TESTS OF CERTAIN TYPES OF IGNORABLE NONRESPONSE IN SURVEYS SUBJECT TO ITEM NONRESPONSE OR ATTRITION

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Abstract

Analysts of cross-sectional or panel surveys often base inferences about relationships between variables on complete units, excluding units that are incomplete due to item nonresponse or attrition. This practice is justifiable if exclusion is ignorable in an appropriate sense. This paper characterizes certain types of ignorable exclusion in surveys subject to item nonresponse and develops tests based on these characterizations. These tests are applied to data from several National Election Study panels and evidence is found of violations of these characterizations. Characterizations and tests of certain types of ignorable attrition in standard panel surveys are also presented.

1. Introduction

Every sample survey is subject to nonresponse. Unit nonresponse occurs when a person, or unit, in the original sampling frame does not respond to the survey. Item nonresponse occurs when a person in the original sampling frame responds to the survey, but fails to provide information on at least one survey variable, or item. Cross-sectional surveys are subject to unit and item nonresponse, while panel surveys are subject to unit nonresponse in the first wave, attrition in subsequent waves,

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and item nonresponse in all waves. In a panel survey, attrition occurs when people who respond in whole or in part in the first wave (or set of waves) do not respond at all in subsequent waves. Thus, attrition can be viewed as a type of unit nonresponse.

Frequently, survey analysts base inferences about relationships between variables on complete units, excluding any unit that is incomplete due to any form of nonresponse. This practice is justifiable provided exclusion is ignorable. For example, exclusion is always ignorable if complete units are a random subsample of an original random sample of the population of interest. In this case, the joint distribution of the complete units is equal to the joint distribution of the original sample. Using the terminology of Little and Rubin (1987, p. 14), we say that incomplete units are *missing completely at random*, or MCAR, for short. In regression and likelihood-based inference, information on the parameters of interest is contained in the conditional distribution of the response variable given the explanatory variables. Here, exclusion is ignorable provided it does not corrupt this conditional distribution. In this case, again using the terminology of Little and Rubin (1987), we say that incomplete units are *missing at random*, or MAR, for short.

Nonignorable exclusion of incomplete units can lead to biased estimators and therefore, to invalid inferences. Brehm (1993) makes this point with respect to unit nonresponse in political polls and surveys. Hausman and Wise (1979) and Hsiao (1986, Section 8.3.2) make similar points with respect to attrition in economic panel surveys. King, Honaker, Joseph, and Sheve (1998) address the problem of item nonresponse in their work on imputation methods, and suggest that item nonresponse may be a more serious problem in surveys on political attitudes and behavior than unit nonresponse.

Hsiao (1986), Little and Rubin (1987), and Brehm (1993) discuss various methods for correcting selection bias resulting from violations of the MAR assumption. These methods include the Heckman (1979) correction, the Achen (1986) correction, and maximum likelihood methods, and involve postulating and estimating a model of the exclusion mechanism. See also the work of King (1989, Chapter 9). Alternatively, imputation techniques can sometimes be applied to "fill in" missing values and so avoid biases associated with item nonresponse. See Rubin (1987,1996), and King, Honaker, Joseph, and Sheve (1998) for recent work on imputation methods.

Before applying a potentially complicated procedure to try to guard against bias stemming from nonignorable exclusion, it is useful to have simple tests at hand to help decide if such action is needed. This paper develops simple odds-ratio tests of characterizations of certain types of ignorable item nonresponse. These tests apply to cross-sectional surveys as well as to panel surveys. We also develop odds-ratio and χ^2 tests of characterizations of certain types of ignorable attrition in standard panel surveys. The natural data structure for most of these tests is a 2-way table. However, as we will show, the tests can be easily adapted to cover data organized into any multi-way table.

Throughout this paper, we shall assume that respondents in the original sampling frame of a survey are representative of the population of interest. It follows that any bias in the set of complete units is due only to excluding incomplete units in a nonrandom fashion. This is an important assumption, since many surveys are not simple random samples, even though they may be designed to approximate such samples. For example, the American National Election Study (NES) surveys have complex sampling designs involving multi-stage stratification and clustering. While NES surveys are designed to approximate simple random samples of the United States population, to the extent that they are not good approximations, to that extent additional bias is introduced into the data. We do not address this important issue in this paper, but rather assume from the outset that the original sampling frame is representative.

In the next section, we define various types of MCAR and MAR data and indicate the types for which we will develop tests. In Section 3, we state characterizations, based on odds-ratios, of MCAR and MAR conditions for survey data subject to item nonresponse. We then develop tests based on these characterizations in the context of American National Election Study (NES) surveys. These are special two-wave panels, where respondents are asked about political attitudes and demographics in a pre-election survey and voting behavior in a post-election survey. In Section 4, we apply the odds-ratio tests to NES data from the presidential election years 1952, 1980, and 1984. Section 5 reports the results of several simulations illustrating the power of the tests and relating power properties to the magnitude of selection bias in a simple binary response model. In Section 6, we develop characterizations and tests for MCAR and MAR conditions in the context of attrition in standard panel surveys where each variable is observed in each wave of the panel. Section 7 summarizes. An Appendix provides proofs of the characterizations on which the odds-ratio tests are based, and also presents the tabulated NES data used in Section 4.

2. MCAR and MAR

Depending on the application, either an MCAR or MAR condition must hold to justify a complete-unit analysis. The MCAR condition is needed in inference problems where the joint distribution of the random variables under study must be preserved, such as in multivariate density estimation, cluster analysis, or factor analysis. In standard regression analysis or likelihood-based inference, the parameters of interest often depend only on the conditional distribution of the response variable given the explanatory variables. In settings such as these, the weaker MAR condition is all that is needed. When the MAR condition is violated, parameter estimates may suffer from selection bias.

In this section, we define different ways in which data can be MCAR or MAR, and identify the types of missingness for which we will develop tests in this paper.

To fix ideas, suppose we survey a random sample of 1000 United States citizens who were registered to vote in the 1998 congressional election. We are interested in the relationship between race (nonblack or black) and turnout (nonvoter or voter) in the election. We view turnout as a response variable and race as an explanatory variable. In this survey, the people are called survey units and the variables race and turnout are called survey items. If some people do not respond, we have unit nonresponse. If people respond, but refuse to reveal either their race or their voting behavior (or both), we have item nonresponse. The missing data set is the set of (race, turnout) pairs for units in the original sampling frame with at least one item missing. The complete data set is the set of (race, turnout) pairs for the remaining units. The union of the complete and missing data sets is the full data set.

We will partition each of the MCAR and MAR conditions into three types. The following notation will be useful in defining these types in the context of the example described in the previous paragraph. Let $n = 1000$. Let Y_{ij} denote the ith observation on the jth variable, $i = 1, \ldots, n$, $j = 1, 2$. The subscript $j = 1$ corresponds to race and $j = 2$ corresponds to turnout. Let M_{ij} denote a missing-data indicator for Y_{ij}. That is, $M_{ij} = 1$ if Y_{ij} is missing, and $M_{ij} = 0$ otherwise. Thus, the complete data are the set of (Y_{i1}, Y_{i2}) pairs for which $M_{i1} = M_{i2} = 0$.

We begin with the notion of MCAR. As stated previously, the MCAR condition holds if the joint distribution of race and turnout for the complete data set is equal to the joint distribution of race and turnout for the full data set.

We say that MCAR(unit) holds if and only if

(i) $M_{i1} = M_{i2}$, $i = 1, \ldots, n$.

(ii) M_{i1}, $i = 1, \ldots, n$, are independent and identically distributed.

Condition (i) says that race is missing if and only if turnout is missing. Thus, MCAR(unit) holds only in the context of pure unit nonresponse, hence the name. The assumption of identical distributions in condition (ii) implies that $P(M_{i1} = 1)$ is constant. In other words, missingness does not depend on race, turnout, or observation number. To illustrate this notion, suppose we remove each unit from the full data set independently with probability p. (One can think of the M_{i1}'s in (ii) as independent coin flips, each with probability p of turning up heads.) Under this missingness mechanism, the joint distribution of race and turnout for the complete data set is equal to the joint distribution of race and turnout for the full data set. Thus, MCAR(unit) holds.

We say that MCAR(item) holds if and only if

(i) M_{ij}, $i = 1, \ldots, n$, $j = 1, 2$, are independent.

(ii) For $j = 1, 2$, M_{ij}, $i = 1, \ldots, n$, are identically distributed.

Condition (i) implies that missingness in race is independent of missingness in turnout. Thus, MCAR(item) holds only in the context of pure item nonresponse, hence the name. Condition (ii) implies that $P(M_{i1} = 1)$ and $P(M_{i2} = 1)$ are constant for all i, but that these constants can be different. To illustrate, suppose we remove each race item from the full data set independently with probability p. Next, independently of race item removal, we remove each turnout item from the full data set independently with probability λ. (One can think of the M_{i1}'s (M_{i2}'s) in (ii) as independent coin flips, each with probability p (λ) of turning up heads. Moreover the M_{i1}'s and the M_{i2}'s are independent.) Under this missingness mechanism, the joint distribution of the complete data set is equal to the joint distribution of the full data set. Thus, MCAR(item) holds.

We say that MCAR(other) holds if the joint distribution of the complete data set is equal to the joint distribution of the full data set, but neither MCAR(unit) nor MCAR(item) holds.

Next, consider the notion of MAR. Refer once again to the example above. Recall that MAR holds if the conditional distribution of turnout (the response variable) given race (the explanatory variable) for the complete data set is equal to the conditional distribution of turnout given race for the full data set.

We say that MAR(unit) holds if and only if

(i) $M_{i1} = M_{i2}$, $i = 1, \ldots, n$.

(ii) M_{i1}, $i = 1, \ldots, n$, are independent, and, within race categories, identically distributed.

Condition (i) says that race is missing if and only if turnout is missing. Thus, MAR(unit) holds only in the context of pure unit nonresponse. Condition (ii) implies that $P(M_{i1} = 1)$ is constant within race categories, but that these constants can be different. For example, suppose we remove each black respondent with probability p and each nonblack respondent with probability τ. If $p \neq \tau$, then the joint distribution of the complete data set is not equal to the joint distribution of the full data set. The corresponding marginal distributions are different, for example. However, the conditional distribution of turnout given race for the complete data set is the same as the conditional distribution of turnout given race for the full data. Thus, MAR(unit) holds.

We say that MAR(item) holds if and only if

(i) M_{ij}, $i = 1, \ldots, n$, $j = 1, 2$, are independent.

(ii) M_{i2}, $i = 1, \ldots, n$, are identically distributed.

(iii) M_{i1}, $i = 1, \ldots, n$, are identically distributed within race categories.

By condition (i), missingness in race is independent of missingness in turnout. Thus, MAR(item) holds only in the context of pure item nonresponse. Conditions (ii) and (iii), respectively, imply that $P(M_{i2} = 1)$ is constant and $P(M_{i1} = 1)$ is constant within race categories. Moreover, all these constants can be different. To illustrate, suppose we remove each race item from the set of black respondents with probability p and each race item from the set of nonblack respondents with probability τ. In addition, independently of race item removal, we remove each turnout item with probability λ. Once again, if $p \neq \tau$, then the joint distribution of race and turnout for the complete data set is different from the joint distribution of race and turnout for the full data. However, the corresponding conditional distributions of turnout given race are the same. Thus, MAR(item) holds.

We say that MAR(other) holds if the conditional distribution of the response variable given the explanatory variables for the complete data is the same as the corresponding conditional distribution for the full data, but neither MAR(unit) nor MAR(item) holds.

In summary, an MCAR condition is needed to justify a complete-unit analysis when the joint distribution of all the variables under study must be preserved. The weaker MAR condition distinguishes between a response variable and explanatory variables. This condition suffices when only the conditional distribution of the response variable given the explanatory variables must be preserved, as in regression or likelihood-based inference. Missingness that depends on the response variable can corrupt this conditional distribution and lead to various types of selection bias.

MCAR(unit) and MAR(unit) conditions hold only in the context of pure unit nonresponse. MCAR(unit) requires that missingness be independent of all variables under study, while MAR(unit) allows missingness to depend on the explanatory variables, but not the response. By comparison, MCAR(item) and MAR(item) conditions hold only in the context of pure item nonresponse. MCAR(item) requires that missingness in one variable be independent of missingness in all other variables, and that missingness in each variable be independent of the values of that variable. MAR(item) requires that missingness in the response variable be independent of missingness in the explanatory variables. Unlike MCAR(item), MAR(item) allows missingness to depend on the explanatory variables, but not on the response variable.

In this paper, we will develop tests of the MCAR(item) and MAR(item) conditions in the context of cross-sectional data and special panels such as the NES panels. In the two-wave NES panels, it seems particularly natural to test for ignorable item nonresponse, since often the explanatory variables are measured in the first wave and the response variables in the second wave. We will also develop tests of the MCAR(unit) and MAR(unit) conditions in the context of standard panels. However, in this panel data context, we will test for ignorable attrition, not for ignorable first wave unit nonresponse.

3. Characterizations and Tests of MCAR(item) and MAR(item)

This section states characterizations of the MCAR(item) and MAR(item) conditions defined in the last section, and develops tests of these conditions based on these characterizations.

The simplest setting for the tests developed in this section is illustrated in Table 1 below. Responses to the NES survey for presidential election year 1952 are cross-classified by the explanatory variable, race, and the response variable, turnout. Race was recorded primarily in the pre-election wave and has two categories: nonblack, denoted $\overline{B}$, and black, denoted B. Turnout was recorded in the post-election wave and has two categories: nonvoter, denoted $\overline{V}$, and voter, denoted V. There are 1899 respondents, and there is incomplete information on 286, or about 15%, of these. We let M_R denote the event missing race, and M_T the event missing turnout.

	Turnout		
Race	M_T	$\overline{V}$	V
M_R	3	44	57
$\overline{B}$	168	314	1142
B	14	105	52

Table 1: Race by Turnout, 1952

Is it justifiable to use the 2×2 subtable of complete units to make inferences about the relationship between race and turnout? For example, is it valid to do a probit analysis of turnout on race using only complete units? The answer is "yes" if either the MCAR(item) or MAR(item) condition holds.

If either the MCAR(item) or MAR(item) condition holds, then certain odds-ratio[2] relations follow. Recall that for an event A with probability p, the odds that A occurs rather than not is $p/(1-p)$. Now, consider the odds that turnout is observed rather than not observed. Intuitively, if either MCAR(item) or MAR(item) holds, then the odds for those whose race is observed should be the same as the odds for those whose race is not observed. Consequently, the ratio of these odds should be unity. Other odds-ratio relations also follow from the MCAR(item) and MAR(item) conditions. Later in this section, we state a minimal set of odds-ratio conditions that characterizes the MCAR(item) condition and another minimal set that characterizes the MAR(item) condition. We use these conditions to develop simple tests of the MCAR(item) and MAR(item) conditions. Proofs of the characterizations are established in the appendix.

Before we state the characterizations, we first define the odds-ratio components of the charac-

[2] See Agresti (1990) for a thorough treatment of odds-ratios as measures of association. See Rosenbaum (1995, Chapter 4) for the use of odds-ratios to measure the degree of hidden bias in observational studies.

terizations. For simplicity and clarity, we begin with the 3×3 table of race by turnout given above, and then generalize to an arbitrary 2-way table.

Code the race outcomes $M_R = 0$, $\overline{B} = 1$, and $B = 2$. Similarly, code the turnout outcomes $M_T = 0$, $\overline{V} = 1$, and $V = 2$. For $i = 0, 1, 2$, $j = 0, 1, 2$, write p_{ij} for the probability of race outcome i and turnout outcome j. For example, $p_{00} = P(M_R, M_T)$. We assume that $p_{ij} > 0$ for all i and j. We get the table of joint probabilities in Table 2.

	Turnout		
Race	M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

Table 2: Joint Probabilities

In Table 2, Row 0 holds the probabilities of turnout outcomes when race is missing, and rows 1 and 2 hold the probabilities of turnout outcomes when race is observed. Similarly, column 0 holds the probabilities of race outcomes when turnout is missing, and columns 1 and 2 hold the probabilities of race outcomes when turnout is observed. Given that race is observed, the odds that turnout is observed rather than not observed is given by $(p_{11}+p_{12}+p_{21}+p_{22})/(p_{10}+p_{20})$. Similarly, given that race is not observed, the odds that turnout is observed rather than not observed is given by $(p_{01} + p_{02})/p_{00}$. The ratio of these odds is our first odds-ratio, denoted Θ. That is,

$$\Theta = \frac{p_{00}(p_{11} + p_{12} + p_{21} + p_{22})}{(p_{01} + p_{02})(p_{10} + p_{20})} .$$

This odds-ratio can be conveniently remembered by considering the following display.

We see that Θ is the product of sums of elements in the diagonal blocks divided by the product of sums of elements in the off-diagonal blocks.

Race	Turnout M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

Return to Table 2. Given that the race outcome is B, the odds that turnout is observed rather than not observed is given by $(p_{21} + p_{22})/p_{20}$. Similarly, given that the race outcome is $\overline{B}$, the odds that turnout is observed rather than not observed is given by $(p_{11} + p_{12})/p_{10}$. The ratio of these odds is our second odds-ratio, denoted R_{12}, since it uses information from rows 1 and 2 in Table 2. We see that

$$R_{12} = \frac{p_{10}(p_{21} + p_{22})}{(p_{11} + p_{12})p_{20}} .$$

This odds-ratio can be conveniently remembered by considering the following display.

Race	Turnout M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

We see that R_{12} is the product of sums of elements in the diagonal blocks divided by the product of sums of elements in the off-diagonal blocks.

Return once more to Table 2. Given that the turnout outcome is V, the odds that race is observed rather than not observed is given by $(p_{12} + p_{22})/p_{02}$. Similarly, given that the turnout outcome is $\overline{V}$, the odds that race is observed rather than not observed is given by $(p_{11} + p_{21})/p_{01}$. The ratio of these odds is our third odds-ratio, denoted C_{12}, since it uses information from columns 1 and 2 in Table 2. We see that

$$C_{12} = \frac{p_{01}(p_{12} + p_{22})}{p_{02}(p_{11} + p_{21})}\,.$$

This odds-ratio can be conveniently remembered by considering the following display.

	Turnout		
Race	M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

We see that C_{12} is the product of sums of elements in the diagonal blocks divided by the product of sums of elements in the off-diagonal blocks.

We are now in a position to state the characterizations of the MCAR(item) and MAR(item) conditions for the race by turnout example.

THEOREM 1. *MCAR(item) holds if and only if* $\Theta = 1$, $R_{12} = 1$, *and* $C_{12} = 1$.

THEOREM 2. *MAR(item) holds if and only if* $\Theta = 1$ *and* $R_{12} = 1$.

The condition $\Theta = 1$ says that the odds of observing rather than not observing turnout for those whose race is observed is equal to the corresponding odds for those whose race is not observed. Similarly, $R_{12} = 1$ says that the odds of observing rather than not observing turnout for blacks is equal to the corresponding odds for nonblacks. Finally, $C_{12} = 1$ says that the odds of observing rather than not observing race for voters is equal to the corresponding odds for nonvoters. These 3 conditions are intuitively obvious implications of the MCAR(item) condition, as are the first 2 conditions for the MAR(item) condition. Formal proofs of these characterizations appear in the appendix.

Theorems 1 and 2 suggest tests of the MCAR(item) and MAR(item) conditions using sample analogues of the population odds-ratios. We now develop the details of these tests. Consider the 3×3 table of counts given below. In this table, n_{ij} is the number of sample points falling in the ijth cell of the table.

	Turnout		
Race	M_T	$\overline{V}$	V
M_R	n_{00}	n_{01}	n_{02}
$\overline{B}$	n_{10}	n_{11}	n_{12}
B	n_{20}	n_{21}	n_{22}

Define the sample analogues of Θ, R_{12}, and C_{12} as follows:

$$\begin{aligned}
\hat{\Theta} &= \frac{(n_{00}+0.5)(n_{11}+n_{12}+n_{21}+n_{22}+0.5)}{(n_{01}+n_{02}+0.5)(n_{10}+n_{20}+0.5)} \\
\hat{R}_{12} &= \frac{(n_{10}+0.5)(n_{21}+n_{22}+.05)}{(n_{11}+n_{12}+0.5)(n_{20}+0.5)} \\
\hat{C}_{12} &= \frac{(n_{01}+0.5)(n_{12}+n_{22}+.05)}{(n_{02}+0.5)(n_{11}+n_{21}+0.5)}.
\end{aligned}$$

Addition of 0.5 to the components of the ratios improves small sample performance without affecting asymptotic behavior (Agresti, 1990, Section 3.4.1). If MAR(item) holds, then $\log \hat{\Theta}$ and $\hat{R}_{12}$ converge in distribution to normal random variables with mean zero and standard errors approximated by the square-root of the sum of the reciprocals of the corresponding block sums. For example, the approximate standard error of $\hat{\Theta}$ is given by

$$ASE \equiv \sqrt{\frac{1}{n_{00}+0.5} + \frac{1}{\sum_{i=1}^{2}\sum_{j=1}^{2} n_{ij}+0.5} + \frac{1}{\sum_{j=1}^{2} n_{0j}+0.5} + \frac{1}{\sum_{i=1}^{2} n_{i0}+0.5}}.$$

This follows from the joint asymptotic normality of the cell counts and an application of the delta method. Agresti (1990, p.425) gives a formal derivation of this result. An analogous result holds for $\log \hat{R}_{12}$. Under MCAR(item), an analogous result also holds for $\log \hat{C}_{12}$.

An asymptotic level α test of MCAR(item) consists of rejecting MCAR(item) if and only if at least one of the test statistics $|\log \hat{\Theta}|$, $|\log \hat{R}_{12}|$, or $|\log \hat{C}_{12}|$, standardized by their ASE's, exceeds the z-score $z_{\alpha/6}$. This is a conservative asymptotic level α test based on Bonferroni's inequality (Bickel and Doksum, 1976, p. 162). If $\alpha = .05$, then $z_{\alpha/6} = 2.39$. An asymptotic level α test of MAR(item) consists of rejecting MAR(item) if and only if either $|\log \hat{\Theta}|$ or $|\log \hat{R}_{12}|$, standardized by their ASE's, exceeds the z-score $z_{\alpha/4}$. If $\alpha = .05$, then $z_{\alpha/4} = 2.24$.

Next, we state the characterizations for general 2-way tables. Consider an explanatory variable X categorized into I levels $X_1, \ldots, X_I$ and a response variable Y categorized into J levels $Y_1 \ldots, Y_J$. ($I = J = 2$ in the race by turnout example.) Let M_X and M_Y denote missing X and missing Y, respectively. Consider the following $(I+1) \times (J+1)$ table of joint probabilities:

		Y		
X	M_Y	Y_1	$\cdots$	Y_J
M_X	p_{00}	p_{01}	$\cdots$	p_{0J}
X_1	p_{10}	p_{11}	$\cdots$	p_{12}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
X_I	p_{I0}	p_{I1}	$\cdots$	p_{IJ}

Define the population odds-ratios

$$
\begin{aligned}
\Theta &= \frac{p_{00}(\sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij})}{(\sum_{j=1}^{J} p_{0j})(\sum_{i=1}^{I} p_{i0})} \\
R_{ij} &= \frac{p_{i0}(\sum_{k=1}^{J} p_{jk})}{(\sum_{k=1}^{J} p_{ik})p_{j0}} \quad i = 1, \ldots, I-1 \quad j = i+1 \\
C_{ij} &= \frac{p_{0i}(\sum_{k=1}^{I} p_{kj})}{(\sum_{k=1}^{I} p_{ki})p_{0j}} \quad i = 1, \ldots, J-1 \quad j = i+1\,.
\end{aligned}
$$

Theorem 1′. *MCAR(item) holds if and only if* $\Theta = 1$, $R_{i,i+1} = 1$, $i = 1, \ldots, I-1$, *and* $C_{i,i+1} = 1$, $i = 1, \ldots, J-1$.

Theorem 2′. *MAR(item) holds if and only if* $\Theta = 1$ *and* $R_{i,i+1} = 1$, $i = 1, \ldots, I-1$.

The sample analogues $\hat{\Theta}$, $\hat{R}_{ij}$, and $\hat{C}_{ij}$ are defined as before, by replacing the p_{ij}'s with n_{ij}'s and adding 0.5 to each component of the sample odds-ratios. The ASE's and tests are also defined as before. Thus, an asymptotic level α test of MCAR(item) consists of rejecting MCAR(item) if and only if at least one of the test statistics $|\log \hat{\Theta}|$, $|\log \hat{R}_{i,i+1}|$ for some $i \in \{1, \ldots, I-1\}$, or $|\log \hat{C}_{i,i+1}|$ for some $i \in \{1, \ldots, J-1\}$, standardized by their ASE's, exceeds the z-score $z_{\alpha/2k}$, where $k = I + J - 1$. An asymptotic level α test of MAR(item) consists of rejecting MAR(item) if and only if either $|\log \hat{\Theta}|$ or $|\log \hat{R}_{i,i+1}|$ for some $i \in \{1, \ldots, I-1\}$, standardized by their ASE's, exceeds the z-score $z_{\alpha/2k}$, where $k = I$.

Remark 1. The asymptotic level α tests of the MCAR(item) and MAR(item) conditions developed in this section are based on Bonferroni's inequality, which bounds the probability of a union of possibly dependent events. In the MAR(item) example for the 3×3 table discussed above, the events in question are A_1 and A_2, where A_i is the event that the ith test statistic exceeds the Bonferroni critical value of $z_{\alpha/4}$. These events are dependent, since the corresponding test statistics are based on the same data set. Bonferroni's inequality guarantees that, asymptotically, the probability of a Type 1 error is less than or equal to α. If this probability is strictly less than α, then the test will reject a true null hypothesis according to this stricter criterion. In this sense, the test is conservative.

Remark 2. Theorems 1 and 2 (and their generalizations Theorems 1′ and 2′) provide characterizations of the MCAR(item) and MAR(item) conditions. In other words, the odds-ratio conditions in these theorems are necessary and sufficient. This is important for interpreting the outcomes of tests of these conditions.

Necessity of the MAR(item) condition, for example, implies that rejection of the MAR(item) test has the usual interpretation that the data do not support the MAR(item) hypothesis. However, this does not necessarily mean that corrective action is required. For example, MAR(unit) or MAR(other) could hold without MAR(item) holding. On the other hand, for data like NES data where item nonresponse is likely to be the sole or primary source of missingness, decisive rejection of MAR(item) when the estimated odds ratios are far from unity is cause for concern and may require corrective action to avoid selection biases. (See, for example, the 1984 income by turnout results in Table 3 in Section 4.)

Sufficiency of the MAR(item) condition, for example, implies that failure to reject may have a more positive interpretation, namely, that the data support the MAR(item) hypothesis. This is especially true when the estimated odds-ratios are close to unity. (See, for example, the 1984 race by turnout results given in Table 3 in Section 4.) In this case, a practitioner doing regression or likelihood-based inference may proceed with caution with a complete-unit analysis.

Remark 3. The odds-ratio tests developed in this section are defined in the context of a special two-wave panel, a single categorical explanatory variable, and a categorical response variable. Extensions of the test in various directions are straightforward.

Consider the case of more than one explanatory variable. For example, suppose we have categorical explanatory variables race (nonblack or black) and income (low or high), and turnout (nonvoter or voter) is the response. Construct a new combined explanatory variable, a race-income variable, with four levels: nonblack-low, ($\overline{B}L$), nonblack-high ($\overline{B}H$), black-low (BL), and black-high (BH). Let M_{RI} denote the event that either race or income is missing for a given respondent. Form the

5×3 table of counts:

	Turnout		
Race-Income	M_T	$\overline{V}$	V
M_{RI}	n_{00}	n_{01}	n_{02}
$\overline{B}L$	n_{10}	n_{11}	n_{12}
$\overline{B}H$	n_{20}	n_{21}	n_{22}
BL	n_{30}	n_{31}	n_{32}
BH	n_{40}	n_{41}	n_{42}

As before, n_{ij} denotes the number of sample points falling in the ijth cell of the table. For example, n_{00} denotes the number of respondents for whom either race or income is missing and turnout is missing. We may now proceed to test for MAR(item) as prescribed in this section, letting the race-income variable play the role of the explanatory variable.

We can also apply the test when either the response or the explanatory variables are continuous by categorizing continuous variables in an appropriate way. Some information may be lost in categorizing, but the test is still useful for spotlighting departures from the MCAR(item) or MAR(item) conditions in the data.

Also, note that nothing intrinsic to the panel structure of the NES surveys was required to apply the odds-ratio tests. The tests would apply just as readily if the data were collected in a single post-election survey. In other words, the tests, as presented in this section, apply immediately to cross-sectional data.

4. Applications to NES Data

In this section, we apply the odds-ratio tests to NES data. We calculate the odds-ratios and the test statistics for the 2-way tables of race by turnout, education by turnout, and income by turnout, for the presidential election years 1952, 1980, and 1984. The results appear in Table 2.

Nonresponse rates are printed in parentheses next to the years. Information on education and income levels are given in the appendix along with the tables on which these calculations are based.

To illustrate the testing procedures, suppose we wish to test the MCAR(item) assumption for the 1952 race by turnout table. The relevant test statistics are the ratios of $|\log \hat{\Theta}|$, $|\log \hat{R}_{12}|$, and $|\log \hat{C}_{12}|$ to their ASE's. The Bonferroni critical value for this test is 2.39 when the significance level is .05. We will reject the MCAR(item) assumption at level .05 if any of the 3 test statistics exceeds 2.39. We see from Table 3 that the test statistic associated with the odds-ratio $\hat{C}_{12}$ equals 3.81, and so we decisively reject MCAR(item) at this level. Suppose that instead of testing MCAR(item) for this table, we test MAR(item). In this case, the relevant test statistics are the ratios of $|\log \hat{\Theta}|$ and $|\log \hat{R}_{12}|$ to their ASE's. The Bonferroni critical value for this test is 2.24 when the significance level is .05. We will reject MAR(item) if either of the test statistics exceeds 2.24. We see from Table 3 that neither test statistic exceeds this value, and so technically we would not reject MAR(item) at level .05. However, note that the test statistic associated with $\hat{\Theta}$ is equal to 2.20. Since the Bonferroni test is conservative and this test statistic is close to the critical value, we might be inclined to reject MAR(item).

As a second example, consider testing MCAR(item) for the 1984 race by turnout table. The relevant test statistics and Bonferroni critical value are the same as in the last example. Since the largest test statistic is 1.15, we do not reject MCAR(item) for this table at level .05. In fact, we can make a more positive statement. Notice that each of the three estimated odds-ratios is fairly close to unity. Recall that the corresponding odds-ratio conditions characterize MCAR(item). That is, these conditions are necessary and sufficient for MCAR(item) to hold. Thus, failure to reject MCAR(item), coupled with the fact that the estimated odds-ratios are close to unity, can be interpreted more positively as support for the hypothesis that MCAR(item) holds. (See also

Race by Turnout

	1952 (.15)		1980 (.13)		1984 (.12)	
	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic
$\hat{\Theta}$	.30	2.16	.97	.02	1.04	.04
$\hat{R}_{12}$	1.26	.80	1.11	.40	1.25	1.15
$\hat{C}_{12}$	2.20	3.81	.35	.69	1.15	.22

Education by Turnout

	1952 (.15)		1980 (.13)		1984 (.12)	
	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic
$\hat{\Theta}$	.47	1.70	2.92	1.09	2.25	1.33
$\hat{R}_{12}$	1.07	.28	1.08	.17	1.73	1.24
$\hat{R}_{23}$	1.05	.21	2.44	1.99	.60	1.37
$\hat{R}_{34}$	1.07	.29	.55	1.58	1.62	1.71
$\hat{R}_{45}$	.97	.10	1.25	1.05	1.46	1.89
$\hat{R}_{56}$	1.07	.17	1.48	1.82	.97	.15
$\hat{R}_{67}$			2.56	2.27	2.51	2.88
$\hat{R}_{78}$			.58	.98	.54	1.41
$\hat{C}_{12}$	2.27	4.01	.35	.69	1.15	.22

Income by Turnout

	1952 (.17)		1980 (.22)		1984 (.21)	
	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic	Odds-Ratio	Test Statistic
$\hat{\Theta}$	1.48	1.56	2.27	4.26	2.61	5.96
$\hat{R}_{12}$	1.32	.91	.84	.65	.83	.94
$\hat{R}_{23}$	.96	.18	.97	.13	1.86	2.64
$\hat{R}_{34}$	.82	.75	1.42	1.36	1.42	1.21
$\hat{R}_{45}$	1.16	.60	.84	.70	.81	.76
$\hat{C}_{12}$	2.05	3.90	1.20	.97	1.25	1.35

Table 3: Odds-Ratio Test Results

Remark 2 in Section 3.)

As a final example, consider testing MAR(item) for income and turnout in 1984. The relevant test statistics are the ratios of $|\log \hat{\Theta}|$ and $|\log \hat{R}_{i,i+1}|$ for $i = 1, \ldots, 4$, to their ASE's. For this test, the Bonferroni critical value is 2.58 when the significance level is .05. We decisively reject MAR(item) based on the test statistics associated with $\hat{\Theta}$ and $\hat{R}_{23}$. Moreover, the fact that $\hat{\Theta}$, for example, is far from unity suggests that trying to determine the relationship between these two variables based on a complete-unit analysis may be problematic. (See also Remark 2 in Section 3.)

It is interesting to note that the odds-ratio entries in Table 3 give precise, interpretable information on the nature and extent of the departures from the MCAR(item) and MAR(item) conditions. For example, consider the results for race and turnout in 1952. The odds-ratio $\hat{C}_{12} = 2.2$ tells us that odds of observing rather than not observing race is 2.2 times greater for voters than for non-voters. For income and turnout in 1984, the odds-ratio $\hat{\Theta} = 2.61$ tells us that the odds of observing rather than not observing turnout is 2.61 times greater for someone whose income is observed than for someone whose income is not observed. Similarly, the odds-ratio $\hat{R}_{23} = 1.86$ says that the odds of observing rather than not observing turnout is 1.86 times greater for someone in income group 3 than for someone in income group 2.

5. Simulation Results

In this section, we briefly summarize the results of eight simulations that explore the power of the proposed odds-ratio tests against various gradually increasing departures from MCAR(item) and MAR(item) conditions. We also summarize results of a ninth simulation experiment assessing the effect of gradually increasing departures from the MAR(item) condition on estimates of regression parameters in a simple probit model. Details of these simulations can be obtained by downloading an expanded version of this paper located with other 1999 working papers at the Political Methodology website *http://www.polmeth.calpoly.edu*.

The first set of four simulations examines departures from MCAR(item), starting from the null case of 2000 observations on two binary variables organized into a 2×2 table. In all these simulations, the odds-ratio test of MCAR(item) exhibits the correct size under the null hypothesis. The first three simulations show that the power of the test approaches unity rapidly against alternatives where missingness in one variable is more and more dependent on missingness in the other variable. The last of these simulations examines MAR(item) departures from MCAR(item). As expected, the test is less powerful against this alternative than against the other three, since under MAR(item), missingness in one variable is independent of missingness in the other.

The second set of four simulations examines departures from MAR(item), using the same setup as that used for the first set of simulations. The first three simulations mimic the previous first three and the conclusions are qualitatively the same. The fourth simulation examines departures from MAR(item) where missingness in the explanatory variable is independent of missingness in the response variable, but missingness is allowed to depend on the values of the response variable as well as the values of the explanatory variable. Again, the results are qualitatively the same as those obtained in the fourth MCAR(item) simulation.

The final set of simulations explores the effect of gradually increasing departures from the MAR(item) condition on estimates of regression parameters in a simple probit model. We find that for one set of alternatives, the test is unlikely to reject the MAR(item) hypothesis when the magnitude of selection bias due to nonrandom exclusion is small, while the test is likely to reject when the magnitude is high. This is certainly a desirable property. However, for two other sets of alternatives, the test is likely to reject even when the magnitude of the selection bias is small, indicating that the odds-ratio test is too powerful in these instances.

6. Characterizations and Tests of Ignorable Attrition in Standard Panel Surveys

As noted in Section 3, NES surveys are special two-period panels in which some variables are observed only in the pre-election survey and others are observed only in the post-election survey. Unlike NES panels, standard panel surveys observe each variable in each period. The odds-ratio tests can be applied as in Section 3 to detect nonignorable exclusion due to item nonresponse within each wave of the panel. However, because more information is available, characterizations and tests of ignorable attrition can also be developed. We illustrate this with a simple hypothetical example.

Consider a panel of married women followed over two time periods. At each period, observations are taken on their work status, employed (E) or unemployed (U), and their husband's income, high (H) or low (L). View work status as a response variable and income as an explanatory variable. In the second period, some women are lost to attrition, some persist. Consider the first period tables of joint probabilities of income by work status given below.

	Work Status	
Income	E	U
L	π_{11}	π_{12}
H	π_{21}	π_{22}

Table 4: All women.

	Work Status	
Income	E	U
L	p_{11}	p_{12}
H	p_{21}	p_{22}

Table 5: Women who persist to the second period.

As indicated in the captions, the first table gives the first period joint distribution of income by work status for all women, whereas the second table gives the first period joint distribution for those women who persist to the second period. For $i = 1, 2$, let R_i denote the odds-ratio $\frac{\pi_{i1}p_{i2}}{\pi_{i2}p_{i1}}$. The following characterizations follow immediately from the definitions of MCAR(unit) and MAR(unit) given in Section 2:

THEOREM 3. *MCAR(unit) holds if and only if $\pi_{ij} = p_{ij}$ for all i and j.*

THEOREM 4. *MAR(unit) holds if and only if $R_i = 1$, $i = 1, 2$.*

These characterizations suggest easy ways to test the MCAR(unit) and MAR(unit) conditions. Theorem 3 implies that we can test MCAR(unit) by applying a simple χ^2 test for equality of distributions (see, for example, Hogg and Tanis, 1983, pp.414-415). It is clear that this characterization and the corresponding test can be generalized to cover data organized into any multi-way table. Theorem 4 implies that we can test MAR(unit) by forming the sample odds-ratios corresponding to R_1 and R_2, and then proceeding as in Section 3. This characterization and the corresponding test generalize to cover 2-way tables of any size. We plan to develop these and related ideas further in future work.

7. SUMMARY

This paper develops a number of simple tests of conditions that justify a complete-unit analysis of survey data subject to item nonresponse or attrition. The tests presuppose that the original sampling frame is representative of the population of interest, and are based on characterizations of certain types of MCAR and MAR conditions. A MAR condition, for example, is needed to justify regression analysis using only complete units. A violation of the MAR condition signals that some corrective action may be required. For example, techniques like those of Heckman (1979) or Achen (1986) may help correct parameter estimation bias introduced by exclusion. Alternatively, imputation or EM methods developed in Little and Rubin (1987), Rubin (1987,1996), and King et al. (1998) may help correct bias as well as increase efficiency in parameter estimation.

Tests of certain MCAR and MAR conditions in the context of item nonresponse are applied to NES panel data and evidence is presented of violations of these conditions. Simulation experiments explore the power of the tests and their relation to the magnitude of selection bias in a simple binary response model. Applications to cross-sectional data and extensions to multi-way tables are discussed.

Appendix

In the first part of the appendix, we restate and prove the odds-ratio characterizations of the MAR(item) and MCAR(item) conditions on which the tests defined in Section 3 are based. We prove the results for the 3×3 example introduced in Section 3. The proof for an arbitrary two-way table will follow in a straightforward manner by mimicking the proof for the 3×3 case. At the end of the appendix, we provide a table containing the two-way tables of NES data used to calculate the results presented in Table 3 in Section 4.

We begin by developing notation needed for the proofs of the odds-ratio characterizations. Let π_{11} denote that probability of being a nonblack nonvoter, π_{12} the probability of being a nonblack voter, π_{21} the probability of being a black nonvoter, and π_{22} the probability of being a black voter. Define $\gamma = \pi_{11} + \pi_{12}$ to be the marginal probability of being nonblack, and $1 - \gamma = \pi_{21} + \pi_{22}$ to be the marginal probability of being black. Similarly, define $\alpha = \pi_{11} + \pi_{21}$ to be the marginal probability of being a nonvoter, and $1 - \alpha = \pi_{12} + \pi_{22}$ the marginal probability of being a voter.

Write $\overline{M}_R$ for the outcome race not missing and $\overline{M}_T$ for the outcome turnout not missing. Define $p = P(M_R \mid \overline{B})$, the probability that race is missing given that a respondent is nonblack, and $\tau = P(M_R \mid B)$, the probability that race is missing given that a respondent is black. Note that under MCAR(item), $p = \tau$. Finally, define $\lambda = P(M_T)$, the probability that turnout is missing.

THEOREM 1: *MCAR(item) holds if and only if* $\Theta = 1$, $R_{12} = 1$, *and* $C_{12} = 1$.

PROOF. We begin by proving that MCAR(item) implies $\Theta = 1$, $R_{12} = 1$, and $C_{12} = 1$. Straightforward calculations yield the following table of joint probabilities:

		Turnout	
Race	M_T	$\overline{V}$	V
M_R	$p\lambda$	$p(1-\lambda)\alpha$	$p(1-\lambda)(1-\alpha)$
$\overline{B}$	$(1-p)\lambda\gamma$	$(1-p)(1-\lambda)\pi_{11}$	$(1-p)(1-\lambda)\pi_{12}$
B	$(1-p)\lambda(1-\gamma)$	$(1-p)(1-\lambda)\pi_{21}$	$(1-p)(1-\lambda)\pi_{22}$

From the table, it is easy to verify that $\Theta = 1$, $R_{12} = 1$, and $C_{12} = 1$.

We now show that $\Theta = 1$, $R_{12} = 1$, and $C_{12} = 1$ imply the MCAR(item) condition. Consider the following 3×3 table of joint probabilities where we assume that each $p_{ij} > 0$:

		Turnout	
Race	M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

Take p_{ij}, $i = 1, 2$, $j = 1, 2$ as known, since these entries correspond to the subtable of complete units to be used for analysis. Define $\sigma = p_{11} + p_{12} + p_{21} + p_{22}$. Fix p_{02} at an arbitrary positive value consistent with the odds-ratio constraints and the constraints that the p_{ij}'s be positive and sum to unity. (If no such value of p_{02} exists, then the result is trivially true.) We have four equations ($\sum_{i,j} p_{ij} = 1$, $\Theta = 1$, $R_{12} = 1$, and $C_{12} = 1$) in the four unknowns p_{00}, p_{10}, p_{20}, and p_{01}. Define, for $i = 1, 2$, $g_i = p_{i1} + p_{i2}$ and $a_i = p_{1i} + p_{2i}$. Next, define the ratios $r = g_1/g_2$ and $c = a_1/a_2$. Finally, define $\kappa = (1+r)(1+c)p_{02}/\sigma$. Straightforward algebra shows that the system of equations defined above has the following unique solution:

$$
\begin{aligned}
p_{00} &= \kappa(1-\sigma-(1+c)p_{02})/(\kappa+1+r) \\
p_{10} &= r(1-\sigma-(1+c)p_{02})/(\kappa+1+r) \\
p_{20} &= (1-\sigma-(1+c)p_{02})/(\kappa+1+r) \\
p_{01} &= cp_{02}\,.
\end{aligned}
$$

All that remains is to show that this solution satisfies the constraints of the MCAR(item) condition. For example, note that $p_{00} = P(M_R, M_T)$, $p_{00}+p_{01}+p_{02} = P(M_R)$ and $p_{00}+p_{10}+p_{20} = P(M_T)$. MCAR(item) requires that the events M_R and M_T be independent, and so $P(M_R, M_T)$ must equal $P(M_R)P(M_T)$. Simple algebra shows that $p_{00} = (p_{00}+p_{01}+p_{02})(p_{00}+p_{10}+p_{20})$, as required. As another example, MCAR(item) requires that $\pi_{11}+\pi_{12}+\pi_{21}+\pi_{22} = 1$. Again, simple algebra shows that $(p_{11}+p_{12}+p_{21}+p_{22})/(1-p)(1-\lambda) = 1$, where $p = p_{00}+p_{01}+p_{02}$ and $\lambda = p_{00}+p_{10}+p_{20}$. The other MCAR(item) constraints can be verified in like manner. *QED.*

THEOREM 2: *MAR(item) holds if and only if* $\Theta = 1$ *and* $R_{12} = 1$*.*

PROOF. We begin by proving that MAR(item) implies $\Theta = 1$ and $R_{12} = 1$. Straightforward calculations yield the following table of joint probabilities:

		Turnout	
Race	M_T	$\overline{V}$	V
M_R	$[p\gamma+\tau(1-\gamma)]\lambda$	$[p\pi_{11}+\tau\pi_{21}](1-\lambda)$	$[p\pi_{12}+\tau\pi_{22}](1-\lambda)$
$\overline{B}$	$(1-p)\gamma\lambda$	$(1-p)(1-\lambda)\pi_{11}$	$(1-p)(1-\lambda)\pi_{12}$
B	$(1-\tau)(1-\gamma)\lambda$	$(1-\tau)(1-\lambda)\pi_{21}$	$(1-\tau)(1-\lambda)\pi_{22}$

From the table, it is easy to verify that $\Theta = 1$ and $R_{12} = 1$.

We now show that $\Theta = 1$ and $R_{12} = 1$ imply the MAR(item) condition. Consider the following 3×3 table of joint probabilities where we assume that each $p_{ij} > 0$.

	Turnout		
Race	M_T	$\overline{V}$	V
M_R	p_{00}	p_{01}	p_{02}
$\overline{B}$	p_{10}	p_{11}	p_{12}
B	p_{20}	p_{21}	p_{22}

Take p_{ij}, $i = 1, 2$, $j = 1, 2$ as known, since these entries correspond to the subtable of complete units to be used for analysis. Define $\sigma = p_{11}+p_{12}+p_{21}+p_{22}$. Fix $\beta = p_{01}+p_{02}$ at an arbitrary positive value consistent with the odds-ratio constraints and the constraints that the p_{ij}'s be positive and sum to unity. (If no such value of β exists, then the result is trivially true.) We have three equations ($\sum_{i,j} p_{ij} = 1$, $\Theta = 1$ and $R_{12} = 1$) in the three unknowns p_{00}, p_{10}, and p_{20}. Define, for $i = 1, 2$, $g_i = p_{i1} + p_{i2}$. Next, define the ratio $r = g_1/g_2$. Simple algebra shows that the system of equations defined above has the unique solution:

$$
\begin{aligned}
p_{00} &= \beta(1-\sigma-\beta)/(\beta+\sigma) \\
p_{10} &= \sigma r(1-\sigma-\beta)/[(1+r)(\beta+\sigma)] \\
p_{20} &= \sigma(1-\sigma-\beta)/[(1+r)(\beta+\sigma)]\,.
\end{aligned}
$$

All that remains is to show that this solution satisfies the constraints of the MAR(item) condition. For example, $p_{00} = P(M_R, M_T)$, $p_{00} + p_{01} + p_{02} = P(M_R)$ and $p_{00} + p_{10} + p_{20} = P(M_T)$. MAR(item) requires that the events M_R and M_T be independent, and so $P(M_R, M_T)$ must equal $P(M_R)P(M_T)$. Simple algebra shows that $p_{00} = (p_{00} + p_{01} + p_{02})(p_{00} + p_{10} + p_{20})$, as required. As another example, MAR(item) requires that $(p_{01} + p_{02})/(1 - \lambda) = p_{00}/\lambda$, where $\lambda = p_{00} + p_{10} + p_{20}$. Again, simple algebra shows that this holds. The other MAR(item) constraints can be verified in like manner. *QED*.

Remark. In the previous proof, notice that if MAR(item) holds but MCAR(item) does not hold, then $p \neq \tau$. In other words, suppose MAR(item) holds. Then MCAR(item) also holds if and only if $p = \tau$.

Table 6 below contains the 2-way tables of counts used to calculate the results in Table 3. The following abbreviations are used: M_R for missing race, M_E for missing education, M_I for missing income, M_T for missing turnout, $\overline{V}$ for nonvoter, V for voter, $\overline{B}$ for nonblack, B for black, S/NG for some or no grade school, G for grade school, SH for some high school, H for high school, SC for some college, C for college, AD for advanced degree, 5K for \$5,000, etc.

Race by Turnout

	1952			1980			1984		
	M_T	$\overline{V}$	V	M_T	$\overline{V}$	V	M_T	$\overline{V}$	V
M_R	3	44	57	0	0	3	1	3	8
$\overline{B}$	168	314	1142	185	348	891	232	448	1315
B	14	105	52	22	55	110	35	74	141

Education by Turnout

	1952				1980			1984		
	M_T	$\overline{V}$	V		M_T	$\overline{V}$	V	M_T	$\overline{V}$	V
M_E	5	46	58	M_E	1	0	3	3	3	8
S/NG	45	169	191	G 0-4	8	12	15	12	19	21
G	35	81	219	G 5-7	15	19	35	12	31	39
SH	36	84	241	G 8	9	36	45	25	31	54
H	40	58	329	H9-11	42	90	109	42	101	129
SC	14	18	116	H	85	151	347	89	216	494
C	10	7	97	SC	34	72	225	63	91	400
				C	7	16	149	12	26	216
				AD	6	7	76	10	7	103

Income by Turnout

	1952				1980				1984		
	M_T	$\overline{V}$	V		M_T	$\overline{V}$	V		M_T	$\overline{V}$	V
M_I	20	55	77	M_I	43	47	100	M_I	62	62	143
0-1K	17	76	62	0-7K	27	96	123	0-9K	53	153	203
1-3K	41	151	281	7-13K	35	74	163	9-17K	66	106	261
3-4K	36	88	276	13-20K	33	64	152	17-25K	29	85	217
4-5K	28	40	193	20-30K	34	73	243	25-35K	22	62	266
5K+	43	53	362	30K+	35	49	223	35K +	36	57	374

Table 6: 2-way tables on which results in Table 3 are based.

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