, 2.1(c), and 2.2(c) allow us to apply a strong law of large numbers for each of the averages in the above formula. Further, since $\mathbb{E}[\varepsilon_{OR,n} | W_{OR,n}] = 0 \Rightarrow \mathbb{E}[W_{OR,n} \varepsilon_{OR,n}] = 0$, the last term $\frac{1}{N} W'_{OR} \varepsilon_{OR} \rightarrow_p 0$, yielding our desired consistency result: $(\hat{\beta}^* - \beta^*) \rightarrow_p 0$. \square

A.3 Asymptotic Normality and Clustered Variances

We now establish asymptotic normality and present a consistent estimator for the variance of $\hat{\beta}^*$. This establishes the proof for Proposition 2:

Proposition 2. *The ORIV estimator satisfies asymptotic normality under Assumptions (1) and (2). The estimated standard errors, when clustered by participant, are consistent for the asymptotic standard errors.*

Proof. We first establish asymptotic normality. Let $\Sigma_{\varepsilon_{OR}} \equiv E[\varepsilon_{OR} \varepsilon'_{OR}]$. By laws of large numbers,

$$S_{X'W} \equiv \text{plim}_{n \rightarrow \infty} \frac{1}{N} X'_{OR} W_{OR}, \quad S_{W'W} \equiv \text{plim}_{n \rightarrow \infty} \frac{1}{N} W'_{OR} W_{OR}, \quad \text{and} \quad \Omega_{OR} \equiv \text{plim}_{n \rightarrow \infty} \frac{1}{N} W'_{OR} \Sigma_{\varepsilon_{OR}} W_{OR}.$$

From the Central Limit Theorem, $N^{-1/2} W'_{OR} \varepsilon_{OR} \rightarrow_d N(0, \Omega_{OR})$.

Asymptotic normality of $\hat{\beta}_{OR}$ now follows since:

$$\begin{aligned}\sqrt{N}(\hat{\beta}^* - \beta^*) &= \left(\frac{1}{N} X'_{OR} W_{OR} \left(\frac{1}{N} W'_{OR} W_{OR} \right)^{-1} \frac{1}{N} W'_{OR} X_{OR} \right)^{-1} \\ &\quad * \frac{1}{N} X'_{OR} W_{OR} \left(\frac{1}{N} W'_{OR} W_{OR} \right)^{-1} N^{-1/2} W'_{OR} \varepsilon_{OR} \\ &\rightarrow_p N(0, \Sigma_{\hat{\beta}^*}), \text{ where} \\ \Sigma_{\hat{\beta}^*} &= (S_{X'W} S_{W'W}^{-1} S'_{X'W})^{-1} S_{X'W} S_{W'W}^{-1} \Omega S_{W'W}^{-1} S'_{X'W} (S_{X'W} S_{W'W}^{-1} S'_{X'W})^{-1}.\end{aligned}$$

A feasible estimator of the asymptotic variance requires an estimate for Ω , which we will show is available using the usual clustered variance-covariance matrix estimator. To achieve this result, we characterize the structure of $\Sigma_{\varepsilon_{OR}}$. Consider

$$\begin{aligned}\mathbb{E}[\varepsilon_s^{i,j} \varepsilon_t^{k,l}] &= \mathbb{E}[(\varepsilon_s^* + \nu_{Y,s}^i - \nu_{X,s}^j \beta^*) (\varepsilon_t^* + \nu_{Y,t}^k - \nu_{X,t}^l \beta^*)] \\ &= \mathbb{E}[\cancel{\varepsilon_s^* \varepsilon_t^*} + \cancel{\varepsilon_s^* \nu_{Y,t}^k} - \cancel{\varepsilon_s^* \nu_{X,t}^l} \beta^* \\ &\quad + \cancel{\nu_{Y,s}^i \varepsilon_t^*} + \nu_{Y,s}^i \nu_{Y,t}^k - \cancel{\nu_{Y,s}^i \nu_{X,t}^l} \beta^* \\ &\quad - \cancel{\nu_{X,s}^j \varepsilon_t^*} - \cancel{\nu_{X,s}^j \nu_{Y,t}^k} + \nu_{X,s}^j \nu_{X,t}^l (\beta^*)^2] \\ &= \mathbb{E}[\varepsilon_s^* \varepsilon_t^* + \nu_{Y,s}^i \nu_{Y,t}^k + \nu_{X,s}^j \nu_{X,t}^l (\beta^*)^2].\end{aligned}$$

Therefore, we have:

$$\mathbb{E}[\varepsilon_s^{i,j} \varepsilon_t^{k,l}] = \begin{cases} 0, & \text{if } s \neq t, \forall i, j, k, l \\ s_{00} \equiv \sigma_{\varepsilon^*}^2, & \text{if } s = t, i \neq k, j \neq l \\ s_{i0} \equiv \sigma_{\varepsilon^*}^2 + \sigma_{\nu_{Y,i}}^2, & \text{if } s = t, i = k, j \neq l \\ s_{0j} \equiv \sigma_{\varepsilon^*}^2 + \sigma_{\nu_{X,j}}^2 (\beta^*)^2, & \text{if } s = t, i \neq k, j = l \\ s_{ij} \equiv \sigma_{\varepsilon^*}^2 + \sigma_{\nu_{Y,i}}^2 + \sigma_{\nu_{X,j}}^2 (\beta^*)^2, & \text{if } s = t, i = k, j = l \end{cases} \quad (\text{A.2})$$

This formula allows us to populate the entries of $\Sigma_{\varepsilon_{OR}}$. Denote by S_Y the diagonal

matrix with entries $\left[\sigma_{\nu_Y^1}^2, \dots, \sigma_{\nu_Y^{K_Y}}^2 \right]$ and, similarly, by S_X the diagonal matrix with entries $\left[\sigma_{\nu_X^1}^2, \dots, \sigma_{\nu_X^{K_X}}^2 \right]$. Letting I_N denote the $N \times N$ Identity matrix and working with Kronecker products, we can write:

$$\begin{aligned} \Sigma_{\varepsilon_{OR}} &= \sigma_{\varepsilon^*}^2 \left[(1_{(K_X K_Y)} 1'_{(K_X K_Y)}) \otimes I_N \right] \\ &\quad + [S_Y \otimes ((1_{K_X} 1'_{K_X}) \otimes I_N)] \\ &\quad + \beta^{*2} [((1_{K_Y} 1'_{K_Y}) \otimes S_X) \otimes I_N]. \end{aligned} \tag{A.3}$$

Importantly, the result indicates that all of the non-zero entries in $\Sigma_{\varepsilon_{OR}}$ correspond to instances where the individual representing the unit of observation is the same in two different regression matrices.

Define $\hat{\varepsilon}_{OR,(i)} \equiv Y_{OR,i} - X_{OR,i} \hat{\beta}^*$ and consider the estimator for $\hat{\Sigma}_{\varepsilon_{OR}}$ that sets its (i, j) entry equal to $\hat{\varepsilon}_{OR,(i)} \hat{\varepsilon}_{OR,(i)}$ if the (i, j) entry in $\Sigma_{\varepsilon_{OR}}$ is non-zero. Then, by the Law of Large Numbers:

$$\begin{aligned} \hat{\Omega} &\equiv \frac{1}{N} W'_{OR} \hat{\Sigma}_{\varepsilon_{OR}} W_{OR} \rightarrow_p \Omega \\ N \hat{\Sigma}_{\hat{\beta}^*} &\equiv N \frac{X'_{OR} W_{OR} (W'_{OR} W_{OR})^{-1} \hat{\Omega} (W'_{OR} W_{OR})^{-1} W'_{OR} X_{OR}}{(X'_{OR} P_{W_{OR}} X_{OR})^2} \rightarrow_p \Sigma_{\hat{\beta}_{OR}}. \end{aligned}$$

□

A.4 Inference on Correlation Coefficients

A.4.1 Consistency and Asymptotic Normality

We now establish consistency and asymptotic normality of the correlation estimator as stated in Proposition 3:

Proposition 3. *$\hat{\rho}_{XY}^*$ is consistent with an asymptotically normal distribution, where standard errors can be derived using the delta method. These standard errors can be consistently*

estimated using a bootstrap to construct confidence intervals.

Proof. Consistency follows from a straightforward application of the Continuous Mapping Theorem. In particular, note that:

$$\begin{aligned} \widehat{\text{Cov}}[X^a, X^b] \rightarrow_p \text{Cov}[X^a, X^b] = \sigma_{X^*}^2 \\ \widehat{\text{Cov}}[Y^a, Y^b] \rightarrow_p \text{Cov}[Y^a, Y^b] = \sigma_{Y^*}^2 \end{aligned} \Rightarrow \sqrt{\frac{\widehat{\text{Cov}}[X^a, X^b]}{\widehat{\text{Cov}}[Y^a, Y^b]}} \rightarrow_p \sqrt{\frac{\text{Cov}[X^a, X^b]}{\text{Cov}[Y^a, Y^b]}} = \frac{\sigma_{X^*}}{\sigma_{Y^*}}.$$

Given that $\hat{\beta}^*$ is consistent for $\beta^* = \frac{\text{Cov}(X^*, Y^*)}{\sigma_{X^*}^2}$, we have:

$$\hat{\rho}_{XY}^* = \sqrt{\frac{\widehat{\text{Cov}}[X^a, X^b]}{\widehat{\text{Cov}}[Y^a, Y^b]}} \hat{\beta}^* \rightarrow_p \frac{\text{Cov}(X^*, Y^*)}{\sigma_{X^*} \sigma_{Y^*}} = \rho_{XY}^*.$$

For the limiting distribution, assume independent sampling and suppose all observed random variables have bounded 8th moments (so their kurtosis can be consistently estimated).

Then $\hat{\beta}^*$, $\widehat{\text{Cov}}[X^a, X^b]$, and $\widehat{\text{Cov}}[Y^a, Y^b]$ are jointly asymptotically normal:

$$\sqrt{n} \begin{bmatrix} \hat{\beta}^* - \beta^* \\ \widehat{\text{Cov}}[X^a, X^b] - \sigma_{X^*}^2 \\ \widehat{\text{Cov}}[Y^a, Y^b] - \sigma_{Y^*}^2 \end{bmatrix} \rightarrow_d N(0, \Psi),$$

where:

$$\Psi = \begin{bmatrix} \sigma_{\hat{\beta}^*}^2 & \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[X^a, X^b]} & \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[Y^a, Y^b]} \\ \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[X^a, X^b]} & \sigma_{\widehat{\text{Cov}}[X^a, X^b]}^2 & \sigma_{\widehat{\text{Cov}}[X^a, X^b], \widehat{\text{Cov}}[Y^a, Y^b]} \\ \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[Y^a, Y^b]} & \sigma_{\widehat{\text{Cov}}[X^a, X^b], \widehat{\text{Cov}}[Y^a, Y^b]} & \sigma_{\widehat{\text{Cov}}[Y^a, Y^b]}^2 \end{bmatrix}$$

reflects the asymptotic covariance matrix of the estimators $\hat{\beta}^*$, $\widehat{\text{Cov}}[X^a, X^b]$, and $\widehat{\text{Cov}}[Y^a, Y^b]$.

Denoting the kurtosis of X^* and Y^* as $\zeta_X = E[(X^* - E[X^*])^4]$ and $\zeta_Y = E[(Y^* - E[Y^*])^4]$,

respectively, algebraic manipulation yields:

$$\begin{aligned}
\sigma_{\hat{\beta}^*}^2 &= \Sigma_{\hat{\beta}_{OR}} \tag{A.4} \\
\sigma_{\widehat{\text{Cov}}[X^a, X^b]}^2 &= \zeta_X - \sigma_{X^*}^4 + \sigma_{X^*}^2 \left(\sigma_{\nu_X^1}^2 + \sigma_{\nu_X^2}^2 \right) + \sigma_{\nu_X^1}^2 \sigma_{\nu_X^2}^2 \\
\sigma_{\widehat{\text{Cov}}[Y^a, Y^b]}^2 &= \zeta_Y - \sigma_{Y^*}^4 + \sigma_{Y^*}^2 \left(\sigma_{\nu_Y^1}^2 + \sigma_{\nu_Y^2}^2 \right) + \sigma_{\nu_Y^1}^2 \sigma_{\nu_Y^2}^2 \\
\sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[X^a, X^b]} &= \beta \frac{X'_1 X_1 \sigma_{\nu_X^1}^2 + X'_2 X_2 \sigma_{\nu_X^2}^2}{X'_1 X_1 + X'_2 X_2} \\
\sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[Y^a, Y^b]} &= \beta \frac{\sigma_{\nu_Y^1}^2 + \sigma_{\nu_Y^2}^2}{2} \\
\sigma_{\widehat{\text{Cov}}[X^a, X^b], \widehat{\text{Cov}}[Y^a, Y^b]} &= \beta^2 \zeta_X
\end{aligned}$$

The delta method approximation builds on a Taylor expansion of $\hat{\rho}_{XY}^*$ at $(\beta^*, \sigma_{X^*}^2, \sigma_{Y^*}^2)$:

$$\begin{aligned}
\hat{\rho}_{XY}^* &= \sqrt{\frac{\widehat{\text{Cov}}[X^a, X^b]}{\widehat{\text{Cov}}[Y^a, Y^b]}} \hat{\beta}^* \approx \sqrt{\frac{\sigma_{X^*}^2}{\sigma_{Y^*}^2}} \beta^* + \sqrt{\frac{\sigma_{X^*}^2}{\sigma_{Y^*}^2}} \left(\hat{\beta}^* - \beta^* \right) \\
&\quad + \frac{\beta^*}{2\sqrt{\sigma_{X^*}^2 \sigma_{Y^*}^2}} \left(\widehat{\text{Cov}}[X^a, X^b] - \sigma_{X^*}^2 \right) \\
&\quad - \frac{\beta^*}{2} \sqrt{\frac{\sigma_{X^*}^2}{\sigma_{Y^*}^6}} \left(\widehat{\text{Cov}}[Y^a, Y^b] - \sigma_{Y^*}^2 \right).
\end{aligned}$$

Since the remainder in the approximation is $o_p(n)$, asymptotic normality for $\hat{\rho}_{XY}^*$ follows:

$$\begin{aligned}
\sqrt{n} (\hat{\rho}_{XY}^* - \rho_{XY}^*) &= \sqrt{\frac{\sigma_{X^*}^2}{\sigma_{Y^*}^2}} \sqrt{n} \left(\hat{\beta}^* - \beta^* \right) \\
&\quad + \frac{\beta^*}{2\sqrt{\sigma_{X^*}^2 \sigma_{Y^*}^2}} \sqrt{n} \left(\widehat{\text{Cov}}[X^a, X^b] - \sigma_{X^*}^2 \right) \\
&\quad - \frac{\beta^*}{2} \sqrt{\frac{\sigma_{X^*}^2}{\sigma_{Y^*}^6}} \sqrt{n} \left(\widehat{\text{Cov}}[Y^a, Y^b] - \sigma_{Y^*}^2 \right) \\
&\rightarrow_d N \left(0, V_{\hat{\rho}_{XY}^*}^\infty \right).
\end{aligned}$$

To derive the asymptotic variance, we square the right-hand side of this expression and solve for its limiting expectation. Using the expressions from (A.4), we can derive the

asymptotic variance as:

$$\begin{aligned}
V_{\hat{\rho}_{XY}^*}^\infty &= \frac{\sigma_{X^*}^2}{\sigma_{Y^*}^2} \sigma_{\hat{\beta}^*}^2 + \frac{\beta^{*2}}{4\sigma_{X^*}^2 \sigma_{Y^*}^2} \sigma_{\widehat{\text{Cov}}[X^a, X^b]}^2 + \frac{\beta^{*2}}{4} \frac{\sigma_{X^*}^2}{\sigma_{Y^*}^6} \sigma_{\widehat{\text{Cov}}[Y^a, Y^b]}^2 \\
&\quad + \frac{\beta^*}{\sigma_{Y^*}^2} \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[X^a, X^b]} - \beta^* \frac{\sigma_{X^*}^2}{\sigma_{Y^*}^4} \sigma_{\hat{\beta}^*, \widehat{\text{Cov}}[Y^a, Y^b]} - \frac{\beta^{*2}}{2\sigma_{Y^*}^4} \sigma_{\widehat{\text{Cov}}[X^a, X^b], \widehat{\text{Cov}}[Y^a, Y^b]}.
\end{aligned} \tag{A.5}$$

Since all the variance and covariance terms on the right hand side of (A.5) are finite, so long as the variances $\sigma_{X^*}^2$ and $\sigma_{Y^*}^2$ are bounded away from zero, this asymptotic variance is also finite. With asymptotic normality, we can construct confidence intervals using asymptotically pivotal t-statistics centered on the null-hypothesis that $\rho_{XY}^* = \hat{\rho}_{XY}^*$ for all $|\hat{\rho}_{XY}^*| < 1$. With this specification, the bootstrap's empirical distribution function converges to the population distribution function. In particular, standard errors can be consistently estimated using a bootstrap to construct confidence intervals. \square

A.4.2 Bootstrapping Correlation Standard Errors for Confidence Regions

Given the complex formula in (A.5) and the difficulty of computing fourth moments using standard statistical tools, we propose using a bootstrap to compute standard errors for constructing confidence intervals. Proposition 3 above assures that estimated standard errors using a bootstrap are consistent. Incorporating the sampling error from $\widehat{\text{Cov}}[X^a, X^b]$ and $\widehat{\text{Cov}}[Y^a, Y^b]$ requires only a small modification to the standard bootstrap procedure, as presented for the case with two replicates of X and Y in Algorithm 1 below. We present the rescaling approach to computing the ORIV correlation in step 3 as it may facilitate bootstrap implementation in some statistical packages. STATA code implementing the bootstrapped standard errors is shown in Appendix C. A STATA package for computing ORIV correlations and bootstrapped standard errors is available from the authors upon request.

Algorithm 1 Bootstrap Algorithm for ORIV Correlation Standard Errors

For $m = 1, \dots, M$:

1. Randomly draw an index of n observations from $\{1, \dots, n\}$ with replacement. Denote this index: $T^{(m)} = \{t_1^{(m)}, t_2^{(m)}, \dots, t_n^{(m)}\}$.

2. Construct $X^{a,(m)}, X^{b,(m)}, Y^{a,(m)}$, and $Y^{b,(m)}$ so that:

$$X^{a,(m)} = [X_{t_1^{(m)}}^a, X_{t_2^{(m)}}^a, \dots, X_{t_n^{(m)}}^a], \quad X^{b,(m)} = [X_{t_1^{(m)}}^b, X_{t_2^{(m)}}^b, \dots, X_{t_n^{(m)}}^b]$$

$$Y^{a,(m)} = [Y_{t_1^{(m)}}^a, Y_{t_2^{(m)}}^a, \dots, Y_{t_n^{(m)}}^a], \quad Y^{b,(m)} = [Y_{t_1^{(m)}}^b, Y_{t_2^{(m)}}^b, \dots, Y_{t_n^{(m)}}^b].$$

3. Compute ORIV Correlations $\hat{\rho}_{XY}^{(m)}$ from $X^{a,(m)}, X^{b,(m)}, Y^{a,(m)}$, and $Y^{b,(m)}$:

- 3.A. Calculate the bootstrapped attenuation factors

$$\bar{X}^{a,(m)} \equiv \frac{1}{n} \sum_{\tau=1}^n X_{t_\tau}^a, \quad \bar{X}^{b,(m)} \equiv \frac{1}{n} \sum_{\tau=1}^n X_{t_\tau}^b, \quad \bar{Y}^{a,(m)} \equiv \frac{1}{n} \sum_{\tau=1}^n Y_{t_\tau}^a, \quad \bar{Y}^{b,(m)} \equiv \frac{1}{n} \sum_{\tau=1}^n Y_{t_\tau}^b$$

$$\hat{\sigma}_{X^*,(m)}^2 = \frac{1}{n} \sum_{\tau=1}^n \left(X_{t_\tau}^a - \bar{X}^{a,(m)} \right) \left(X_{t_\tau}^b - \bar{X}^{b,(m)} \right),$$

$$\hat{\sigma}_{Y^*,(m)}^2 = \frac{1}{n} \sum_{\tau=1}^n \left(Y_{t_\tau}^a - \bar{Y}^{a,(m)} \right) \left(Y_{t_\tau}^b - \bar{Y}^{b,(m)} \right).$$

- 3.B. Rescale the resampled data to define $\tilde{X}^{a,(m)}, \tilde{X}^{b,(m)}, \tilde{Y}^{a,(m)}$, and $\tilde{Y}^{b,(m)}$.

$$\tilde{X}^{a,(m)} \equiv \frac{X^{a,(m)}}{\sqrt{\hat{\sigma}_{X^*,(m)}^2}}, \quad \tilde{X}^{b,(m)} \equiv \frac{X^{b,(m)}}{\sqrt{\hat{\sigma}_{X^*,(m)}^2}}, \quad \tilde{Y}^{a,(m)} \equiv \frac{Y^{a,(m)}}{\sqrt{\hat{\sigma}_{Y^*,(m)}^2}}, \quad \tilde{Y}^{b,(m)} \equiv \frac{Y^{b,(m)}}{\sqrt{\hat{\sigma}_{Y^*,(m)}^2}}$$

- 3.C. Compute and store $\hat{\rho}_{XY}^{(m)}$ from the ORIV Regression of $\tilde{Y}^{(m)}$ on $\tilde{X}^{(m)}$.

The bootstrap variance estimator is:

$$\hat{V}_{BS}(\hat{\rho}_{XY}) \equiv \frac{1}{M} \sum_{m=1}^M \left(\hat{\rho}_{XY}^{(m)} - \hat{\rho}_{XY} \right)^2$$

A.4.3 Finite-Sample Hypothesis Tests and Confidence Interval Coverage

We performed a series of simulations to ensure hypothesis tests and confidence intervals using asymptotic and bootstrapped standard errors have appropriate size and coverage in finite samples. The simulations specify a data generating process sampling independently in t :

$$\begin{bmatrix} X_t^* \\ Y_t^* \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right).$$

We generated two contaminated measures of both X_t^* and Y_t^* with independent normally distributed measurement error, defining

$$X_t^a = X_t^* + \eta_t^{X,a} \quad X_t^b = X_t^* + \eta_t^{X,b} \quad Y_t^a = Y_t^* + \eta_t^{Y,a} \quad Y_t^b = Y_t^* + \eta_t^{Y,b},$$

where each $\eta_t \sim N(0, \sigma_\eta^2)$ is drawn independently of all other measurement errors.

Different simulations are used to generate data sets of size $n \in \{100, 250, 500, 1000\}$. We estimate models with correlations $\rho \in \{0, 0.25, 0.5, 0.95, 0.99\}$ and measurement error variances $\sigma_\eta^2 \in \{0, 0.1/0.9, 0.25/0.75, 0.5/0.5\}$. Hypothesis tests and confidence intervals are constructed using t-Statistics at the 1%, 5%, and 10% levels, where standard errors are calculated using the asymptotic approximation in (A.5) and the bootstrap algorithm with 10,000 replications.

Table A.1 presents the finite-sample estimated standard errors from 1,000 simulations. Across all specifications, the estimated standard errors display only moderate bias. The only notable bias appears in extreme cases featuring small samples, where measurement error variability and the underlying correlation are both high. In such settings, the bootstrap procedure occasionally generates samples in which the estimated variance $\hat{\sigma}_{Y^*}^2$ is nearly zero. The explosive impact of these draws on the estimated correlations leads to bootstrapped standard errors that greatly exceed the population standard errors. Outside of these extreme cases, the bootstrap demonstrates negligible bias in estimated standard errors. In fact, for reasonable levels of measurement errors, of the order we observe in our data, we see very little bias.

To evaluate the finite-sample performance of hypothesis tests, table A.2 reports rejection frequencies using t-Tests. As with the standard errors, when measurement error constitutes less than 50% of the variance in the underlying variables or correlations are lower than 0.95, inference using bootstrapped or asymptotic standard errors appears reasonable for any sample size. We expect these to be the ranges that are relevant for most experimental work.

Table A.1: Simulated Bias in Estimated Standard Errors for Correlations

Panel A: Bootstrap Standard Error Bias (*1,000)					Panel B: Asymptotic Standard Error Bias (*1,000)				
$n = 100$					$n = 100$				
ρ			σ_η^2		ρ		σ_η^2		
	0	0.1/0.9	0.25/0.75	0.5/0.5		0	0.1/0.9	0.25/0.75	0.5/0.5
0	-1.535	-1.237	-1.132	6.380	0	-1.818	-1.375	-1.543	-2.774
0.25	0.016	-0.239	-0.962	3.110	0.25	-0.665	-0.948	-1.663	-3.177
0.5	1.329	0.633	0.998	9.208	0.5	-1.079	-1.822	-1.329	-5.909
0.95	0.376	-0.982	-0.250	36.380	0.95	-3.332	-3.371	-5.157	-2.167
0.99	0.481	0.792	-0.149	48.505	0.99	5.123	7.613	-0.748	-0.565
$n = 250$					$n = 250$				
ρ			σ_η^2		ρ		σ_η^2		
	0	0.1/0.9	0.25/0.75	0.5/0.5		0	0.1/0.9	0.25/0.75	0.5/0.5
0	0.842	0.752	-0.671	0.504	0	0.830	0.719	-0.708	-0.135
0.25	-0.011	0.122	-0.461	-0.792	0.25	-0.047	0.101	-0.521	-1.367
0.5	-1.460	-0.721	-2.570	-1.323	0.5	-1.873	-1.101	-2.851	-2.017
0.95	-0.010	-0.097	-0.784	-0.841	0.95	-2.368	-2.043	-3.512	-3.558
0.99	0.113	0.224	0.241	0.470	0.99	2.318	2.790	-1.190	-3.186
$n = 500$					$n = 500$				
ρ			σ_η^2		ρ		σ_η^2		
	0	0.1/0.9	0.25/0.75	0.5/0.5		0	0.1/0.9	0.25/0.75	0.5/0.5
0	-1.171	-2.095	-0.517	-2.247	0	-1.130	-2.106	-0.525	-2.445
0.25	-0.215	-0.092	0.200	0.837	250	-0.281	-0.166	0.150	0.643
0.5	-0.906	-1.109	-0.120	-0.387	500	-1.084	-1.276	-0.262	-0.592
0.95	0.312	0.173	0.188	0.938	950	-1.546	-1.610	-1.763	0.540
0.99	0.018	0.046	0.532	-0.067	990	1.390	1.249	-0.537	-1.166
$n = 1,000$					$n = 1,000$				
ρ			σ_η^2		ρ		σ_η^2		
	0	0.1/0.9	0.25/0.75	0.5/0.5		0	0.1/0.9	0.25/0.75	0.5/0.5
0	0.653	0.215	1.076	-0.068	0	0.632	0.223	1.076	-0.063
0.25	0.175	0.437	-0.443	-0.193	0.25	0.150	0.431	-0.431	-0.212
0.5	0.727	0.764	0.242	1.639	0.5	0.658	0.649	0.166	1.688
0.95	-0.201	0.009	-0.146	-0.447	0.95	-0.989	-0.783	-0.716	-0.494
0.99	0.005	0.002	-0.422	0.756	0.99	0.594	0.656	-1.682	1.239

This table reports simulated finite-sample bias in estimated standard errors relative to simulated standard errors for the ORIV correlations in 1,000 simulations with normally distributed variables for a range of sample sizes (n), true correlations (ρ), and variability of measurement error (σ_η^2). Bootstrap standard errors (Panel A) are calculated using the bootstrap algorithm presented in section A.4.2 with 10,000 resampling draws. Asymptotic standard errors (Panel B) are calculated using the formula in (A.5).

For completeness, we explore more extreme cases to identify where the asymptotic approximations may not be valid. When $n = 100$, hypothesis tests typically reject the null hypothesis more often than expected based on the nominal size though the distortion is not extreme. If the true correlation is as high as 0.99 or the variance of measurement error matches or exceeds that of the underlying X^* and Y^* , the asymptotic approximation breaks down in samples with fewer than 1,000 observations. The bootstrap overstates the variance of the estimator in these settings, which results in overly conservative tests that fail to reject the null hypothesis with sufficient frequency.

Table A.2: Simulated Rejection Frequencies for Bootstrapped Correlation t-Tests

Panel A: 1% Level					Panel B: 5% Level					Panel C: 10% Level							
$n = 100$					$n = 100$					$n = 100$							
ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%
0	1.9%	1.1%	1.3%	1.3%	0	6.2%	6.3%	6.9%	5.7%	0	11.3%	10.6%	12.6%	10.8%			
0.25	1.9%	1.9%	1.9%	1.5%	0.25	6.3%	5.9%	6.6%	4.5%	0.25	11.4%	11.0%	10.3%	9.3%			
0.5	1.7%	2.2%	1.7%	0.9%	0.5	7.3%	6.8%	5.9%	3.6%	0.5	11.6%	12.0%	10.0%	8.3%			
0.95	1.8%	1.3%	1.1%	0.2%	0.95	5.8%	5.0%	4.6%	1.8%	0.95	9.2%	9.0%	8.6%	5.3%			
0.99	0.1%	0.0%	0.2%	0.3%	0.99	1.1%	2.4%	2.8%	2.1%	0.99	2.9%	6.2%	7.5%	4.3%			
$n = 250$					$n = 250$					$n = 250$							
ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%
0	1.2%	1.0%	1.3%	0.9%	0	4.5%	5.0%	5.2%	5.0%	0	10.2%	10.4%	10.4%	10.4%			
0.25	1.0%	1.1%	1.0%	1.9%	0.25	6.0%	5.6%	5.7%	5.1%	0.25	10.8%	10.9%	10.3%	9.9%			
0.5	1.3%	1.1%	1.2%	1.9%	0.5	5.4%	5.7%	5.7%	6.0%	0.5	10.1%	11.6%	10.4%	11.1%			
0.95	1.2%	0.7%	0.7%	0.3%	0.95	4.3%	4.1%	4.1%	4.0%	0.95	8.6%	7.8%	8.2%	9.3%			
0.99	0.3%	0.4%	0.7%	0.1%	0.99	2.9%	4.0%	3.9%	3.4%	0.99	5.6%	7.9%	8.7%	7.1%			
$n = 500$					$n = 500$					$n = 500$							
ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%
0	0.6%	0.6%	0.8%	1.2%	0	4.2%	4.2%	4.0%	6.3%	0	9.4%	9.6%	8.6%	10.0%			
0.25	1.0%	1.0%	1.6%	1.2%	0.25	5.5%	4.7%	5.5%	5.7%	0.25	10.3%	10.8%	10.3%	11.3%			
0.5	0.7%	0.7%	0.8%	1.6%	0.5	5.0%	4.4%	5.7%	6.1%	0.5	8.6%	9.8%	9.4%	10.8%			
0.95	1.4%	1.4%	1.6%	0.6%	0.95	6.3%	5.7%	5.0%	5.3%	0.95	11.2%	10.7%	8.2%	10.1%			
0.99	0.7%	0.9%	0.7%	0.2%	0.99	3.6%	4.8%	4.6%	2.5%	0.99	7.8%	9.8%	9.5%	8.9%			
$n = 1,000$					$n = 1,000$					$n = 1,000$							
ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%	ρ	0	10%	σ_η^2	25%	50%
0	0.3%	0.6%	0.5%	1.4%	0	5.0%	5.0%	4.8%	5.4%	0	10.0%	9.8%	10.3%	9.6%			
0.25	0.9%	1.0%	1.1%	1.5%	0.25	5.3%	5.3%	5.8%	5.6%	0.25	10.3%	9.7%	11.3%	9.9%			
0.5	1.2%	1.2%	1.3%	1.1%	0.5	4.9%	5.6%	5.6%	5.6%	0.5	11.5%	10.0%	11.0%	10.3%			
0.95	0.9%	1.2%	0.6%	0.8%	0.95	5.5%	5.5%	4.1%	4.5%	0.95	10.4%	9.7%	9.6%	9.2%			
0.99	0.8%	1.0%	0.6%	1.0%	0.99	3.7%	4.6%	4.4%	4.9%	0.99	7.1%	10.2%	10.2%	9.9%			

This table reports simulated rejection frequency of the null hypothesis using t-Tests with bootstrapped standard errors (presented in section A.4.2 with 10,000 resampling draws) for ORIV correlations. The results report on 1,000 simulations with normally distributed variables for a range of sample sizes (n), true correlations (ρ), and variability of measurement error (σ_η^2). Critical values are based on nominal test sizes of 1% (Panel A), 5% (Panel B), and 10% (Panel C).

Overall, these results affirm the obvious caution when performing inference in relatively small samples in which most of the variability in observations is due to measurement error. More extreme cases may require additional data or a more refined test statistic accounting for boundary issues that complicate the sampling distribution of the estimator as $\rho_{XY}^* \rightarrow 1$ (a complex statistical problem beyond the scope of the current paper).

A.5 Equivalent Estimators

The ORIV estimator provides a convenient and intuitive representation for consolidating all the information available to the experimenter. This convenient representation can be motivated by many representations and so we highlight here a couple of special cases yielding numerically equivalent estimators to ORIV. These equivalences also motivate an interpretation of ORIV as a model combination estimator that will be useful in characterizing the estimator's efficiency.

A.5.1 Averaging Left-Side Variables: $K_Y \geq 2$

As mentioned in the text, averaging of observations is common in experimental work. As it turns out, when measurement error occurs on both left- and right-side variables, a numerically equivalent estimator to ORIV can be derived by averaging observations of the left-side variable and stacking only right-side variables.

Proposition 4. *Suppose $K_Y \geq 2$, let $\bar{Y} = \frac{1}{K_Y} \sum_{k=1}^{K_Y} Y^k$, and define:*

$$\begin{aligned} \bar{Y}_{OR} &= \mathbf{1}_{K_X} \otimes \bar{Y} \\ \bar{X}_{OR} &= [X^1, \dots, X^{K_X}]' \end{aligned} \quad \bar{W}_{OR} = \begin{bmatrix} W^1 & 0 & \dots & 0 \\ 0 & W^2 & \ddots & 0 \\ 0 & \dots & \ddots & \vdots \\ 0 & \dots & 0 & W^{K_X} \end{bmatrix}.$$

Then $\bar{\beta}_{OR} = (\bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{X}_{OR})^{-1} \bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{Y}_{OR} = \hat{\beta}^*$.

Proof. The first step in the proof shows that $X'_{OR} P_{W_{OR}} X_{OR} = K_Y \bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{X}_{OR}$, which comes immediately from the SUR structure ORIV imposes on the first stage regressions of X^k on W^k . Due to the blocked structure of \bar{W}_{OR} ,

$$\bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{X}_{OR} = \sum_{k=1}^{K_X} X^{k'} P_{W^k} X^k.$$

Due to the stacked structure of \tilde{X}^k and \tilde{W}^k , $\tilde{X}^{k'} P_{\tilde{W}^k} \tilde{X}^k = K_Y X^{k'} P_{W^k} X^k$, which when blocked across the measurements of X^* , gives:

$$X'_{OR} P_{W_{OR}} X_{OR} = \sum_{k=1}^{K_X} \tilde{X}^{k'} P_{\tilde{W}^k} \tilde{X}^k = \sum_{k=1}^{K_X} K_Y X^{k'} P_{W^k} X^k = K_Y \bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{X}_{OR}.$$

We complete the proof using a parallel argument to show $X'_{OR} P_{W_{OR}} Y_{OR} = K_Y \bar{X}'_{OR} P_{\bar{W}_{OR}} \bar{Y}_{OR}$. □

A.5.2 Averaging Estimates of Individual IV Models: $K_Y = 1$

In the special case where $K_Y = 1$, either because there is only one measurement for Y^* or because multiple measurements have been concentrated into \bar{Y} , the ORIV estimator is numerically equivalent to a weighted average of estimates from the individual IV specifications.

Proposition 5. *Let $\hat{\beta}_k = (X^{k'} P_{W^k} X^k)^{-1} X^{k'} P_{W^k} Y$ and $\omega_k = (X^{k'} P_{W^k} X^k)$, then:*

$$\tilde{\beta}^* = \frac{1}{\sum_{k=1}^{K_X} \omega_k} \sum_{k=1}^{K_X} \omega_k \hat{\beta}_k = \hat{\beta}^*.$$

Proof. We begin with the observation that $\tilde{W}^k = W^k$ and $\tilde{X}^k = X^k$ when $K_Y = 1$. From the previous subsection, we can write:

$$\hat{\beta}^* = \left(\sum_{k=1}^{K_X} X^{k'} P_{W^k} X^k \right)^{-1} \sum_{k=1}^{K_X} X^{k'} P_{W^k} Y = \left(\sum_{k=1}^{K_X} \omega_k \right)^{-1} \sum_{k=1}^{K_X} X^{k'} P_{W^k} X^k \hat{\beta}_k = \frac{\sum_{k=1}^{K_X} \omega_k \hat{\beta}_k}{\sum_{k=1}^{K_X} \omega_k} = \tilde{\beta}^*.$$

□

A further specialized case of Proposition 5 applies when $K_X = 2$. In this setting,

$$\omega_1 = X^{1'} P_{W^1} X^1 = X^{1'} P_{X^2} X^1 = X^{2'} P_{X^1} X^2 = X^{2'} P_{W^2} X^2 = \omega_2.$$

Consequently, in the setting where $K_Y = 1$ and $K_X = 2$, the ORIV estimator can be

computed by taking the simple average of the two IV estimators $\hat{\beta}_1$ and $\hat{\beta}_2$.

A.6 GMM Representation, Efficiency, and GLS Estimation

We close our discussion of the ORIV estimator with a brief comment on the estimator's efficiency. First, we present an unweighted GMM representation for the ORIV estimator to establish ORIV's efficiency in a homoskedastic model where measurement error has a constant variance across replicates. In the special case where measurement errors are homoskedastic, the ORIV estimator is asymptotically efficient. In general, when measurement errors are characterized by heteroskedastic variances, the efficiently weighted GMM estimator corresponds to a GLS implementation of ORIV that can improve efficiency.

A.6.1 GMM Representation of the ORIV Estimator

ORIV's efficiency derives from combining all available exclusion restrictions when estimating the regression model. Let W_t^j denote the t^{th} row of the matrix W_t , construct the $K_Y K_X$ identifying moment conditions:

$$g_t^{k,j} \equiv E[(Y_t^k - X_t^j \beta) W_t^j] = 0, \quad k = 1, \dots, K_Y, \quad j = 1, \dots, K_X.^2$$

We can consolidate these conditions using a vector representation:

$$g_t \equiv \left[g_t^{1,1}, \dots, g_t^{1,K_X}, g_t^{2,1}, \dots, g_t^{2,K_X}, \dots, g_t^{K_Y,1}, \dots, g_t^{K_Y,K_X} \right]'$$

²This mean independence assumption is implied by the conditional independence assumptions in Assumptions 1 and 2. However, the conditional independence assumptions provide additional restrictions on higher order moments such that $E[(Y_t^k - X_t^j \beta) f(W_t^j)] = 0$ for any function f . Under conditional independence, these additional restrictions are informative and could be used to enhance estimator efficiency or test the validity of the independence assumptions.

The GMM objective function weights the quadratic loss for the sample-estimated moment conditions $\bar{g}_n \equiv \frac{1}{n} \sum_{t=1}^n g_t$ so that:

$$\hat{\beta}_{GMM} = \operatorname{argmin} \bar{g}_n' \mathcal{W} \bar{g}_n$$

where \mathcal{W} is a $K_Y K_X \times K_Y K_X$ weighting matrix.

In our leading specification, which features homoskedastic errors and homogeneous variances across replicated measures, the efficient choice for \mathcal{W} is the identity matrix (since all moment restrictions are equally informative). The GMM objective function in this case exactly matches the objective function minimized by the ORIV estimator, establishing ORIV's efficiency as demonstrated in Section A.6.2. More generally, in the presence of heteroskedasticity or heterogeneous contamination variances, the identity-weighting matrix may be inefficient. In these cases, discussed in Sections A.6.3 and A.6.4, the efficiently-weighted GMM estimator is equivalent to an FGLS implementation of ORIV that also achieves efficiency.

A.6.2 Efficiency in Homoskedastic Setting: $K_Y = 1$

In this subsection, we consider efficiency when $K_Y = 1$ while maintaining the following homogeneity assumption for variances in the model.

Assumption 3 (Identical Variances Across Replicates). *Suppose measurement error has the same variance for all replicates, so that:*

1. $\sigma_{\nu, X, k} = \sigma_{\nu, X, j} = \sigma_{\nu, X}, \forall j, k$
2. $\sigma_{\nu, Y, k} = \sigma_{\nu, Y, j} = \sigma_{\nu, Y}, \forall j, k$.

Intuitively, efficiency obtains under Assumption 3 because each observation is equally informative and, as such, should be weighted equally. To verify efficiency, we analyze the variance-covariance matrix for estimates from individual models. Let $\hat{B} = [\hat{\beta}_1, \dots, \hat{\beta}_{K_X}]'$. The asymptotic variance-covariance matrix for this vector is defined by the limit of the scaled

covariances:

$$\begin{aligned} \text{NCov} \left[\hat{\beta}_i, \hat{\beta}_j \right] &= N \frac{X^i P_{W^i} P_{W^j} X^j}{(X^i P_{W^i} X^i) (X^j P_{W^j} X^j)} s_{1(\{i=j\})} \\ &= \frac{\frac{1}{N} X^i P_{W^i} P_{W^j} X^j}{\left(\frac{1}{N} X^i P_{W^i} X^i \right) \left(\frac{1}{N} X^j P_{W^j} X^j \right)} s_{1(\{i=j\})} \xrightarrow{p} \zeta_{i,j}. \end{aligned}$$

We will show below that the homoskedastic measurement error guaranteed by Assumption 3 implies $\zeta_{i,j} = \zeta_{k,l}, \forall i \neq j, k \neq l$, and $\zeta_{i,i} = \zeta_{j,j}, \forall i, j$, allowing us to express the asymptotic variance-covariance matrix for \hat{B} as:

$$\Sigma_{\hat{B}}^{\infty} = 1_{K_X} 1'_{K_X} \zeta_{1,2} + I_{K_X} \zeta_{1,1}.$$

If we want to form an efficient linear combination of the estimators in \hat{B} while maintaining consistency, we would be looking for a vector of weights, $w = [w_1, w_2, \dots, w_{K_X}]'$, that sum to one and minimize:

$$\begin{aligned} w^* &= \arg \min_w w' \Sigma_{\hat{B}}^{\infty} w, \text{ such that, } w' 1_{K_X} = 1 \\ &\Rightarrow w^* = K_X^{-1} 1_{K_X}. \end{aligned}$$

That is, in the homoskedastic setting, equally weighting the estimates of the individually valid IV estimators is asymptotically efficient.

Proposition 6 (Two-Parameter Covariance Matrix for Individual IV Estimators). *Under Assumptions (1)–(3), the variance-covariance matrix for estimates from the individual IV models for β^* features constant correlations and homoskedastic variances. That is,*

$$\Sigma_{\hat{B}}^{\infty} = 1_{K_X} 1'_{K_X} \zeta_{1,2} + I_{K_X} \zeta_{1,1}.$$

Proof. By virtue of the homoskedastic measurement error, in the limit:

$$\begin{aligned} \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} P_{W^i} X^i &= \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{j'} P_{W^j} X^j \equiv \omega_{11}, \text{ and} \\ \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} P_{W^i} P_{W^j} X^j &= \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{k'} P_{W^k} P_{W^l} X^l \equiv \omega_{12}, \text{ and, } \forall i \neq j, k \neq l \end{aligned}$$

The exact formulas for the constants ω involve some algebraic manipulation. The relatively simple constant for the norm of X^i projected onto its instruments W^i is just the expected R^2 of the first stage regression:

$$\omega_{11} = \frac{(K_X - 1) \mathbb{E}[(X^*)^2]^2}{(K_X - 1) \mathbb{E}[(X^*)^2] + \sigma_{\nu_X}^2}.$$

With respect to ω_{12} , notice that

$$\begin{aligned} \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} W^i &= \mathbb{E}[X_n^i W_n^{i'}] = \mathbb{E}[(X^*)^2] 1'_{K_X-1} \\ \text{plim}_{N \rightarrow \infty} \frac{1}{N} W^{i'} W^i &= \mathbb{E}[W_n^i W_n^{i'}] = \mathbb{E}[(X^*)^2] 1_{K_X-1} 1'_{K_X-1} + \sigma_{\nu_X}^2 I_{K_X-1} \\ \text{plim}_{N \rightarrow \infty} \frac{1}{N} W^{i'} W^j &= \mathbb{E}[W_n^i W_n^{j'}] = \mathbb{E}[(X^*)^2] 1_{K_X-1} 1'_{K_X-1} + \sigma_{\nu_X}^2 E_i' E_j \end{aligned}$$

Here E_i is a $K_X \times (K_X - 1)$ matrix constructed by removing the i^{th} column from the $K_X \times K_X$ identity matrix. Establishing the probability limit result for $\text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} P_{W^i} P_{W^j} X^j$ requires chaining together and inverting a series of these expressions:

$$\text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} P_{W^i} P_{W^j} X^j = \text{plim}_{N \rightarrow \infty} \frac{1}{N} X^{i'} W^i \left(\frac{1}{N} W^{i'} W^i \right)^{-1} \frac{1}{N} W^{i'} W^j \left(\frac{1}{N} W^{j'} W^j \right)^{-1} \frac{1}{N} W^{j'} X^j$$

Importantly, the dependence on which measurements are involved, i and j is entirely wrapped up in inner products of the form $1'_{K_X-1} E_i' E_j 1_{K_X-1} = K_X - 2$ □

A.6.3 Efficiency in Heteroskedastic Settings: $K_Y = 1$

We now consider efficiency in the heteroskedastic setting, relaxing assumption 3 to allow for the case where $\sigma_{\nu_X^k} \neq \sigma_{\nu_X^j}$, while maintaining the assumption that $K_Y = 1$. In this setting, the heteroskedastic errors will admit a more efficient GLS estimator. Here, S_X is the diagonal matrix with $\sigma_{\nu_X^k}^2$ in the $(k, k)^{th}$ entry, so that:

$$\Sigma_{\varepsilon_{OR}}^{K_Y=1} = (\sigma_{\varepsilon^*}^2 + \sigma_{\nu_Y}^2) [(1_{K_X} 1'_{K_X}) \otimes I_N] + \beta^{*2} (S_X \otimes I_N)$$

Importantly, even though the off-diagonal terms retain a homoskedastic structure, now the diagonal terms reflect the heteroskedasticity in the measurement error for X . It is this heteroskedasticity that allows for enhanced efficiency. For efficient estimation in the presence of heteroskedasticity, the usual formulas for GLS estimation can be used for ORIV.

To characterize the efficient weights associated with each of the individual IV models, we can again consider the model combination exercise pertaining to the combination of estimates in \hat{B} to minimize variance. $\Sigma_{\hat{B}}$ is no longer going to have constant variances and covariances. Without a homogeneity assumption, we cannot further simplify its representation beyond the results above:

$$\begin{aligned} w^* &= \arg \min_w w' \Sigma_{\hat{B}}^\infty w, \text{ such that, } w' 1_{K_X} = 1 \\ \Rightarrow w^* &= \frac{\Sigma_{\hat{B}}^{\infty-1} 1_{K_X}}{1'_{K_X} \Sigma_{\hat{B}}^{\infty-1} 1_{K_X}}. \end{aligned}$$

Without reweighting observations, the ORIV estimator will assign weights of $w_{i,OR} = \frac{X^i P_{W^i} X_i}{\sum_{k=1}^K (X^k P_{W^k} X_k)}$. Therefore, consider reweighting observations for the i^{th} model to achieve

the optimal weights. Specifically, defining $\lambda_i \equiv \sqrt{\frac{w_i^*}{w_{i,OR}}}$ and $\lambda \equiv [\lambda_1, \dots, \lambda_{K_X}]'$, let

$$\begin{aligned} \tilde{Y}_{OR,\lambda} &= \lambda \otimes Y^1 \\ \tilde{X}_{OR,\lambda} &= [\lambda_1 X^1, \dots, \lambda_{K_X} X^{K_X}]' \end{aligned} \quad \tilde{W}_{OR,\lambda} = \begin{bmatrix} \lambda_1 W^1 & 0 & \dots & 0 \\ 0 & \lambda_2 W^2 & \ddots & 0 \\ 0 & \dots & \ddots & \vdots \\ 0 & \dots & 0 & \lambda_{K_X} W^{K_X} \end{bmatrix}.$$

Then $\hat{\beta}_{GLS}^* = \left(\tilde{X}'_{OR,\lambda} P_{\tilde{W}_{OR,\lambda}} \tilde{X}_{OR,\lambda} \right)^{-1} \tilde{X}'_{OR,\lambda} P_{\tilde{W}_{OR,\lambda}} \tilde{Y}_{OR,\lambda}$ is the efficient, asymptotically unbiased estimator for β^* .

A.6.4 Efficiency in Heteroskedastic Settings: $K_Y \geq 2$

The efficiency arguments in the homoskedastic setting when $K_Y = 1$ extend immediately to models with $K_Y > 1$ by the equivalence result in Proposition 4. The heteroskedastic setting is slightly complicated because the simple average is no longer the most efficient way to combine the information in the different measurements of Y . Instead, we replace the simple average with the efficient average, $\bar{Y}_{GLS} = \frac{1}{\sum_{k=1}^{K_Y} \sigma_{\nu_Y^k}^{-2}} \sum_{k=1}^{K_Y} \sigma_{\nu_Y^k}^{-2} Y^k$.

Having minimized measurement error in the left-side variable, Y^* , we can construct the weighted GLS estimator from the previous subsection. Define $\tilde{Y}_{OR,\lambda} = \lambda \otimes \bar{Y}_{GLS}$. Then $\hat{\beta}_{GLS}^* = \left(\tilde{X}'_{OR,\lambda} P_{\tilde{W}_{OR,\lambda}} \tilde{X}_{OR,\lambda} \right)^{-1} \tilde{X}'_{OR,\lambda} P_{\tilde{W}_{OR,\lambda}} \tilde{Y}_{OR,\lambda}$ is the efficient, asymptotically unbiased estimator for β^* .

A.6.5 Estimating Measurement Error Variance and FGLS Efficiency: $K_Y = 1$

Feasibly implementing the GLS adjustments proposed in the previous two subsections requires estimating the weighting parameters λ . Clearly, the weights for $w_{i,OR}$ can be readily estimated from sample data. As such, we focus on estimating w^* , an exercise that turns to estimating Σ_B^∞ . In the heteroskedastic setting,

$$\text{Cov} \left[\hat{\beta}_i, \hat{\beta}_j \right] = \begin{cases} \frac{X^{i'} P_{W^i} P_{W^j} X^j}{(X^{i'} P_{W^i} X^i)(X^{j'} P_{W^j} X^j)} s_{10}, & \text{if } i \neq j \\ \frac{1}{X^{i'} P_{W^i} X^i} s_{1i}, & \text{if } i = j \end{cases}.$$

Feasible implementation then requires estimating s_{10} and s_{1i} . Recalling their definition from (A.2), a feasible estimator for s_{10} would average over the cross-products between residuals corresponding to two different elicitation of X^* for the same individual. The feasible estimator for s_{1i} then just sums those squared residuals corresponding to the i^{th} model's specification.

$$\hat{s}_{10} = \frac{1}{N} \sum_{n=1}^N \frac{2}{(K_X - 1)(K_X - 2)} \sum_{i=1}^{K_X-1} \sum_{j=i+1}^{K_X} \hat{\varepsilon}_n^i \hat{\varepsilon}_n^j, \text{ and, } \hat{s}_{1i} = \frac{1}{N} \sum_{n=1}^N \hat{\varepsilon}_n^{i2}.$$

Applications of the Law of Large Numbers then ensure that $\hat{s}_{1i} \rightarrow_p s_{1i}, i = 0, \dots, K_X$. Substituting these values into the formula for Σ_B^∞ defines a consistent estimator $\hat{\Sigma}_B \rightarrow_p \Sigma_B^\infty$, which can be used to consistently estimate the efficient weights, $\hat{w}^* = \frac{\hat{\Sigma}_B^{-1} 1_{K_X}}{1'_{K_X} \hat{\Sigma}_B^{-1} 1_{K_X}}$.

A.6.6 Estimating Measurement Error Variance and FGLS Efficiency: $K_Y \geq 2$

Implementing feasible efficient estimators with multiple measurements of Y^* , each of which has different variances, requires estimating the variance-covariance matrix for the different measurements. This setting is straightforward, as the objective of inference can be directly estimated. Let $\hat{\Sigma}_Y \rightarrow \Sigma_Y$ denote the sample and population covariance matrix for the different measurements of Y . Recalling that S_Y is the diagonal matrix of measurement error variances for Y^* , it is immediately apparent that:

$$\Sigma_Y = \sigma_{\varepsilon^*}^2 1_{K_Y} 1'_{K_Y} + S_Y.$$

Consequently, a consistent estimate of $\sigma_{\nu_Y^k}^2$ could estimate $\sigma_{\varepsilon^*}^2$ by averaging the off-diagonal entries of Σ_Y , and subtracting this average from the k^{th} diagonal entry.

A.6.7 A Note of Caution on FGLS Implementations

When implementing FGLS for ORIV, it is best to exploit the structure of the model to its fullest extent. Simply applying FGLS to the full ORIV specification will perform very poorly. Since the population covariance matrix for residuals is singular, in finite samples, its inverse is not well scaled. Even attempting to simply combine elicitation for Y efficiently can result in extreme weights without imposing homogeneity on the off-diagonal entries in Σ_Y . While we do not present detailed results to this effect, our simulation analysis suggests that FGLS estimators perform poorly relative to the simple ORIV estimator. Unless there is good reason to model substantial heterogeneity in measurement error, our informal recommendation is to avoid FGLS corrections.

B Other Ways of Using Controls

B.1 Principal Components

The sixth column of Table 3 contains 76 control variables (including the non-parametric controls for perceived rank and performance). With a dataset the size of ours, this does not present a challenge, as can be seen from the fact that standard errors of the coefficient on gender are stable across specifications. However, this many controls would be infeasible in a sample on the order of the original NV experiment. Moreover, a potential concern with the final column of Table 3 is that adding too many controls measured with error may bias the coefficient on gender downwards. Although this is unlikely to be the case here due to the size of our dataset and the fact that we get similar results with only five controls, it may be a concern in smaller datasets. This raises the question of how to use a relatively exhaustive control strategy with available degrees of freedom.

A growing literature in statistics and econometrics considers inference in the presence of a high-dimensional set of sparse controls.³ One way to cope with this issue is to perform model selection after rotating the controls into their principal components, following the strategy proposed by Belloni, Chernozhukov, and Hansen (2013). This transformation concentrates the information from the controls into relatively few factors, effectively controlling for a rich characterization of risk preferences and overconfidence without giving up too many degrees of freedom.

Table B.1 illustrates this approach. We first conduct a principal components analysis of all 76 controls used in the last column of Table 3. We then enter these principal components sequentially in the columns of Table B.1. As can be seen, the first principal component causes the coefficient on gender to fall by approximately half, the third by a further half,

³This literature's roots lie in machine learning techniques for automating model selection, including the LASSO (Tibshirani, 1996), SCAD (Fan and Li, 2001), and the Dantzig Selector (Candes and Tao, 2007). Performing inference after model selection presents a non-trivial statistical problem (Leeb and Pötscher, 2005) with recent innovations from Belloni, Chernozhukov, and Wang (2011), Fan and Liao (2014), Belloni, Chernozhukov, and Hansen (2013), and Van de Geer et al. (2014) establishing new techniques for robust inference after model selection.

Table B.1: Principal components can be used to save degrees of freedom.

Dependent Variable:	Chose to Compete ($N = 783$)					
Male	0.19*** (.034)	0.10*** (.034)	0.10*** (.034)	0.054* (.033)	0.054* (.033)	0.041 (.033)
First Principal Component		0.14*** (.017)	0.14*** (.017)	0.15*** (.016)	0.15*** (.016)	0.15*** (.016)
Second Principal Component			0.020 (.016)	0.020 (.015)	0.019 (.015)	0.019 (.015)
Third Principal Component				-0.14*** (.015)	-0.13*** (.015)	-0.14*** (.015)
Fourth Principal Component					-0.0094 (.015)	-0.0093 (.015)
Fifth Principal Component						0.034** (.015)
Adjusted R^2	0.038	0.12	0.12	0.20	0.20	0.20

Notes: ***, **, * denote statistical significance at the 1%, 5%, and 10% level, with standard errors in parentheses. Coefficients and standard errors on all non-dichotomous measures are standardized. $N = 783$.

and the fifth by an additional 25%. The second and fourth have no impact on the coefficient on gender. The adjusted- R^2 begins to drop when the 12th component is entered, at which point the coefficient on male is 0.0431 (s.e. 0.0330)—that is, the coefficient on male does not change meaningfully as components 6 through 11 are added. Importantly, this suggests one can control for all relevant variation using only five controls. Moreover, this strategy allows the use of non-parametric or semi-parametric versions of the controls we enter linearly.

As can be seen throughout the table, the second and fourth principal components are statistically insignificant, indicating the potential for using LASSO, or similar variable selection techniques, when using principal components. The Belloni, Chernozhukov, and Hansen (2013) approach would imply a two-stage model selection strategy, using first a LASSO regression to select the principal components of the controls that correlate with tournament participation and then using a second LASSO regression to select the components that correlate with gender. We refer interested readers to their paper for details of the algorithm.

B.2 Measurement Error in Binary Choices

As mentioned in the text, Niederle and Vesterlund (2007) attempt to control for preferences with another competition choice. Namely, in the last stage of the experiment (their Task 4), participants are given a second opportunity to be paid for their performance in the piece-rate task (Task 1). They may choose whether to be paid as a piece rate, or to enter their Task 1 performance into a tournament with the other three participants in the group (this decision affected participants' payoffs when Task 4 was randomly chosen for payment by the experimenters). This choice has the same payoff features as the competitive choice given in their main task (Task 3). The idea is that the choice in Task 4 would control for risk aversion, overconfidence, and feedback aversion, so that different choices in Task 3 and Task 4 would be explained by a difference in preferences for competition per se. However, in the presence of measurement error, this control is subject to the issues highlighted in Section 3. Here, we explore the effect of measurement error in binary controls theoretically. We then show that an analysis of NV's data that properly accounts for measurement error leads to the same conclusion we arrive at, that gender differences in competition are driven by gender differences in fundamental attitudes such as risk aversion and overconfidence.

Although this section is motivated by the approach in NV, it is developed in generality as it applies to any case where a binary elicitation, measured with error, is used as a control. The next subsection carries out this general analysis, and the following subsection gives a numerical example that is closely related to NV.

B.2.1 Stochastic Choice with Binary Preference Control

This section provides the formal arguments underlying Section 3.3. In the general formulation, let individual i have latent characteristics X_i (risk and overconfidence), and be identified by treatment D_i (gender). Each such individual faces two binary decision tasks in which they report $Y_i^a \in \{0, 1\}$ (competition) as well as its replicate $Y_i^b \in \{0, 1\}$ (entering piece-rate performance into a competition). Each individual answers 1 to both the decision task and

the replicate with probability $p_i(X_i)$ that depends on X_i but is independent of treatment, D_i , so that

$$\mathbb{E}[Y_i^a|X_i, D_i] = \mathbb{E}[Y_i^a|X_i] = p_i(X_i) = \mathbb{E}[Y_i^b|X_i] = \mathbb{E}[Y_i^b|X_i, D_i].$$

Though $Y_i^a|X_i \perp D_i \perp Y_i^b|X_i$, the unconditional statement is not generally true due to potential dependence between D_i and X_i . Consequently, p_i is correlated with D_i only because both are correlated with X_i .

By a standard application of the Frisch-Waugh-Lovell Theorem, we can estimate the effect of D on Y^a conditional on a set of controls in two stages. That is, we can get rid of the dependence on X_i , through $p_i(X_i)$, to directly understand how measurement error would produce a biased estimate of the effect of D on Y^a , even controlling for Y^b . In the first stage, we regress Y^a and D on the controls and recover the residuals from both regressions:

$$\begin{aligned} Y^a &= \pi_{Y^a,0} + \pi_{Y^a,1}p_i(X_i) + u_{Y^a} \\ D &= \pi_{D,0} + \pi_{D,1}p_i(X_i) + u_D. \end{aligned}$$

In the second stage, we regress the residual variation in outcomes, u_{Y^a} , on the residual variation in treatment, u_D .

$$u_{Y^a} = \beta u_D + \varepsilon.$$

Without measurement error, the estimate of β would be zero.

However, when Y_i^b as a proxy for $p_i(X_i)$ this introduces measurement error. In this case, the first-stage estimates for $\pi_{Y^a, \cdot}$ and $\pi_{D, \cdot}$ will be biased towards zero by measurement error. Consequently, in the second stage regression of u_{Y^a} on u_D , both residuals will be tainted by persistent variation in the controls due to measurement error. Further, the contamination in both residuals will be correlated. This correlated contamination is what drives distorted inference in the second stage regression.

Replacing each individual's choice probability p_i with the replicate Y_i^b provides just this form of measurement error. To show this formally, denote this error as $\nu_i = Y_i^b - p_i$, which takes the value $-p_i$ with probability $1 - p_i$ and the value $1 - p_i$ with the complementary probability p_i . Then we have

$$\begin{aligned} Y^a &= \tilde{\pi}_{Y^a,0} + \tilde{\pi}_{Y^a,1}Y^b + \tilde{u}_{Y^a} = \tilde{\pi}_{Y^a,0} + \tilde{\pi}_{Y^a,1}(p + \nu) + \tilde{u}_{Y^a} \\ D &= \tilde{\pi}_{D,0} + \tilde{\pi}_{D,1}Y^b + \tilde{u}_D = \tilde{\pi}_{D,0} + \tilde{\pi}_{D,1}(p + \nu) + \tilde{u}_D. \end{aligned}$$

The residuals for this regression clearly differ from those in the regression without measurement error. Let $\gamma_{Y^a} = \frac{\text{Var}[Y^a]}{\text{Var}[Y^a] + \text{Var}[\nu]}$ and $\gamma_D = \frac{\text{Var}[D]}{\text{Var}[D] + \text{Var}[\nu]}$, so that $\tilde{\pi}_{\cdot,1} = \gamma \cdot \pi_{\cdot,1}$. Note that $\pi_{1,\cdot} - \tilde{\pi}_{1,\cdot} = (1 - \gamma)\pi_{1,\cdot}$ and

$$\begin{aligned} \pi_{0,\cdot} = \mathbb{E}[Y^a] - \pi_{1,\cdot}\mathbb{E}[p] &\Rightarrow \tilde{\pi}_{0,\cdot} = \pi_{0,\cdot} + (\pi_{1,\cdot} - \tilde{\pi}_{1,\cdot})\mathbb{E}[p] \\ \tilde{\pi}_{0,\cdot} = \mathbb{E}[Y^a] - \tilde{\pi}_{1,\cdot}\mathbb{E}[Y^b] &= \pi_{0,\cdot} + (1 - \gamma)\pi_{1,\cdot}\mathbb{E}[p]. \end{aligned}$$

We can now relate the residuals \tilde{u}_{\cdot} to their uncontaminated counterpart u_{\cdot} :

$$\begin{aligned} u_{\cdot} - \tilde{u}_{\cdot} &= \tilde{\pi}_{0,\cdot} - \pi_{0,\cdot} + (\tilde{\pi}_{1,\cdot} - \pi_{1,\cdot})p + \tilde{\pi}_{1,\cdot}\nu \\ &= (1 - \gamma)\pi_{1,\cdot}\{\mathbb{E}[p] + p\} + \gamma\pi_{1,\cdot}\nu. \end{aligned}$$

Regressing the contaminated residual in outcomes on the contaminated residual in treatment then yields a spurious correlation, as

$$\begin{aligned} \text{Cov}[\tilde{u}_{Y^a}, \tilde{u}_D] &= \text{Cov}[u_{Y^a} + (1 - \gamma_{Y^a})\pi_{1,Y^a}p + \gamma_{Y^a}\pi_{1,Y^a}\nu, u_D + (1 - \gamma_D)\pi_{1,D}p + \gamma_D\pi_{1,D}\nu] \\ &= \text{Cov}[u_{Y^a}, u_D] + (1 - \gamma_{Y^a})\pi_{1,Y^a}(1 - \gamma_D)\pi_{1,D}\text{Var}[p] + \gamma_{Y^a}\pi_{1,Y^a}\gamma_D\pi_{1,D}\text{Var}[\nu], \end{aligned}$$

where constant expectations are dropped for compactness. We also see that:

$$\text{Var}[\tilde{u}_D] = \text{Var}[u_D] + (1 - \gamma_D)^2\pi_{1,D}^2\text{Var}[p] + \gamma_D^2\pi_{1,D}^2\text{Var}[\nu].$$

Consequently, even though $\text{Cov}[u_{Y^a}, u_D] = 0$, when we test the second stage regression:

$$\tilde{u}_{Y^a} = \tilde{\beta}\tilde{u}_D,$$

we are likely to get biased results indicating a significant treatment effect in the contaminated data because:

$$\mathbb{E}[\tilde{\beta}] = \frac{(1 - \gamma_Y^a)\pi_{1,Y^a}(1 - \gamma_D)\pi_{1,D}\text{Var}[p] + \gamma_{Y^a}\pi_{1,Y^a}\gamma_D\pi_{1,D}\text{Var}[\nu]}{\text{Var}[u_D] + (1 - \gamma_D)^2\pi_{1,D}^2\text{Var}[p] + \gamma_D^2\pi_{1,D}^2\text{Var}[\nu]} \neq 0.$$

B.2.2 Numerical Example

Note that choosing to compete in NV is tantamount to choosing a lottery that pays a fixed amount with some probability—the probability of winning the tournament—and zero otherwise, over a sure thing—the piece-rate payment.⁴ We use this observation to present a numerical example.

In the setup of the competition task, the choice will be driven by the interaction of subjective probabilities of winning (overconfidence) with risk aversion. To simplify, we consider only variations in risk aversion for a fixed lottery and certainty equivalent. Participants are given the choice between a lottery that pays \$100 with a 25% probability and receiving a \$20 payment with certainty. Each participant has CRRA utility with risk aversion parameter θ_i . Conditional on risk aversion, participants' choices are governed by logistic choice probabilities:

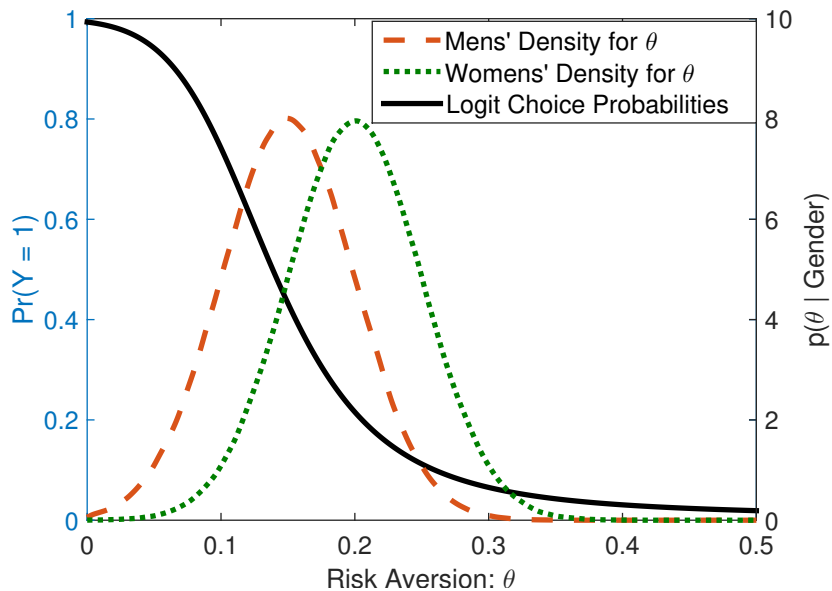
$$p_i = \text{Pr}\{\text{Choose Lottery}|\theta_i\} = \frac{\exp\{\frac{0.25}{1-\theta_i}100^{1-\theta_i}\}}{\exp\{\frac{0.25}{1-\theta_i}100^{1-\theta_i}\} + \exp\{\frac{1}{1-\theta_i}20^{1-\theta_i}\}}$$

To synthetically calibrate preferences, participants who have $\theta_i = \frac{\log(4/5)}{\log(1/5)} \approx 0.14$ are indifferent between the lottery and the certainty equivalent, choosing each with equal probability.

We assume the risk-aversion parameter is distributed normally for men and women, with

⁴Indeed, participants were informed of the number of correct sums they solved during each task.

Figure B.1: Example Illustrating Lottery Choice Probabilities and Gender-Differences in Risk Aversion



each distribution having standard deviation 0.05. To calibrate choices to the 19.0% gap in competition we observe in our data, we assume the distribution for men has mean 0.15, and for women, 0.2. This results in men and women who will choose the lottery about 46% and 27% of the time, respectively. Figure B.1 illustrates the probability of choosing the lottery conditional on risk aversion and the distributions over risk aversion for men and women.

Let $Y_i^a = 1\{\text{Player } i \text{ Chooses Lottery in Main Decision}\}$ represent the observed choice for participant i in the main decision task. If we knew θ_i and could compute each individual's p_i , then $p_i \equiv \mathbb{E}[Y_i^a | \theta_i]$ would be the best predictor for Y_i^a and the ideal explanatory variable and control. As such, following NV, we describe p_i as an individual's *preferences*. A regression of Y_i^a on gender and preferences p_i would fail to find any relationship between gender and lottery take-up (competition) in a large sample.

If we only observe a replicate $Y_i^b = 1\{\text{Player } i \text{ Chooses Lottery in Replicate Task}\}$, that replicate will have classical, mean-zero measurement error for the proper control p_i . Letting $\eta_i = Y_i^b - p_i$, the variance of the realization of this measurement error, $p_i(1 - p_i)$ correlates

with gender because the distribution of p_i depends on gender.

Numerically, if we simulate the choices Y_i^a , Y_i^b for 800 individuals (calibrated to our sample size) and regress the choice of the lottery in Y_i^a on gender 10,000 times, we observe an average coefficient of 0.16 (or 16%, s.e. 0.034, $p < 0.01$). This can be understood to occur because although the coefficient on Y^b should be unity, it is instead 0.19 (s.e. 0.036, $p < 0.01$) due to measurement error.

This suggests that by forcing the coefficient on Y_i^b to be 1, the resulting regression can produce a valid test. That is, by regressing $Y_i^a - Y_i^b$ on gender, the resulting coefficient will be a conservative test of the effect of gender controlling for the second choice.⁵ Following this prescription in our simulated data gives a very accurate and precise estimate of -0.000078 (s.e. 0.034, $p = 1.00$).

C Example STATA

C.1 Using Principal Components as Controls

```
tab sumsRank, gen(ss)
tab sumsCorrectCompete, gen(scc)
tab performanceDiff, gen(pdd)

#delimit;
pca ss* scc* pdd* *RiskyProjectAllocation riskyUrn*0MaxValue compoundUrn*
*Over* CRTCorrect CRTPercentile ravenCorrect ravenPercentile guess*Confidence
gpaPercentile;
#delimit cr

predict p1-p76

//standardize principal components to make output easily interpretable
foreach x of varlist p1-p5 {
sum 'x'
```

⁵The test is conservative because although the left side will now properly correspond to $Y - \mathbb{E}[Y|X]$, the right side will consist of D rather than $D - \mathbb{E}[D|X]$. This test is asymptotically efficient, but it will be inefficient in small samples.

```
replace 'x' = ('x'-r(mean))/r(sd)
}
```

```
reg sumsCompete male p1-p5
```

C.2 ORIV When Y is Measured without Error

This subsection and the next implement ORIV assuming that the X and Y variables are on the same scale. If they are not, one should first put them on the same scale. If there is an obvious way to do this, as in the case of certainty equivalents of lotteries with the same probabilities, but a different high option, this should be done. Otherwise standardization of the variables may be attractive.

Note that if one estimates 2SLS in stages, the estimated standard errors from the second stage, in (7), would be incorrect, as they do not take into account the fact that \hat{X}^a and \hat{X}^b are estimated. Therefore, it is preferable to estimate (6) directly, using a statistical package's 2SLS command as this will give correct asymptotic standard errors.

```
keep id highLowValue firstProjectValue secondProjectValue
//standardize variables
for each var of varlist highLowValue-secondProjectValue {
    quietly sum 'var'
    replace 'var' = ('var' - r(mean))/r(sd)
}
```

```
expand 2, generate(replicant)
gen mainVar = firstProjectValue if replicant == 0
replace mainVar = secondProjectValue if replicant == 1
gen instrument = secondProjectValue if replicant == 0
replace instrument = firstProjectValue if replicant == 1
```

```
//One constant per stack
forvalues x=0/1 {
    gen constant'x' = replicant == 'x'
}
```

```
ivregress 2sls highLowValue (mainVar = instrument) control*, cluster(id) nocons
```

C.3 ORIV Estimates of a Correlation

```
keep id riskyUrn20Value riskyUrn30Value compoundUrn20Value compoundUrn30Value
replace id = _n
//standardize variables
for each var of varlist riskyUrn20Value-compoundUrn30Value {
    quietly sum `var'
    replace `var' = (`var' - r(mean))/r(sd)
}

//Duplicate data four times for each way of stacking data
expand 4
sort id
gen replicant = mod(_n,4)

//Created stacked variables
gen LHS = riskyUrn20Value if replicant <= 1
replace LHS = riskyUrn30Value if replicant >= 2
gen mainVar = compoundUrn20Value if replicant == 0 | replicant == 2
replace mainVar = compoundUrn30Value if replicant == 1 | replicant == 3
gen instrument = compoundUrn30Value if replicant == 0 | replicant == 3
replace instrument = compoundUrn20Value if replicant == 1 | replicant == 2

//One constant per stack
forvalues x=1/4 {
    gen constant`x' = replicant == `x'-1
}

//Compute ORIV coefficients and correlations
ivregress 2sls LHS (mainVar = instrument) constant*, cluster(id) nocons
local correctedCoefficient = _b[mainVar]
qui corr riskyUrn20Value riskyUrn30Value if replicant == 0, cov
local correctedYVar = r(cov_12)
qui corr compoundUrn20Value compoundUrn30Value if replicant == 0, cov
local correctedXVar = r(cov_12)
local correctedCorrelation = ///
    `correctedCoefficient'*sqrt(`correctedXVar'/'correctedYVar')

display "The ORIV Correlation is: `correctedCorrelation'"
```

C.4 Bootstrapped Standard Errors

```
//note: should already have dataset generated above in memory
local reps = 10000

keep if replicant == 0
drop replicant

capture program drop orivbstrap
program define orivbstrap, rclass
    preserve
    bsample, cluster(id)

    quietly ivreg LHS (mainVar = instrument) control*, nocons
    scalar BSCorrectedCoefficient = _b[mainVar]

    quietly corr riskyUrn20Value riskyUrn30Value, cov
    scalar BSCorrectedYVar = r(cov_12)
    quietly corr compoundUrn20Value compoundUrn30Value, cov
    scalar BSCorrectedXVar = r(cov_12)

    return scalar BSCorrectedCorrelation = ///
        BSCorrectedCoefficient*sqrt(BSCorrectedXVar/BSCorrectedYVar)
    restore
end

simulate BSCorrectedCorrelation = ///
    r(BSCorrectedRho), reps('reps') seed(10) nodots: orivbstrap
bstat, stat(correctedCorrelation)
```

D Comparison of Participant Pools

We now compare responses to standard choice tasks on the Caltech Cohort Study (CCS) to other participant pools. The participant's responses on the CCS resemble those previously reported. That is, the participants in the CCS are not dramatically different from those in other participant pools. This is in line with the replication results reported in the paper: before correcting for measurement error, our data yield virtually identical conclusions to those reported in the original experiments.

D.1 The Beauty Contest

Each installment of the CCS contained a beauty contest game following Nagel (1995). Each participant is asked to choose a number between 0 and 100. They are told that they will be grouped with 9 randomly-chosen participants, and if their choice is closest to $2/3$ of the average of the choices in that group, they will receive a \$5 reward, otherwise nothing.⁶

This task is often viewed as a proxy for cognitive sophistication. Any choice above 66.66 is dominated. If everyone uses undominated strategies, any choice above 44.44 will not pay off. In this way, each iteration can be associated with a greater level of sophistication. In the limit, this iterated elimination of dominated strategies yields the unique equilibrium of the game—everyone should choose 0. Nagel (1995) reported numbers very far from 0 when the game was played for the first time by Bonn University students. In her data, the mean number chosen was 36.73 and the median was 33. Data from the first installment of the CCS in the Fall of 2013 yields similar results: In our sample of 806 participants, the mean was 37.17 with a median of 33.⁷

D.2 The Cognitive Reflection Test

Next we compare responses on the Cognitive Reflection Test (CRT), described briefly in Section 2.1. The CRT was introduced by Frederick (2005) and consists of three quantitative questions, each having a seemingly intuitive answer that is wrong. In the Spring 2015 installment of the CCS, we included the three questions proposed by Frederick (2005), but with different wording and scaling.

The number of correct answers on the CRT is viewed as a proxy for meta-cognition. It is associated with time preferences: participants who score higher are more “patient”. In the CCS, the fraction of participants with 0, 1, 2, and 3 correct answers are 19%, 25%, 28%, and 27%, respectively. Frederick (2005) reports responses from 11 participant populations

⁶If they tie with a subset of k participants in their group, each gets $1/k$ of the \$5 reward.

⁷While Nagel (1995) does not report her raw data, the distribution of selected numbers in our data visually resembles the distribution depicted in her Figure 1B.

with aggregate proportions of 33%, 28%, 23%, and 17%. In the CCS, participants were less likely to answer no question correctly, and more likely to answer all three questions correctly, while the rates for 1 or 2 correct answers are similar.

D.3 Dictator Giving

All installments of the CCS included the dictator game, in which participants have to divide a fixed allocation between themselves and another randomly chosen participant. We have noted that generosity declines markedly the second time a student participates in the CCS, so we focus on the initial, Fall 2013, installment. The mean amount given away is 22%, with 43% giving away nothing, 13% giving away a third, and 27% giving away half.

Engel (2011) compiles data from 83 papers, consisting of 616 treatments and 20,813 individual observations. The mean amount given away across all treatments was 28%. At the individual level, he reports that 36% give away nothing, 9% give away a third, and 17% give away half. These numbers are very close to those we observe. Participants in the CCS are slightly more extreme in the sense that they are more likely to give away nothing, but when they do give, they are more generous.

D.4 Risky Projects

We conclude in Section 4 that a particularly appealing risk elicitation is the “Project” measure, described in Section 2.1, and based on Gneezy and Potters (1997). Here we focus on a particular implementation of this elicitation in the Fall 2014 installment of the CCS. This implementation allowed participants to allocate 200 tokens between a safe option and a risky project, the latter returning 2.5 tokens with 50% probability per token allocated.⁸ In our data, the mean allocation to the risky project is 74%, with a median of 75%.

In comparison, Agranov and Yariv (2015) elicit precisely the same measure in 8 sessions

⁸We focus on this version of the task since it is the most commonly used and therefore allows for a natural comparison with existing data.

with 80 students at the University of California, Irvine (UCI). There, the mean allocation was 72% with a median of 70%. The distribution of allocations in their data looks very similar to that in the CCS. For example, in the CCS approximately 30% of participants allocated half or less of their endowment to the risky project, while at UCI approximately 35% of participants did.

E Question Wordings

Screenshots and design details of the CCS can be found in Section 2. Here we present the question wordings that were used in this paper. Throughout this section, comments in square brackets are meant to express information that is not found on the screen, but is useful in understanding the flow of the survey. A new item number generally indicates another screen (even though there may not be a particular question associated with it).

E.1 Competition

This question, meant to elicit competitiveness, follows NV, and was used on the Spring 2015 CCS. It consists of several parts, some of which are used by both LV and us as controls.

1. This next task asks you to add together series of numbers. You will be given three minutes to complete as many sums as possible. When all surveys are submitted, we will randomly group you with 3 other people (so you will be in a group of 4). **You will be paid only if you achieve the highest number of completed sums within this group, in which case you will be paid 40 tokens per sum completed.**

In case of a tie between those who completed the highest number of sums, we will randomly determine the participant who will be paid.

2. [3 minutes in which to do as many sums as the participant can]
3. In the randomly determined group of 4 (you and 3 others), where do you think you rank in terms of the number of sums completed (1 corresponding to the highest number of sums completed in the group, 4 corresponding to the lowest number of sums completed in the group). You will earn an additional 50 tokens if your guess is correct.

[radio buttons next to numbers 1 through 4]

4. You will be given an additional three minutes to complete as many sums as possible. Please pick how you would like to be paid from the following two options:

- (a) 10 tokens per sum completed; or
- (b) When all surveys are submitted, we will randomly group you with 3 other people (so you will be in a group of 4). We will compare the number of sums you complete now with the number of sums the other 3 completed in the previous stage you just concluded. **You will be paid only if you achieve the highest number of completed sums within this group, in which case you will be paid 40 tokens per sum completed.**

In case you tie with another participant(s), we will randomly determine whom of these gets first place, and you will be paid only if it is you who is declared the winner.

- (c) [3 minutes in which to do as many sums as the participant can]

Some other details of our implementation are worth mentioning. As these are identical to the design choices of NV, we address them here, rather than in the main text.

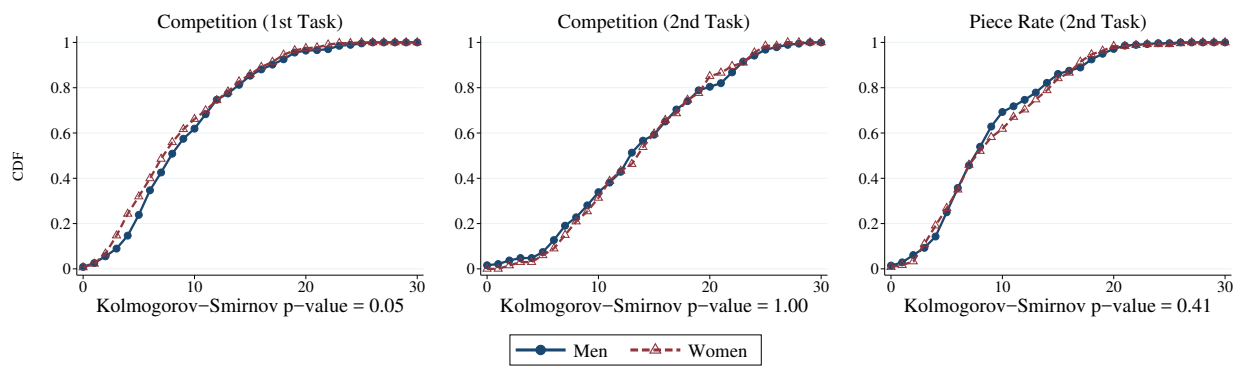
Generation of sums: The five two-digit numbers used for the tasks were generated randomly.

Payment for guessed tournament rank: If there were ties in a group, Question 3 was paid in the way that was most favorable to participants. For example, if a participant correctly solved 14 sums, while others in the group solved 14, 12, and 11 sums, the participant would get the 50 token reward with a guessed rank of either 1 or 2.

Selection of task: This task was chosen by NV because it was found to elicit similar performance between men and women. This may not hold for all tasks: see Gneezy et al. (2003).

Figure E.2 illustrates the performance of men and women in both tasks, splitting participants in the second task into those who chose to compete and those who chose the piece-rate scheme. There is no significant difference between the performance of men and women in the second task, regardless of the compensation scheme.

Figure E.2: Performance in different tasks by gender



However, in the first task there is a difference in performance. This difference occurs among those that complete fewer than 10 sums. In that group, women solve slightly fewer sums, on average, than men. This may appear to differ from the findings of NV, who observe statistically identical performance. However, we have roughly 10 times as many participants as they do. If we draw a group of 40 men and 40 women—the number of participants in NV—10,000 times randomly from our data, we observe a p-value lower than 10% only 8.3% of the time. Thus, performance data seems consistent with NV.

A final note on our implementation is warranted: because our experiment was conducted on a survey rather than in the lab we could not enforce our prohibition on the use of calculators. To mitigate this, sums were presented as images so they would be impossible to cut and paste into an online calculator. Participants could not use the back button during the task, and logging out and returning caused them to skip the task altogether. Participants were asked to sign an acknowledgement that they would abide by the Caltech Honor Code. Despite all this, we cannot eliminate the possibility that some students used calculators. However, in order for this to affect our results, it would have to be the case that women believed men used calculators more often, and that this belief affected not only their decision to compete, but also their choices in unrelated risk aversion and overconfidence elicitation.

Moreover, as noted in the text, we later had 98 Caltech students take the survey in the lab, where they could not use calculators. All results were substantially the same, but with

larger standard errors driven by the smaller sample.

E.2 Overconfidence

These questions were used as controls in the regressions on competitiveness, and elicited on the Spring 2015 CCS.

The first three questions are used as a control for overprecision, and are based on Ortoleva and Snowberg (2015).

1. We will now measure your ability to assess numbers quickly.
We will show you three pictures of jars of jellybeans. Please give us your best guess as to the number of jellybeans in each jar.
2. [With random picture of a jar of jellybeans] Please enter the number of jellybeans you think are in this jar (between 1 and 1000).
3. How confident are you of your answer to this question?
 - (a) No confidence at all
 - (b) Not very confident
 - (c) Somewhat unconfident
 - (d) Somewhat confident
 - (e) Very confident
 - (f) Certain

[Repeat three times.]

The next set of questions ask the participant to complete a set of tasks, and then asks the participant to guess their performance (as a measure of estimation / overestimation), and then their guess as to how their performance compares with their peers (placement / overplacement). Participants are given 30 seconds to answer each logical question.

4. This next task asks you to solve five logical puzzles. You will be given thirty seconds to complete each puzzle, and will be paid 20 tokens for each puzzle solved correctly.
5. [5 Raven's matrices from Condon and Revelle (2014). All participants executed the same matrices in the same order.]
6. How many of the 5 puzzles do you think you solved correctly?

7. Out of 100 other randomly picked survey participants, what percentage do you think solved more puzzles correctly than you?

The next set follows the same structure, but uses questions from the cognitive reflection test (CRT).

8. This next task asks you to answer five logical questions. You will have up to 30 second to answer each question and will be paid 20 tokens for each question answered correctly.
9. A monitor and a keyboard cost \$350 in total. The monitor costs \$300 more than the keyboard. How much does the keyboard cost?
10. It takes 10 computers 10 minutes to run 10 simulations. How long does it take 200 computers to run 200 simulations?
11. In the pond in front of Baxter Hall, there is a patch of lily pads. The patch doubles in size every day. If it takes 36 days for the patch to cover the entire pond, how many days would it take to cover half the pond?
12. Professor Wiseman spent one-fourth of his life as a boy, one-eighth as a youth, and one-half as an active man. If Professor Wiseman spent 8 years as an old and wise man, how many years did he spend as an active man?
13. A 4 foot pole casts a shadow that is 2 feet long on the ground. If the pole was 16 feet in height, how long would the shadow be?
14. How many of the 5 questions do you think you answered correctly?
15. Out of 100 other randomly picked survey participants, what percentage do you think answered more questions correctly than you?

E.3 Risk Elicitations

The Fall, 2014 CCS contained a large number of risk elicitation. The Spring, 2015 CCS included variants of the risky projects, the risky urns, and the qualitative risk question, which were used as controls for the competition task.

E.3.1 Projects

These risky projects are based on Gneezy and Potters (1997). Note that they were separated by several questions on the survey.

1. You are endowed with 200 tokens (or \$2) that you can choose to keep or invest in a risky project. Tokens that are not invested in the risky project are yours to keep.

The risky project has a 40% chance of success.

If the project is successful, you will receive 3 times the amount you chose to invest.

If the project is unsuccessful, you will lose the amount invested.

Please choose how many tokens you want to invest in the risky project. Note that you can pick any number between 0 and 100, including 0 or 100:

2. You can invest in another risky project if you would like. You can invest up to 200 tokens, or you can choose to keep them.

The risky project has a 50% chance of success.

If the project is successful, you will receive 2.5 times the amount you chose to invest.

If the project is unsuccessful, you will lose the amount invested.

Please choose how many tokens you want to invest in the risky project. Note that you can pick any number between 0 and 200, including 0 or 200:

E.3.2 Urn MPL

These are standard urn gambles, with certainty equivalents elicited using a multiple price list. Note that the order of the lotteries was randomized, as was the order (spaced throughout the survey) with the compound and ambiguous urn lotteries.

4. In the next task we will ask you to assess the value of several gambles. The gambles will be designed using virtual urns filled with red and black balls. We will give you some information on the composition of each urn.

We'll let you pick which color ball you would like to pay off for each gamble. If you choose a gamble on an urn, we will draw one ball from it. If that ball is the color you chose, you will receive a reward. If it is the other color, you will receive nothing.

The mechanism for selecting the gamble may take some getting used to. We will provide you with a list of rewards from 0 tokens to 150 tokens in increments of 5 tokens, and for each one, we ask that you think whether you prefer that amount, or taking the urn gamble. Once you click on an option, we'll fill in the rest of the choices for you so that they make sense (this will save you time!).

However, you should keep clicking on options that you prefer until the choice in each line is what you would like. You should do so because at the end of the survey, we will randomly select one row from the list, and you will get whatever you chose on that line. If you specified you prefer a sure amount on that line, you'll get that amount. If you specified that you preferred the gamble, then we will draw a ball from the urn, and pay you according to the description above.

5. The next choice will involve an urn containing 20 balls, 10 of which are red and 10 of which are black.

Which color ball would you like to paid 100 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

6. Urn with Equal Number of Red and Black Balls

The urn from which we can draw a ball is composed of 10 red balls and 10 black balls.

The urn gamble pays 100 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 100.]

7. The next choice will involve an urn containing 30 balls, 15 of which are red and 15 of which are black.

Which color ball would you like to paid 150 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

8. Urn with Equal Number of Red and Black Balls

The urn from which we can draw a ball is composed of 15 red balls and 15 black balls.

The urn gamble pays 150 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 150.]

E.3.3 Lottery Menu

This question is based on Eckel and Grossman (2002).

9. Which of the following gambles would you prefer?

Each of the gambles will give you a 50% chance of the Low Payoff, and a 50% chance of the High Payoff.

The gamble you chose will be run at the end of the survey, and we will tell you your payoff then.

	<u>Low Payoff</u>	<u>High Payoff</u>
Gamble 1:	140	140
Gamble 2:	120	180
Gamble 3:	100	220
Gamble 4:	80	260
Gamble 5:	60	300
Gamble 6:	10	350

E.3.4 Qualitative Risk Assessment

This qualitative assessment of risk comes from Dohmen et al. (2011). This was also elicited on the Spring 2015 CCS, and that question was used to instrument the question on the Fall 2014 CCS.

10. How do you see yourself: are you generally a person who is fully prepared to take risks or do you try to avoid taking risks?

Please tick a box on the scale, where the value 0 means: not at all willing to take risks and the value 10 means: very willing to take risks

[radio buttons, presented horizontally, with numbers from 0 to 10 next to each option.]

E.4 Compound and Ambiguous Lotteries

Compound and ambiguous lotteries follow the same format as the risky urn lotteries above. These three blocks of urn gambles were spaced out throughout the survey, and their order was randomized. The compound lotteries were also used as a control (risk-aversion measure) in the regressions regarding competitiveness.

1. The next choice will involve an urn containing 20 red and black balls. The composition of the urn is randomly determined. That is, we will first randomly draw a number between 0 and 20 (all equally likely). That number will be the number of red balls. The remaining balls will be black.

Which color ball would you like to paid 100 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

2. Urn with Uncertain Number of Red and Black Balls

The composition of the urn is randomly determined. That is, we will first randomly determine a number between 0 and 20 (all equally likely). That number will be the number of red balls, the rest of the 20 balls (if any) will be black.

The urn gamble pays 100 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 100.]

3. The next choice will involve an urn containing 30 red and black balls. The composition of the urn is randomly determined. That is, we will first randomly draw a number between 0 and 30 (all equally likely). That number will be the number of red balls. The remaining balls will be black.

Which color ball would you like to paid 150 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

4. Urn with Uncertain Number of Red and Black Balls

The composition of the urn is randomly determined. That is, we will first randomly determine a number between 0 and 30 (all equally likely). That number will be the number of red balls, the rest of the 30 balls (if any) will be black.

The urn gamble pays 150 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 150.]

5. The next choice will involve an urn containing 30 balls, each of which could be red or black. Dean Dabiri has chosen the exact composition of the urn: the balls could all be red, they could all be black, or there could be any combination of red and black balls.

Which color ball would you like to paid 100 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

6. Urn with Unknown Number of Red and Black Balls

The urn has a combination of 20 red and black balls chosen by Dean Dabiri.

The urn gamble pays 100 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 100.]

7. The next choice will involve an urn containing 30 balls, each of which could be red or black. Dean Dabiri has chosen the exact composition of the urn: the balls could all be red, they could all be black, or there could be any combination of red and black balls.

Which color ball would you like to paid 150 tokens for (if it is drawn from the urn in the following questions)? Note that this means you will be paid 0 tokens if the other color is drawn.

- (a) red
- (b) black

8. Urn with Unknown Number of Red and Black Balls

The urn has a combination of 20 red and black balls chosen by Dean Dabiri.

The urn gamble pays 150 tokens if the ball drawn is [red].

What do you prefer? (make sure a radio button in each row is selected)

[Multiple price list with “Urn Gamble” on the left side, and “X tokens” on the right side, with X in increments of 10 from 0 to 150.]

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