A Multi-Dimensional Signaling Model of Campaign Finance

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Abstract

We develop a dynamic multi-dimensional signaling model of campaign finance in which candidates can signal their ability by enacting policy and/or by raising and spending campaign funds, both of which are costly. Our model departs from the existing literature in that candidates do not need to exchange policy influence for campaign contributions, rather, they must decide how to allocate their efforts between policymaking and fundraising. If high-ability candidates are better policymakers and better fundraisers then they will raise and spend campaign funds even if voters care only about legislation. Voters’ inability to reward or punish politicians based on past policy allows fundraising to be used to signal ability at the expense of voter welfare. Campaign finance reform alleviates this phenomenon and improves voter welfare at the expense of politicians. Thus, we expect successful politicians to oppose true campaign finance reform. We also show our model is consistent with findings in the empirical and theoretical campaign finance literature.

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When we were spending so much time raising money, we simply could not devote quality time to thoughtful decisions and debate. It lowered the substance of our work.

- US Senator Alan Simpson (R-WY)\(^1\)

## 1 Introduction

Proponents of campaign finance reform claim that campaign contributions corrupt politicians and bias policy. While conventional wisdom suggests the existence of such an effect, there is a dearth of systematic supportive evidence. This absence is not due to a lack of study. If one examines closely all of the arguments in favor of restricting campaign contributions, a more defensible criticism emerges: raising campaign funds corrupts the time allocation of politicians. They spend too much time and energy raising money and too little time and energy governing. In this paper we explain how this inefficiency persists, and demonstrate how reform would alleviate it.

Most of the academic literature agrees with the proponents of campaign finance reform. In this literature it is assumed that “parties bias their policy choices to attract money from interest groups and then use this money to attract the votes of the uninformed.”\(^2\) Recent empirical work calls both of these assumptions into question. Ansolabehere, de Figueiredo and Snyder (2003) argue that campaign contributions appear to have no effect on legislative behavior. Additionally, if advertising is informative enough to justify the money spent on it, we should expect that changing levels of advertising would have profound effects on the outcomes of elections. This too lacks empirical support (Levitt, 1994).

It is clear, however, that the pursuit of campaign contributions distorts the allocation of politicians’ time, attention, and effort. In his dissent to Randall v. Sorell, Supreme Court Justice John Paul Stevens writes, “Fundraising devours the time and attention of political leaders, leaving them

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\(^1\)Taken from: http://www.boston.com/news/globe/editorial_opinion/oped/articles/2006/03/10/limit_campaign_fund_raising/

\(^2\)Quote from Coate (2004a). For models of this type see: Baron (1989), Snyder (1990), Prat (2002a,b) and Coate (2004a). Prat (2006) and Coate (2004b) make a distinction between service-induced contributions, which are defined as above, and position-induced contributions, where contributions are given based on past voting records. For the purposes of this discussion the two are observationally equivalent. Funds increase the chance of (re-)election and candidates must take a certain position to obtain them. For a complete theoretical treatment of this point see Fox (2007).
too busy to handle their public responsibilities effectively.” (528 U.S. ___ (2006)) Many politicians echo his statement. Former Senator, Vice-President and presidential candidate Walter Mondale (D-MN) asserts, “Candidates and officeholders spend far too much time raising money [in order] to run for office.” Former Senator Nancy Kassebaum (R-KS) states, “Here we are forced to raise money all the time. I don’t worry about money influencing our votes. But I worry about the energy it takes. I just don’t know how people find time to think or reflect.” Former Senator Ernest “Fritz” Hollings (D-SC) says that fundraising “distracts us from the people’s business ... It corrupts and degrades the entire political process.”

We incorporate this bias in effort into a dynamic signaling model that does not rely on vote buying and the accompanying biases in policy. In every election cycle politicians exert effort toward implementing policy and toward raising and spending campaign funds. These activities are costly to politicians. We assume that high-ability candidates are better policymakers and better fundraisers, so they can use these activities to signal their privately known ability. A high-ability politician wants to signal her type at minimum cost. When the marginal costs of each of these activities are increasing, it will generally be cheaper to signal through both channels than to signal through only one.

The assumption that high-ability candidates are better fundraisers is supported by both proponents of loosely regulated campaign financing and the scholarly literature. The amount of money raised by a candidate is often used as a measure of politician quality (see, for example, Squire (1992, 1989) and Bond, Covington and Fleisher (1985)). The equation of fundraising with candidate ability is so pervasive in the popular press that it was recently bemoaned by a columnist for The Philadelphia Inquirer. Moreover, this is a common, although implicit, assumption in the theoretical literature on campaign finance (Coate, 2004b; Prat, 2006).

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4 Our model is theoretically similar to the one-shot, multi-dimensional signaling models of Wilson (1985) and Milgrom and Roberts (1986), and could be easily extended to incorporate any number of signaling channels.

Voters cast their ballots based solely on the expected quality of candidates’ future policies. Voters are not fooled or bought by campaign expenditures, nor do they have preferences for any innate characteristics a politician may posses. Why then are candidates concerned with signaling their ability? The answer follows directly from the dynamic structure of the repeated election setup. Every period high-ability politicians will signal their ability through implementing better policy (and raising more funds) than low-ability politicians. Therefore, voters prefer to have high-ability politicians in office.

A rational electorate must base their decisions only on future expectations, so voters cannot commit to ignoring fundraising or to rewarding politicians for past policies. Because voters cannot commit to incentive contracts with their elected officials, politicians continue to signal through fundraising, although voters would prefer they signaled solely through making better policy.

Limiting campaign expenditures forces candidates to expend less effort on signaling through the raising and spending of campaign funds. In order to preserve separation in equilibrium, this effort (and more) will be expended on making higher quality policy, which will increase voter welfare. While previous models also predict that campaign finance reform would increase voter welfare, these predictions follow directly from the assumption that campaign funds influence policy.

One might ask, “If the literature is so sure that campaign finance reform would increase voter welfare, why has campaign funding grown at the same rate as GDP over the last 100 years?” Our model provides an answer to this question as well: campaign finance reform makes incumbent politicians worse off. Thus, we would not expect them to enact legislation that would curtail campaign spending.

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6In order to focus on the effort allocation decision of politicians, we assume quality of policy is single-dimensional and all voters prefer higher quality. Adding heterogeneous policy positions does not affect the results as long as the standard conditions are met for platforms to converge to the ideal point of the median voter (Hotelling, 1929; Downs, 1957).

7This feature is unique within models with office-motivated candidates, and provides a foundation for candidate valence in one-period games. Valence is defined as the direct utility that a voter gets from having a high-ability politician in office. For a foundation of valence when candidates can also be policy-motivated, see Callander (2005).

8While campaign finance legislation is a hot topic, we note that 13 major campaign finance bills have been passed since 1867, and yet campaign spending has risen ten times faster than inflation and on par with GDP (Ansolabehere, de Figueiredo and Snyder, 2003, p. 120). Note that there have been large swings in the per GDP level of spending, but long-term decreases are generally not preceded by the adoption of campaign finance laws. Indeed, it often seems to be the other way around.
In addition, our model encapsulates many findings from a diverse set of theoretical and empirical work. Incumbents are more likely to win than their challengers. This is due to a selection effect already identified in the literature. We also predict that high-ability challengers may be deterred by incumbents’ high fundraising in the previous period.

Finally, in vote-buying models, campaign spending has two opposing effects on voter welfare. Spending plays an informational role by helping voters decipher which politicians will be best if elected. This is good for voter welfare. However, spending also entails that policy choice is biased away from the preferences of the electorate. This is bad for voter welfare, and thus there is a tradeoff. In our model we eliminate this tradeoff by retaining only the informational role. Yet, even though campaign contributions do not buy policy influence, we find that limiting campaign spending enhances voter welfare by shifting politicians’ efforts from fundraising to governing.

2 The Model

We model a repeated election process. There are time periods \( t = 1, 2, 3, \ldots \) where an election is held at the beginning of each period \( t \geq 2 \). The players are voters and two candidates in each election: the incumbent and the challenger. The candidate who wins the election in period \( t - 1 \) is the incumbent in period \( t \), and the candidate who loses never runs again. Each period, each candidate has privately known ability (or type), \( \theta_t^i \), which can be either low or high, \( \theta_t^i \in \{L, H\} \). The probability of candidate \( i \) being type \( H \) is \( p_t^i \in (0, 1) \), which is common knowledge among all players. Challengers are high types with probability \( p \), as are incumbents who were low types in the previous period. The probability that an incumbent who was a high type in the last period is a high type again is \( \bar{p} > p \). The substantive reason for these switching probabilities is that things change. As time progresses, political circumstances alter and certain politicians may no longer be as effective as they once were. Others may become more effective.

The actions available to voters are voting for the incumbent or the challenger in a given election.\footnote{The model can be extended to encompass more complex switching processes, or a wider array of potential challengers. The ideas will be precisely the same, but the algebra more cumbersome and less transparent.}
The actions available to candidate \( i \) in each period are putting effort into increasing the quality of policy, \( q^t_i \in \mathbb{R}_+ \), and/or putting effort into raising and spending funds, \( f^t_i \in \mathbb{R}_+ \).\(^{10}\) That is, in each period, candidates simultaneously choose pairs \( (q^t_i, f^t_i) \in \mathbb{R}^2_+ \). These activities are costly, but more costly for a low type. Denote \( c(q, f|\theta) \) as the cost function for a candidate of type \( \theta \) taking actions \( q \) and \( f \). We assume:

**Assumption 1** For each \( \theta \), \( c(q, f|\theta) \) is continuous, convex, and strictly increasing. Additionally \( c(0, 0|\theta) = 0 \), and for all \( c \in \mathbb{R}_+ \) there exists \( q(c, \theta) \) and \( f(c, \theta) \) such that \( c(q(c, \theta), 0|\theta) = c = c(0, f(c, \theta)|\theta) \).

**Assumption 2** \(-c(q, f|\theta)\) satisfies strict increasing differences in \( (q, f) \) and \( \theta \). That is, for all \( (q', f') \geq (q, f), (q', f') \neq (q, f) \): \( c(q', f'|H) - c(q, f|H) < c(q', f'|L) - c(q, f|L) \).

The first assumption implies that isocost curves for both types are continuous, intersect both axes and have convex lower contour sets (the set of points that are no more costly than the points on the isocost curve). The second assumption states that any increase in the quality of policy \( q \) and/or fundraising \( f \) costs a low type incrementally more than a high type.\(^{11}\) Together these assumptions imply that for a given cost \( c > 0 \), a high type’s isocost curve at that level lies everywhere above a low type’s isocost curve for the same level.\(^{12}\)

Note that the candidates’ cost functions are very general. In particular, they do not rule out complementarities between fundraising and making better policy—we allow for the possibility that additional fundraising activity will lower the marginal cost of creating quality policy. For example, campaign fundraising events may also serve to educate a candidate about issues that are important to her constituents.

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\(^{10}\)Challengers are assumed to hold other political offices prior to the election. In Section 5, we comment on what happens if this does not hold. If only incumbents signal through policymaking all welfare claims still hold. Nothing would be changed if the efforts of a high type are also more productive.

\(^{11}\)Assumption 2 is not a necessary condition for positive fundraising in equilibrium. Without Assumption 2 positive fundraising can exist in equilibrium even if both types find fundraising equally costly.

\(^{12}\)The model can easily be extended to accommodate any finite number of types since Assumption 2 guarantees that cost functions are such that each type only needs to worry about preventing imitation by the type immediately below it.
Candidates value winning office and minimizing their cost of doing so. The per-period value of being in office is common among all candidates, and normalized to 1. Voters receive utility from higher quality policy, which is directly observable. The per-period utility of a candidate $u^t$ who selects policy and fundraising levels $(q^t, f^t)$ and voters $v^t$ for having policy $q^t$ and a politician of ability $\theta^t$ during period $t$ are:

$$
u^t(q^t, f^t|\theta^t) = I_{\{\text{in office}\}} - c(q^t, f^t|\theta^t)
$$

$$
v^t(q^t) = q^t
$$

All candidates have a common positive discount factor $\delta \leq 1$. Each individual voter can have any positive discount factor. We define our equilibrium concept as follows.

**Definition 1** We use the term *equilibrium* to mean a sequential equilibrium that satisfies the intuitive criterion.\(^{13}\)

We restrict the number of equilibria by assuming throughout that when voters are indifferent between electing either candidate, each is equally likely to win. We focus on symmetric Markov Perfect equilibria, so a pure strategy is a map from a candidate’s type into an action.\(^{14}\) An equilibrium is symmetric if all candidates use the same strategy.

Denote $\pi_\theta(p_i)$ as a candidate’s probability of winning an election if types will be revealed, given that her type is $\theta$ and her opponent has probability $p_i$ of being a high type. Let $x_\theta$ be a candidate

\(^{13}\)Sequential equilibrium is from Kreps and Wilson (1982b) and the intuitive criterion is from Cho and Kreps (1987). To handle a continuum of possible actions, sequential equilibrium is adapted as in Cho and Kreps (1987). To use the intuitive criterion in a model with multiple senders, we impose that voter beliefs regarding candidate $i$’s type are independent of the actions of any other player $j \neq i$. The intuitive criterion is then applied each period to voter beliefs regarding each candidate $i$, holding their beliefs about her opponent $j$’s type as fixed. The criterion is applied in each period as follows. Fix a proposed equilibrium. If in any period $t$, for either candidate $i$, there exists a signal $(q^t_i, f^t_i)$ such that any best response of the voters for the rest of the game to beliefs that $i$ is a high type for sure in period $t$ given that $i$ chose $(q^t_i, f^t_i)$ makes $(q^t_i, f^t_i)$ a profitable deviation for the high type, but not for the low type, then the proposed equilibrium fails the intuitive criterion. This version of the criterion actually has little bite because it does not restrict beliefs in periods $s > t$. However, when voters’ best responses are also assumed to satisfy the Markov property, the intuitive criterion works similarly to how it does in a one-period game.

\(^{14}\)In richer setups of the model, such as when there are more than two candidates, the probability distributions over each candidate’s type can be payoff relevant. See the Appendix for a discussion of state variables in Markov Perfect equilibria.
of type $\theta$’s gain from being perceived as a high type, rather than a low type, which is just the increase in the probability of being elected times the value of being in office. The value of being in office, $\eta(\theta)$, is both the immediate reward, 1, and the possibility of being re-elected:

$$x_\theta = (\pi_H - \pi_L) \eta(\theta) = (\pi_H - \pi_L) (1 + \rho(\theta))$$

where $\rho(\theta)$ represents the present-discounted value of the flow of benefits from the possibility of being re-elected, which is dependent on $\theta$. \(^{15}\) High types are more likely to remain high types ($p < \overline{p}$), and we will show high types are more likely to be elected, therefore $\rho(H) > \rho(L)$, and $x_H > x_L$.

Before investigating our repeated elections model, it is useful to study a simpler, one-shot game. Much of the intuition, central themes and analysis will easily carry over from the one-shot to the repeated game.

### 3 A Single Election Model

In our single election model there are two periods, $t \in \{1, 2\}$. The incumbent holds office in period 1. An election is held at the beginning of period 2, with the winner holding office for the remainder of the game. The actions available to candidates and their per-period cost functions and utility functions are the same as in the repeated election model. We alter voter’s preferences so they now receive utility from both higher quality policy, which is directly observable, and from having a high-ability leader. We will see in the repeated elections model that high types will offer higher quality policies than low types in subsequent periods. The direct benefit to voters of a high type is meant to approximate this phenomenon in the single election context. Voters now have the per-period utility function:

$$v^t(\theta^t, q^t) = 1_{\{\theta = H\}} + q^t$$

\(^{15}\)For a stationary equilibrium it is only necessary to ensure that no type can profit from a one step deviation. This is why $\rho(\theta)$ is the proper component in $x_\theta$. 

7
The equilibrium strategies in the second period are simple: whichever candidate $i$ was elected will not enact policy in the second period ($q_i^2 = 0$), since it is costly and the world ends immediately after. In the second period, voters obtain $v^2(\theta_i^2, 0) = \mathbb{I}_{\{\theta_i^2 = H\}}$. The voters' objective is thus to select the candidate more likely to be a high type after the election. Since $\bar{p} > \underline{p}$, voters will elect the candidate shown to be more likely to be a high type in the first period. We now analyze the first period, omitting the time superscripts, and refer to the first period of the single election model as the one-shot game. Referring to equation (1), note in the one-shot game with two candidates $\pi_H(p_i) = (1 - p_i) + \frac{p_i}{2}$, $\pi_L(p_i) = \frac{1-p_i}{2}$. Thus, $x_\theta$, the gain to a candidate of type $\theta$ from being perceived as a high type, is equal to $\frac{1}{2}$ for all $p_i \in (0, 1)$ and all $\theta \in \{L, H\}$.

We show that a symmetric least cost separating equilibrium always exists, and characterize its properties. Further, we show that all equilibria must be least cost separating, and find conditions under which there exists a unique equilibrium.

### 3.1 No Campaign Finance Reform

Let $(q_\theta, f_\theta)$ denote the equilibrium action of a candidate with type $\theta \in \{L, H\}$. Our equilibrium concept dictates that in any separating equilibrium low type candidates do nothing $(q_L, f_L) = (0, 0)$ as expending effort will not help them gain office, and they dislike it. High types set their levels of policy and fundraising to minimize costs while preventing imitation by low-type candidates. That is, high types solve the following minimization problem:

$$
\min_{q_H, f_H} \quad c(q_H, f_H|H) \quad \text{s.t.} \quad c(q_H, f_H|L) \geq x_L
$$

The continuity of cost functions implies the constraint must bind. Since $c(q, f|H)$ is continuous and Assumption 1 guarantees that $\{q, f : c(q, f|L) = x_L\}$ is a compact set, the minimum exists and is obtained. The solution is most easily seen graphically. Figures 1.a–1.g show candidate’s

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16Equilibrium here is as in Definition 1. We restrict to Markov Perfect equilibrium only in the repeated game.
isocost curves. Points up and to the right are more costly. The thick curve is a low type’s isocost for \( c(q, f|L) = x_L \). The solution lies on this curve. The thinner curves are the isocosts belonging to a high type—whose objective is to find the cheapest of her curves that touches the thick one.

Figures 1.a and 1.b show cases where a high type’s problem has a unique interior solution. This occurs when \( c(q, f|H) \) is “more convex” than \( c(q, f|L) \).\(^{17}\) Figure 1.a illustrates a case where both cost functions are symmetric in \( q \) and \( f \). Figure 1.b shows when the two types have different relative trade-offs between \( f \) and \( q \), the solution will be unique and interior with \( q_H \neq f_H \). Figures 1.c and 1.d show cases where \( c(q, f|H) \) is “no more convex” than \( c(q, f|L) \), but the two functions have different trade-offs between \( f \) and \( q \). This leads to a corner solution. In these figures, a high type’s trade-off favors fundraising and policymaking, respectively, so a high type signals only through these channels. Figure 1.e shows that when a low type and a high type have the same isocost curves (varying only by the values attached to those curves), any point on the thick curve is a solution. Figure 1.f shows that a point of tangency is not always a solution.

When the cost functions are differentiable we can use the well-known techniques of producer theory to find interior solutions. By writing down a high type’s Lagrangian and solving we obtain a familiar “bang-for-the-buck” condition:

\[
\left( \frac{\partial c(q_H, f_H|H)}{\partial q} \right) \left( \frac{\partial c(q_H, f_H|H)}{\partial f} \right) = \left( \frac{\partial c(q_H, f_H|L)}{\partial q} \right) \left( \frac{\partial c(q_H, f_H|L)}{\partial f} \right) \tag{3}
\]

This condition states that the ratio of marginal costs for both types must be the same at any interior solution. Suppose this were not true and the right side of (3) was greater than the left. Then a high type could divert a bit of her effort from \( q \) to \( f \), which would both decrease her total cost of effort and make it no easier for a low type to imitate her. This is an improvement for a high type, and hence the original allocation was not optimal. Notice that (2) may have a corner solution as in Figures 1.d and 1.e, or a boundary may provide the minimum even when (3) has a solution, as in Figure 1.f.

\(^{17}\)The term “more convex” is used informally only. A sufficient condition to ensure that any point of tangency is indeed a minimum of (2) is that \( c(q, f|H) - c(q, f|L) \) is pseudo-convex.
To complete the equilibrium analysis, we need to verify that a high type wants to separate. This fact is immediate from Assumption 2. Any solution \((q_H, f_H)\) lies on the curve \(\{q, f : c(q, f|L) = x_L\}\). Assumption 2 implies then that any solution must also lie on a curve \(\{q, f : c(q, f|H) = c\}\) for some \(c < x_L\). Since \(c < x_L = x_H\), separation is worth the cost to a high type.

**Proposition 1**  In the one-shot game, all equilibria are separating with low types choosing \((q_L, f_L) = (0,0)\) and high types choosing a solution to (2). A symmetric equilibrium always exists, and it is the unique equilibrium if and only if there is a unique solution to (2).

All proofs are in the appendix. In the proof we construct and verify the set of equilibria described above. The intuitive criterion and Assumption 2 rule out pooling equilibria.\(^{18}\) Finally, as outlined above, low types must choose \((0,0)\), and the intuitive criterion implies that a high type need not signal any more than necessary to prevent imitation. No other separating equilibrium can exist. If (2) has multiple solutions, there are multiple equilibria: some symmetric, some not. In any equilibrium, low types of both candidates do nothing, voters believe a candidate is a high type if and only if she chooses a \((q, f)\) pair such that \(c(q, f|L) \geq x_L\). Voters select high types over low types and select either candidate with equal probability if they are believed to be the same type. High-ability candidates choose any solution to (2), be their choices the same or not. For simplicity, we focus on symmetric equilibria. This is weaker than assuming that (2) has a unique solution.

### 3.2 Campaign Finance Reform

There are many proposals regarding campaign finance reform. They range from banning outright the raising and spending of campaign funds, to completely deregulating the sources and uses of funds. In common parlance campaign finance reform is used to refer to additional restrictions on campaign fundraising and spending. A proposal endorsed by many advocates for reform is to

\(^{18}\)Of course, it is also necessary that there exist signals costly enough to deter the low type from imitation (Assumption 1).
publicly fund elections after a candidate raises a small amount of money, say \( \hat{f} \geq 0 \), to signal that she is a serious candidate.\(^\text{19}\)

We model campaign finance reform in exactly this manner. The only funds with signaling value are those raised below the threshold \( \hat{f} \), since all other funds are given exogenously at no cost to a candidate. We first analyze the game under full reform, \( \hat{f} = 0 \). As always, low-ability candidates do nothing, setting \( q_L = 0 \). Now, a high type’s problem is:

\[
\min_{q_H} c(q_H, 0 | H) \\
\text{s.t. } c(q_H, 0 | L) \geq x_L
\]

The solution is \( \{q_H : c(q_H, 0 | L) = x_L\} \), which is unique. Assumption 2 still guarantees that separation is worth its cost to a high type.

In the games depicted by Figures 1.a–1.c, a high type must relocate to her isocost curve that touches the thick curve right at its \( q \)-intercept. This is a more costly action for a high type to undertake than her action without reform. In Figure 1.e, however, this relocation is equally costly to any of the solutions under no reform. The reason is that when the two types have the same curves, all that matters is how much cost was incurred to separate—regardless of how that cost was incurred. The problem becomes unidimensional, and thus there is no loss to a high type. Figure 1.d shows a corner solution when a high type’s trade-off is more favorable toward policymaking than is a low type’s. This leads to the corner solution of signaling solely through policymaking even without reform. Hence, reform will not alter anything here.

Of course, reform need not be full (\( \hat{f} = 0 \)) to be meaningful. Conversely, as Figures 1.d and 1.e show, full reform need not be meaningful. We formalize this in the definition below. Consider

\(^{19}\text{In the United States, spending by a candidate on elections is considered protected speech under the First Amendment, so it is not possible to prevent candidates from spending as much money as they would like on their campaigns. The matching funds proposals alluded to above are designed such that if any candidate chooses to raise and spend more than the threshold amount, public funds are either removed from the offending candidate or more are given to the other candidate such that they both have equal funds. In the context of our model these additional funds would be costly to raise and have no signaling value so no candidate would choose to exceed the cap. See Ansolabehere, de Figueiredo and Snyder (2003) for an in-depth discussion of campaign finance and reform.}\)
an equilibrium, of which there can be many, of a game without reform where a high type selects a pair \((q_H, f_H)\). For a given election game and equilibrium level \(f_H\):

**Definition 2** Reform \(\hat{f}\) is **meaningful** if \(\hat{f} < f_H\). Reform \(\hat{f}\) is **meaningless** if \(\hat{f} \geq f_H\).\(^{20}\)

![Diagram](image)

**Figure 2:** Meaningful Campaign Finance Reform Increases Quality of Policy.

Consider the cost structure in Figure 2, which is the same as Figure 1.a with meaningful reform. There is only one equilibrium in Figure 1.a, and hence only one \(f_H\) to worry about. If \(\hat{f}\) is set at or above \(f_H\) the game is unaffected. If \(\hat{f}\) is set below \(f_H\), the reformed game has a unique equilibrium where high types select \((\tilde{q}_H, \hat{f})\), satisfying \(c(\tilde{q}_H, \hat{f}|L) = x_L\). It is clear from the figure that as \(\hat{f}\) is lowered, the quality of policy chosen by a high type increases. Greater reform leads to better policymaking.

Notice that a level of reform \(\hat{f}\) can be meaningful for some equilibria and meaningless for other equilibria of the same game. Therefore, when considering a reform, it is important to know which equilibrium is expected to be played. Henceforth we consider only meaningful reform.

\(^{20}\)Under this model, reducing the cap on individual or Political Action Committee (PAC) contributions to candidates does not fall under our rubric of meaningful vs. meaningless reform. By changing the cost of raising funds, restrictions on PAC giving changes the structure of the game rather than constraining its play.
3.3 Comparison

Since all equilibria (both with and without reform) are separating, an elected official is always of the highest ability available in her election. Reform does not lead to an increase in the (expected) quality of the elected politician. Therefore, utility in the second period is the same with or without reform for all players.

The difference under reform comes from increasing the (expected) quality of the policy made in the first period. If the incumbent is a high type, any meaningful reform will increase her level of $q$ in the first period. This is costly to her, but beneficial to voters. If the incumbent is a low type, reform will not matter; she does nothing in either case. Further, if challengers are high-ability politicians in other government posts, their selection of $\tilde{q}_{H}$ under reform in the first period increases the utility of the constituency of that position.

4 Repeated Elections

4.1 No Campaign Finance Reform

We now return to the repeated elections model and dispense with the direct benefit to voters of having a high-type politician in office. That is, voters only value high-quality policy ($v^t = q^t$). We find the set of symmetric Markov Perfect equilibria where, in each election, low types do nothing and high types separate as cheaply as possible. Further, voters always vote for high types over low types and vote for either candidate with equal probability in elections between candidates of the same type. Of course a degenerate equilibrium, in which voters expect that candidates will never signal, always exists. If a candidate who is never expected to signal does, voters will not change their behavior and elect her with certainty since they still expect she will not signal in the future. Campaign finance reform will have no effect on the degenerate equilibrium.

For expositional clarity it is useful to define $T(x)$, the minimum cost to a high type of preventing imitation when low types gain $x$ from imitation.
Definition 3 Let the function $T : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be the value function of the minimization problem

\[
T(x) = \min_{q_H,f_H} c(q_H, f_H|H) \\
\text{s.t. } c(q_H, f_H|L) \geq x
\]

The $T$ function is found in exactly the same manner as in the one-shot model. Recall that $x_\theta$ is defined as the incremental gain of being perceived as a high type relative to a low type, not taking costs into account. However, in the one-shot election $x_\theta$ was common for both types, and it was a simple change in probability that we could calculate before investigating the actions of each type. With an infinite horizon, $x_\theta$ is different for different types and endogenous to the system.

Proposition 2 A symmetric separating Markov Perfect equilibrium always exists. All symmetric Markov Perfect equilibria either entail repeated least cost separation or are degenerate.

The non-degenerate equilibria are virtually identical to repetitions of the equilibria in the single election model. In each period, as required by our equilibrium concept, low types do nothing and high types differentiate themselves as cheaply as possible. Voters select high types over low types because high types have a greater chance than low types of being high types in the next period.

4.2 Campaign Finance Reform

Again, campaign finance reform is a limit, $\hat{f}$, to the amount candidates can signal through fundraising in each period. We argue that reform will lead to an increase in the equilibrium level of policy chosen by high-type candidates each period. In the one-shot game, such a claim was straightforward to show. In the repeated election setting it is no longer so clear. Let $T$ be the value function without reform and $\tilde{T}$ be the value function with reform. Since reform restricts what high types can do, it must be that $\tilde{T}(x) \geq T(x)$ for all $x$.

Recall that $x_L$ is a low type’s gain from imitating a high type. We saw above that $x_L$ depends on the value of the possibility of being re-elected, $\rho(L)$. Since a low type might become a high type in a subsequent period, $\rho(L)$ depends on what a high type will expend in subsequent periods,
Figure 3: The Post Reform Equilibrium is in the Shaded Region.

\[ T(x_L) \]. If high types must expend more to differentiate themselves, it is not worth as much to low types to imitate them. However, if the low type’s gain to being seen as a high type goes down, high types need not work as hard to differentiate themselves: decreasing \( x_L \) slackens the constraint. Hence, instituting reform has two effects working in opposite directions. It raises the cost of achieving separation for any given \( x_L \), but it lowers the equilibrium level of \( x_L \).

**Proposition 3**  
*Meaningful Campaign Finance Reform leads to an increase in the equilibrium level of policy chosen by high-type candidates.*

A graph is helpful to understand the intuition of the proof. Figure 3 depicts the same isocosts as Figure 1.a and Figure 2. The bold curve represents the isocost \( c(q, f|L) = x \), where \( x \) is the level of \( x_L \) in the repeated game without reform. The thin curve represents the isocost \( c(q, f|H) = T(x) \), and \( q_H \) is the equilibrium level of \( q \) chosen by a high type in the game without reform. Reform then caps fundraising at \( \tilde{f} \). Let \( \tilde{x} \) be the level of \( x_L \) in the repeated game with this reform.

What will a high type’s new equilibrium action \((\tilde{q}_H, \tilde{f}_H)\) be? Obviously, fundraising will be at or below \( \tilde{f} \). There are three potential regions for the solution to lie in: points on or inside the high type’s isocost curve, \( c(\tilde{q}_H, \tilde{f}_H|H) \leq T(x) \), points on or outside the low type’s isocost curve,
\( c(\tilde{q}_H, \tilde{f}_H|L) \geq x \), or points in the shaded region between the two isocosts, \( c(\tilde{q}_H, \tilde{f}_H|H) > T(x) \) and \( c(\tilde{q}_H, \tilde{f}_H|L) < x \). The new solution must lie in this shaded region.

Suppose that at the new solution \( c(\tilde{q}_H, \tilde{f}_H|L) \geq x \). Then after reform a low type is willing to pay at least as much to imitate a high type as under no reform, \( \tilde{x} \geq x \). But high types are paying strictly more to separate themselves, \( \tilde{T}(\tilde{x}) > T(x) \). However, if high types are paying more to separate, it is less profitable for low types to imitate them. This means that low types are willing to pay at least as much for something that is worth strictly less, an obvious contradiction.

Suppose that at the new solution \( c(\tilde{q}_H, \tilde{f}_H|H) \leq T(x) \). Then after reform high types are paying no more to separate themselves than without reform, \( \tilde{T}(\tilde{x}) \leq T(x) \). But a low type is willing to pay strictly less to imitate a high type, \( \tilde{x} < x \). However, if high types are paying no more to separate, it is at least as profitable for low types to imitate them. This means that low types are willing to pay strictly less for something that is worth at least as much, also a contradiction.

In the shaded region high types pay more to separate and low types are willing to pay less to imitate high types. Thus \( \tilde{q}_H \), the level of policy a high type enacts after reform, must be greater than \( q_H \), the level without reform. In fact, \( \tilde{q}_H > q \) where \( q \) satisfies \( c(q, \hat{f}|L) = x \), implying \( \tilde{q}_H > q > q_H \). Thus, for any meaningful reform we can give a lower bound on the necessary increase in \( q_H \), and this bound is strictly larger than zero.

As in Section 3.3, the increase in the equilibrium level of \( q_H \) prompted by reform increases the per-period effort of high-ability politicians. Thus, reform harms high-ability politicians each period and lowers their continuation value. Unlike Section 3.3, which considered a one shot game, reform also harms low-ability politicians. Low-ability politicians may become high ability in the future, so the continuation values of low-ability politicians decrease as well. This allows us to establish our central result:

**Theorem 1** *Meaningful campaign finance reform increases voter welfare and decreases the welfare of politicians.*
4.3 Can Voters do Better?

How can it be that voters care only about the quality of policy, but from their perspective even high-ability politicians perpetually offer too little quality policy and too much fundraising? The answer lies in the dual role of quality policy and voters’ failure to commit to vote on past performance rather than future expectations. When casting his vote, a rational voter cares only about the expected level of policy to be provided by the winning candidate: $q_{\text{tomorrow}}$. To a candidate, policy $q_{\text{today}}$ is only a way to signal her ability. Every period high-ability politicians in office signal in the cheapest way that will guarantee separation. All players know that whomever is elected, the pattern will start anew. Signaling through fundraising amounts to a vicious cycle from which voters cannot escape.

Voters would gain from the ability to reward or punish politicians that provide high or low quality policy with promises of a high or low probability of re-election. There are two ways, both of which our model disallows, to make such promises credible in a repeated game. First, we assume that if voters receive the same continuation value from both candidates they must elect each with equal probability. If voters broke ties according to a history dependent rule, they could squeeze higher quality policy out of their politicians. However, as the number of types increases, the mechanics of our model will not change, but any tie-breaking rule will have only a second-order effect on equilibrium behavior. A candidate’s sole concern will be to credibly signal her type, as it is in our model.

Second, the Markov property rules out equilibria in which high-ability candidates are punished for not providing more $q$ than required for least-cost separation. To support repeated game equilibria of this sort voters must face punishment, for example by moving to a lower payoff equilibrium, if they fail to punish a deviating candidate. This punishment must be either self-induced, which lacks credibility, or requires significant coordination between candidates. However, if candidates are able to coordinate a move to an equilibrium that punishes voters (and therefore benefits themselves), they should do so regardless of any past play.

Campaign finance reform provides an institutional change that benefits voters, though not as
much as an optimal commitment contract would. Obviously, an optimal contract will not provide any rewards to fundraising.\footnote{Recall from contract theory that an optimal contract is a contract that is optimal for the principle (in this case, the electorate). In general this will differ from the contract that maximizes the total surplus of all players.} Full campaign finance reform delivers this property. However, in general, an optimal contract will be a menu of election probabilities $\pi(q)$ such that $1 \geq \pi(q_H) \geq \pi(q_L) \geq 0$, where $q_H \geq q_L \geq 0$, but $q_L$ (and therefore $\pi(q_L)$) need not, and in general will not, be equal to zero. So the voters would strictly prefer an optimal contract to reform, and strictly prefer reform to no reform.

## 5 Extensions

In this section we demonstrate how our model can be extended to explain many of the literature’s stylized facts and encapsulate existing theories.

### 5.1 Lack of Reform

The most obvious stylized fact our model explains is why true campaign finance reform is never implemented. Reform harms politicians. Politicians govern the rules of their own re-election contests, and therefore choose to keep the system most favorable to them.

### 5.2 Incumbency Advantage

#### 5.2.1 Selection Effect

Our model also captures the well known advantage incumbents have over their challengers in re-election races. In our model the incumbency advantage is purely a selection effect.\footnote{Gowrisankaran, Mitchell and Moro (2006) estimate that half of the incumbency advantage in the U.S. Senate can be explained by the selection effect and the other half by deterrence of high-quality challengers. For more on the incumbency advantage see Ansolabehere and Snyder (2002).} Let $\alpha^t$ be the probability an incumbent is a high type in period $t$. In steady state $\alpha^t = \alpha^{t+1} = \alpha$ is defined by the probability that the winner was a high type last period multiplied by $\overline{p}$, plus the probability
that the winner was a low type last period multiplied by $p$:

$$\alpha = (1 - (1 - \alpha)(1 - p))\bar{p} + (1 - \alpha)(1 - p)p$$

$$= \frac{p(1 + p - \bar{p})}{1 + (1 - p)(p - \bar{p})} \in (p, \bar{p})$$

The probability that an incumbent wins an election is therefore:

$$\text{Prob}(I \text{ wins}) = \alpha \pi_H(p) + (1 - \alpha)\pi_L(p)$$

$$= \frac{\alpha - p}{2} + \frac{1}{2} > \frac{1}{2}$$

Notice that the incumbency advantage obtains regardless of whether reform is instituted or not.

5.2.2 Differential fundraising

Incumbents will raise more campaign funds than their challengers for two reasons. One, incumbents are more likely to be high types. High types raise funds, and low types do not. Second, in elections with more than two candidates a candidate’s equilibrium level of effort expenditure is strictly decreasing in her opponents’ likelihood of being high types. Thus, in elections with more than two candidates (no matter how unlikely it is that the third candidate will win), even high type challengers will raise less funds on average than high type incumbents since incumbents are more likely to be high types than challengers are ($\alpha \geq \frac{p}{2}$).

5.2.3 Deterring Challengers

Epstein and Zemsky (1995) (in a theoretical study) and Levitt and Wolfram (1997) (in an empirical study) argue that deterring high-quality challengers is an important source of the incumbency advantage. Our model does not allow candidates the choice of whether or not to run. However, we can employ an extension to analyze this phenomenon. Consider a type $\theta$ politician holding government position $X$, which she values at $u_X$. She may either run for re-election or challenge the incumbent in government position $Y$, who was observed to be a high type last period. She
values position $Y$ at $u_Y$. If she runs for re-election her expected utility is $u_X \pi_\theta(p) - c(q, f|\theta)$. If she runs for position $Y$, her expected utility is $u_Y \pi_\theta(\overline{p}) - c(q, f|\theta)$. Thus, she will run for office $Y$ if and only if:

$$u_X \pi_\theta(p) - c(q, f|\theta) \leq u_Y \pi_\theta(\overline{p}) - c(q, f|\theta)$$

$$\frac{u_X \pi_\theta(p)}{\pi_\theta(\overline{p})} \leq u_Y$$

$p < \overline{p}$ implies that $\frac{\pi_\theta(p)}{\pi_\theta(\overline{p})} \geq 1$. So, unless the value of position $Y$ is sufficiently larger than the value of position $X$, high-ability incumbents will deter challengers.

However, high-ability incumbents deter low-ability challengers more easily than high-ability ones that hold the same office. See that:

$$\frac{\pi_L(p)}{\pi_L(\overline{p})} = \frac{1 - p}{1 - \overline{p}} > \frac{2 - p}{2 - \overline{p}} = \frac{\pi_H(p)}{\pi_H(\overline{p})}$$

which means that the difference between $u_Y$ and $u_X$ required for low types to challenge for position $Y$ is greater than the difference required for high types to challenge. However, it is reasonable to assume that on average high types will hold more coveted offices than low types. That is, we would see high types in offices $X$ and low types in offices $Z$, where $u_X > u_Z$. Then there exist values such that:

$$u_X \frac{2 - p}{2 - \overline{p}} > u_Y \text{ and } u_Z \frac{1 - p}{1 - \overline{p}} \leq u_Y$$

implying high-type challengers will be deterred because they already hold reasonably valued offices, while low types will run for position $Y$ because they hold lesser offices. The assumption of values satisfying these inequalities is equivalent to the common assumption that high types have higher outside options than low types do. Of course, low types who luck into $X$ offices will be disinclined to challenge for $Y$, and high types who hold very low offices will be inclined to challenge for higher positions.

This extension raises a further issue regarding the welfare effects of reform. Let us assume that
high types have higher outside options. Recall that reform decreases that value of being in office. Hence, a reasonable first-pass conclusion is that reform makes office less attractive, which decreases the number of high types who run for office. This would lower the value of $p$, the probability of a challenger being a high type.\textsuperscript{23} Now, the effect of reform on voter welfare is ambiguous. Reform raises the level of quality policy high types produce, but it lowers the probability that a high type will be in office. These forces pull in opposite directions, and thus whether voters are better or worse off will depend on candidates’ cost functions and outside options.

5.3 Bloomberg Signaling

If we assume that challengers hold other offices, they can use policymaking to signal their abilities just as incumbents can. Suppose this were not the case. A challenger who holds no office can only signal through fundraising. Thus, non-office-holding challengers will, in general, need to raise and spend more funds to demonstrate they are high quality than the incumbents they run against.\textsuperscript{24}

We see that our model encapsulates many findings from campaign finance and electoral politics. This framework could be used to discuss political phenomena that are not obviously connected to campaign finance, such as the explosion of pork in recent years (see, for example, Evans (2004)). If we posit that pork has value in signaling a candidate’s ability (which it certainly does) and that there has been a shock to the relative costs of signaling through campaign finance or pork, this would produce results qualitatively similar to what has been observed in recent years.

\textsuperscript{23}If the values of outside options are independent of the number of potential candidates taking them, then $p$ will either decrease to zero or be unchanged. If the values of outside options are decreasing in the number of potential candidates that take them, then $p$ can decrease, but remain positive. Take the latter to be the case.

\textsuperscript{24}For example, New York City Mayor Michael Bloomberg and New Jersey Senator and now Governor John Corzine. Note that throughout we have not distinguished between raising and spending funds. It is not enough to have money, a candidate must spend it on their campaign.
6 Conclusion

We have investigated a model in which campaign fundraising is purely a signaling activity. Even though campaign contributions are not buying political favors, we still find that voters will benefit from campaign finance reform, while politicians will oppose it. There is a simple intuition behind the result, which is more general than this single application. When senders have multiple signaling channels at their disposal, and receivers cannot credibly commit to ignoring any of these channels, senders will use the least costly signals to reveal their types—even if receivers would prefer different signals were used. Further, even if senders only use channels preferable to receivers, receivers cannot credibly commit to respond in pre-specified ways. Policies that restrict signaling channels that are less preferable to receivers can partially substitute for commitment mechanisms, improving receiver welfare at the expense of senders.
Appendix

Proposition 1 In the one-shot game, all equilibria are separating with low types choosing \((q_L, f_L) = (0, 0)\) and high types choosing a solution to (2). A symmetric equilibrium always exists, and is the unique equilibrium if and only if there is a unique solution to (2).

Proof. First, we construct equilibria where candidates play as prescribed. Let \((q_\theta, f_\theta)\) denote the action chosen by a candidate with type \(\theta \in \{L, H\}\), and \(\mu(q, f)\) denote the probability voters assign to a candidate being an \(H\) type given her action \((q, f)\). Let \(\mu(q, f) = 1\) if \(c(q, f|L) \geq x\), and \(\mu(q, f) = 0\) otherwise. Given \((q_L, f_L) = (0, 0)\) and \((q_H, f_H)\) is a solution to (2), the proposed beliefs are consistent with candidates’ actions and our equilibrium definition. If \(\mu^i > \mu^j\), voters select candidate \(i\). If \(\mu^i = \mu^j\), voters select either candidate with probability \(\frac{1}{2}\). Voter behavior is clearly optimal given their beliefs. Since the solution to (2) exists, an \(H\) type’s action is well specified. Fix one candidate playing as prescribed, and consider deviations by the other candidate. \(\mu(q_H, f_H) = 1\) implies an \(H\) type cannot gain by deviating to any \((q, f)\) such that \(c(q, f|H) \not\geq c(q_H, f_H|H)\). By \((q_H, f_H)\) a solution to (2), a deviation to any \((q, f)\) such that \(c(q, f|H) < c(q_H, f_H|H)\) results in \(\mu(q, f) = 0\). \(c(q_H, f_H|L) = x\) and Assumption 2 guarantee that this cannot be optimal. If an \(L\) type deviates to a \((q, f)\) such that \(0 < c(q, f|L) < x\), she will be strictly worse off since \(\mu(0, 0) = \mu(q, f) = 0\), but her costs have increased. If she deviates to a \((q, f)\) such that \(c(q, f|L) \geq x\) then \(\mu(q, f) = 1\). She is no better off since \(x\) is exactly the utility gain of moving from \(\mu = 0\) to \(\mu = 1\). This completes the construction.

We show that pooling is impossible by contradiction. Consider a potential equilibrium in which the two types of candidate \(i\) pool: each plays \((q_p, f_p)\) with positive probability, and \(i\) wins with probability \(\alpha^i_p\) when she plays \((q_p, f_p)\). Let \(\alpha^i_H\) be the probability \(i\) wins if she is believed to be type \(H\) with probability \(1\).\(^{25}\) When there is pooling, either \(\alpha^i_p < \alpha^i_H\) or \(\alpha^j_p < \alpha^j_H\) (or both) must hold. Assume \(\alpha^i_p < \alpha^i_H\). Consider \(\mu(q', f')\) for a \((q', f')\) such that \((q', f') \geq (q_p, f_p)\) and

\[
c(q', f'|H) - c(q_p, f_p|H) < \alpha_H - \alpha_p < c(q', f'|L) - c(q_p, f_p|L)
\]

\(^{25}\)\(\alpha^i_H\) may be different from \(\pi_H\) since we have not assumed that candidate \(j\) is using a separating strategy.
A \((q', f')\) pair that satisfies this property is worse for an \(L\) type and better for an \(H\) type than \((q_p, f_p)\) if \(\mu(q', f') = 1\). The intuitive criterion mandates that \(\mu(q', f') = 1\) for any such \((q', f')\). Assumptions 1 and 2 and continuity of the cost functions ensure such a pair exists. Hence, an \(H\) type has a profitable deviation and there cannot be pooling in equilibrium.

Since neither candidate’s types can pool, both candidates’ types will be revealed preceding the election. Voters select candidates based only on their revealed types. We now rule out non-least cost separating equilibria. Consider a proposed separating equilibrium with \((q_L, f_L) \neq (0, 0)\). Then \(\mu(0, 0) \geq \mu(q_L, f_L) = 0\) and \(c(q_L, f_L|L) > c(0, 0|L) = 0\). So \((0, 0)\) is a profitable deviation for an \(L\) type, contradicting the equilibrium hypothesis. Hence, \((q_L, f_L) = (0, 0)\). Fixing \((q_L, f_L) = (0, 0)\), consider a proposed separating equilibrium with \((q_H, f_H)\) such that \(c(q_H, f_H|L) > x\). Then, by continuity, there exists a \((q'_H, f'_H) \leq (q_H, f_H)\) such that \(c(q'_H, f'_H|H) < c(q_H, f_H|H)\) but \(c(q'_H, f'_H|L) > x\). The intuitive criterion mandates that \(\mu(q'_H, f'_H) = 1\). Then \((q'_H, f'_H)\) is a profitable deviation for an \(H\) type, contradicting the equilibrium hypothesis. Now consider a proposed separating equilibrium with \((q_H, f_H)\) such that \(c(q_H, f_H|L) < x\). Then by deviating to \((q_H, f_H)\) a low type’s expected payoff is \(x - c(q_H, f_H|L) > 0\), making it a profitable deviation and contradicting the equilibrium hypothesis.

Therefore, all equilibria must be least cost separating—the set of equilibria constructed above. If (2) has a unique solution, this set is singleton and both candidates must use the same strategy.

\[\square\]

**On Markov state variables:** There is considerable subtly in choosing what are the allowable state variables in Markov Perfect equilibria (see Mailath and Samuelson (2006) and Maskin and Tirole (2001)). In our game we identify the state variables as the types of the two candidates, \(\{\theta_1, \theta_2\}\). In other dynamic settings it is reasonable to allow for the uninformed players’ beliefs about the informed players’ types be part of the Markov state (Kreps and Wilson, 1982a; Gul, Sonnenschein and Wilson, 1986; Abreu and Gul, 2000; Bar-Isaac, 2003; Phelan, 2006). However, in these games, the unknown type has direct effect on the uninformed player’s payoff, that is, the
unknown type is directly payoff relevant. Including players’ beliefs in the state is the only way to generate reasonable stationary equilibria.

In our model, this is not the case. If beliefs are included in the state, players can use them as coordination devices. For example, consider the following equilibrium: when the incumbent is believed to be a high type with probability \( p \) or \( \bar{p} \) both types of candidates pool at the signal \((0, 0)\), otherwise both candidates play separating strategies.\(^{26}\) Voters always elect high types over low types in separating periods, they vote randomly after pooling when the beliefs are \((p, p)\), and they elect the incumbent for sure after pooling when the beliefs are \((\bar{p}, p)\). The pooling periods survive in equilibrium because if a candidate were to break the pooling and identify herself as either a high type or a low type for sure, she would not be elected—electing her guarantees that the system transitions to a pooling period, which is worse for voters. We believe that such equilibria violate the spirit of Markov Perfect equilibrium as summarized by Maskin and Tirole (2001), “[In Markov Perfect equilibria] strategies depend on as few variables as possible; they involve no complex ‘bootstrapping’ in which each player conditions on a particular variable only because others do the same.”

**Proposition 2** A symmetric separating Markov Perfect equilibrium always exists. All symmetric Markov Perfect equilibria either entail repeated least cost separation or are degenerate.

**Proof.** The set of symmetric Markov Perfect equilibria where in each election \((q_L, f_L) = (0, 0)\), \(H\) types separate as cheaply as possible, voters always vote for \(H\) types over \(L\) types and vote for either candidate with equal probability in elections between same types are characterized by:

\[
V(H|p) = \left(\frac{2-p}{2}\right) \delta \left(1 + \bar{p}V(H|p) + (1-p)V(L|p)\right) - T(x_L) \quad (V.1)
\]
\[
V(L|p) = \left(\frac{1-p}{2}\right) \delta \left(1 + pV(H|p) + (1-p)V(L|p)\right) \quad (V.2)
\]
\[
V(H|\bar{p}) = \left(\frac{2-\bar{p}}{2}\right) \delta \left(1 + \bar{p}V(H|\bar{p}) + (1-\bar{p})V(L|\bar{p})\right) - T(x_L) \quad (V.3)
\]
\[
V(L|\bar{p}) = \left(\frac{1-\bar{p}}{2}\right) \delta \left(1 + pV(H|\bar{p}) + (1-p)V(L|\bar{p})\right) \quad (V.4)
\]

\(^{26}\)The challenger always has probability \( \bar{p} \) of being a high type, so there is no variation to condition on.
\[ V(L|p) = \left( \frac{2-p}{2} \right) \delta \left( 1 + pV(H|p) + (1-p)V(L|p) \right) - \bar{x}_L \]  
\[ V(L|\bar{p}) = \left( \frac{2-\bar{p}}{2} \right) \delta \left( 1 + \bar{p}V(H|\bar{p}) + (1-\bar{p})V(L|\bar{p}) \right) - \bar{x}_L \]

Equations (V.1) through (V.4) are standard dynamic programming value functions. \( V(\theta|p) \) is the value of being type \( \theta \) and facing an opponent who is an \( H \) type with probability \( p \). \( \bar{x}_L \) and \( \bar{x}_L \) are the costs \( L \) types would have to incur to successfully imitate \( H \) types when facing opponents with priors \( p \) and \( \bar{p} \) respectively. Equations (IC.5) and (IC.6) are the binding IC constraints. From Section 4, we know that the \( T \) function is well defined. Solving the system gives that \( \bar{x}_L = \bar{x}_L \) and yields a specific value for this term.\(^{27}\)

A solution always exits and is unique. First, the system (V.2), (V.4), (IC.5), (IC.6) consists of four linear equations and five unknowns (\( V(H|\bar{p}) \) does not appear). From these four equations, we obtain unique solutions for \( V(H|p), V(L|p), V(L|\bar{p}) \), and \( \bar{x}_L \) in terms of \( \bar{x}_L \). \( V(H|\bar{p}) \) is also determined uniquely in terms of \( \bar{x}_L \) from (V.3). Hence, our solution exists, and is unique, if and only if there exists a unique \( \bar{x}_L \) satisfying the updated (V.1).

Substituting into (V.1), then rearranging, gives:

\[
\frac{2\bar{x}_L - \delta \bar{x}_L (1 + (p-1)^2)}{V(H|\bar{p})} = \left( \frac{2-p}{2} \right) \delta \left( 1 + \bar{p} \frac{2\bar{x}_L - \delta \bar{x}_L (1 + (p-1)^2)}{V(H|\bar{p})} \right) + (1-\bar{p}) \bar{x}_L (1-p) - T(\bar{x}_L) \\
0 = k + a\bar{x}_L - T(\bar{x}_L) \tag{A.7}
\]

Where \( k \) and \( a \) are constants:

\[
k = \frac{2 - \delta(2-p)(\bar{p}-p)}{2\bar{p}} > 0
\]
\[
a = \frac{1}{2\delta \bar{p}} \left( \delta^2(2-3\bar{p} + p^2)(\bar{p}-\bar{p}) + 2\delta(1 + p^2 + 2\bar{p} - p(2 + \bar{p})) - 4 \right) < 0
\]

\(^{27}\)As in the one-shot game, \( x_L \) is constant with respect to the opponent’s priors \( p \in (0, 1) \), hence \( \bar{x}_L = \bar{x}_L \). In richer settings this will not be the case.
\(k > 0\) gives that the right-hand side of (A.7) is positive when \(\bar{x}_L = 0\). \(a < 0\) and \(T\) strictly increasing give that the right-hand side of (A.7) is strictly decreasing, and is negative for \(\bar{x}_L\) large enough. Therefore (A.7) has a unique solution, and thus so does the entire system.

We now construct the repeated least-cost separating equilibria. Since the equilibrium is stationary, we specify play and beliefs for a single arbitrary time period. Consider the play of a candidate facing an opponent who plays as prescribed and has prior \(p\). In equilibrium \((q_L, f_L) = (0, 0)\), \(H\) types choose a pair satisfying: \(c(q_H, f_H|H) = T(\bar{x}_L)\) and \(c(q_H, f_H|L) = \bar{x}_L\). Voters’ beliefs are \(\mu(q, f) = 1\) if \(c(q, f|L) \geq \bar{x}_L\), and \(\mu(q, f) = 0\) otherwise. These beliefs are correct given the candidates’ strategies. Voters then select \(H\) types over \(L\) types and select either candidate with equal probability if they are equally likely to be \(H\) types. By stationarity, voters know that an \(L\) type in office next period will set \(q_L = 0\), but an \(H\) type will set \(q_H > 0\). Voters prefer an \(H\) type to be in office. However, in the next period a candidate may no longer be the same type she revealed this period. By \(\bar{p} > p\), an \(H\) type is at least as likely to be an \(H\) type again as an \(L\) type is become an \(H\) type. Thus, voters prefer to select candidates they perceive to be \(H\) types before the election. The specified voting behavior is optimal given beliefs. \(\mu(q_H, f_H) = 1\) implies \(H\) types cannot gain from deviating to any \((q, f)\) such that \(c(q, f|H) \geq c(q_H, f_H|H)\). By \(c(q_H, f_H|H) = T(\bar{x}_L)\) and \(c(q_H, f_H|L) = \bar{x}_L\), a deviation to any \((q, f)\) such that \(c(q, f|H) < c(q_H, f_H|H)\) results in \(\mu(q, f) = 0\). Since \(H\) types value office at least as much as \(L\) types \((\bar{x}_H \geq \bar{x}_L)\), and \(L\) types are indifferent between imitating \(H\) types or not (equation (IC.5)), Assumption 2 guarantees that being perceived as an \(L\) type cannot be optimal for an \(H\) type. If an \(L\) type deviates to a \((q, f)\) such that \(0 < c(q, f|L) < \bar{x}_L\), she will be strictly worse off since \(\mu(0, 0) = \mu(q, f) = 0\) but her costs have increased. If she deviates to a to a \((q, f)\) such that \(c(q, f|L) \geq x_L\) then \(\mu(q, f) = 1\). She is no better off since \(\bar{x}_L\) is exactly the utility gain of moving from \(\mu = 0\) to \(\mu = 1\). Everything is analogous for candidates facing opponents with prior \(\bar{p}\), using \(T(\bar{x}_L)\) and \(\bar{x}_L\) in place of \(T(\bar{x}_L)\) and \(\bar{x}_L\).

We now show that all non-degenerate symmetric equilibria are least cost separating. A strategy for a candidate is simply a map from her type to some (possible mixed) action \(\sigma(\theta)\). Denote the
expected per period \( q \) selected by a candidate of type \( \theta \) as \( E[q|\theta] \). One of the following must hold: \( E[q|H] > E[q|L] \), \( E[q|H] = E[q|L] \), or \( E[q|H] < E[q|L] \). Suppose that \( E[q|H] > E[q|L] \). Then voters wish to elect high types over low types. Hence, pooling equilibria and non-least cost separating equilibria are ruled out by the same arguments given in the proof of Proposition 1. Suppose that \( E[q|H] = E[q|L] \). Then voters receive the same continuation value from any candidate regardless of type. If the equilibrium does not specify that \( \sigma = (0, 0) \) for both types, then there is a profitable deviation. A candidate of either type can deviate to \( (0, 0) \), save costs and still be elected with the same probability, \( \frac{1}{2} \). Hence, \( E[q|H] = E[q|L] \) in equilibrium if and only if the equilibrium is degenerate. Suppose that \( E[q|H] < E[q|L] \). Then voters prefer low-type candidates over high-type candidates. Then a high type will choose to deviate and imitate a low type (which she can always do profitably since she has a cost advantage over a low type), contradicting the hypothesis.

Proposition 3  Meaningful Campaign Finance Reform leads to an increase in the equilibrium level of policy chosen by high-type candidates.

Proof. Solving the system of equations (V.1) through (IC.6) yields:

\[
\bar{x}_L = x_L = \frac{2\delta - 2T(x_L)p\delta + 2p\delta^2 - 2\bar{p}\delta^2 + p\overline{p}\delta^2 - p^2\delta^2}{(4-2\delta) + (\overline{p} - 2)(\overline{p} - p)(2\delta + \delta^2(\overline{p} - 1))} \tag{V.8}
\]

\[
\frac{\partial x_L}{\partial T(x_L)} = \frac{-2p\delta}{(4-2\delta) + (\overline{p} - 2)(\overline{p} - p)(2\delta + \delta^2(\overline{p} - 1))} < 0 \tag{V.9}
\]

To see that (V.9) is negative note that the numerator is negative. Also: \( (4 - 2\delta) > 0, (\overline{p} - 2) < 0, (\overline{p} - p) > 0, \) and \( (2\delta + \delta^2(\overline{p} - 1)) > 0 \). The first term in the denominator is positive and the second is negative. The first term is minimized at \( \delta = 1 \), and the second term is minimized (i.e. made maximally negative) at \( \delta = 1, \overline{p} = 0, \overline{p} = 1 \). At these parameter values, the denominator is zero. Hence, the denominator is positive for all \( \delta \) in \([0, 1]\) and \( \overline{p} < p \) both in \((0, 1)\).

Let \( x \) be the level of \( x_L \) prior to reform, and \( \tilde{x} \) be its level after reform is implemented. From the text we know \( \tilde{T}(x) \geq T(x) \) for all \( x \). Let \( q_H \) and \( \tilde{f}_H \) be the post reform levels of
$q$ and $f$ selected by an $H$ type. Assume, by way of contradiction, that $\tilde{T}(\tilde{x}) < T(x)$. By $T$ and $\tilde{T}$ non-decreasing and $\tilde{T} \geq T$, $\tilde{T}(\tilde{x}) < T(x)$ implies $x > \tilde{x}$. But this contradicts (V.9). Hence, the amount a high type is willing to pay for separation must not decrease following reform: $\tilde{T}(\tilde{x}) \geq T(x)$. Since $c(q, f|H)$ is strictly increasing, and the no reform level of fundraising was $f_H > \tilde{f}_H$, $\tilde{T}(\tilde{x}) = c(\tilde{q}_H, \tilde{f}_H|H) \geq c(q_H, f_H|H) = T(x)$ implies $\tilde{q}_H > q_H$.  

**Theorem 1** *Meaningful campaign finance reform increases voter welfare and decreases the welfare of politicians.*

**Proof.** The proof follows directly from the previous proposition and the discussion in Section 4.2.  

■
References


Fox, Justin. 2007. “Electoral Donations and Interest Group Influence.” Yale University, *mimeo*.


