**NUCLEAR
INSTRUMENTS
& METHODS
IN PHYSICS
RESEARCH**
Section A

Light attenuation length of barium fluoride crystals *

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This report presents a deduction of a formula which can be used to calculate the light attenuation length of barium fluoride crystals based on the transmittance (or absorbance) data measured by a spectrophotometer.

1. Introduction

The light attenuation length (λ) is an important parameter in evaluating the quality of barium fluoride (BaF_2) crystals [1]. An approximated formula, eq. (5) of ref. [1], was suggested by one of the authors to calculate the light attenuation length of a BaF_2 crystal by use of its measured transmittance (T) and an ideal theoretical transmittances (T_s) which takes care of the loss at two surfaces of the crystal:

$$\lambda = \frac{l}{\ln(T_s/T)}, \tag{1}$$

where l is the path length, i.e. the crystal length.

However, this equation suffers from two approximations

1) the ideal theoretical transmittances (T_s) was calculated according to an extrapolation of measured refractive index by Malitson [2], while the original measurement was carried out for wavelengths of longer than 260 nm only; and,

2) the equation does not take into account multiple bounces between two end surfaces of the crystal.
The consequence of these two approximations is

1) at the wavelength of BaF_2's fast component (220 nm), the ideal theoretical transmittances (T_s) is not accurate; and

2) the light attenuation length calculated by using eq. (1) is not accurate.
In this note, we present a deduction of an accurate formula which can be used to calculate the light attenuation length of BaF_2 crystals based on the transmittance (or absorbance) data measured by a spectrophotometer.

2. Determination of T_s

As defined above, T_s is an ideal transmittance of a BaF_2 crystal with infinite light attenuation length and two parallel end surfaces. The loss of light in each surface can be calculated by using Fresnel's law:

$$R = \frac{(n - n_{air})^2}{(n + n_{air})^2}, \tag{2}$$

where n and n_{air} are the refractive index of BaF_2 and air which are function of the wavelength. Note, for wavelengths of longer than 200 nm, $n_{air} - 1$ is less than 0.0004.

Taking into account multiple bounces between two end surfaces, the ideal transmittance (T_s) can be written as

$$T_s = (1 - R)^2 + R^2(1 - R)^2 + \cdots$$
$$= (1 - R)/(1 + R). \tag{3}$$

Assuming the crystal has a light attenuation length of λ, which is also a function of wavelength, the light intensity after traversing a path length of l is:

$$L = e^{-l/\lambda}. \tag{4}$$

Taking into account both multiple bounces between two end surfaces and light attenuation loss, the real transmittance (T) measured can be written as

$$T = (1 - R)^2 L + R^2(1 - R)^2 L^3 + \cdots$$
$$= \frac{L(1 - R)^2}{1 - L^2 R^2}. \tag{5}$$

For a thin crystal with good quality (typical light attenuation length of > 200 cm), the l/λ is very small. Eq. (4) thus can be written as
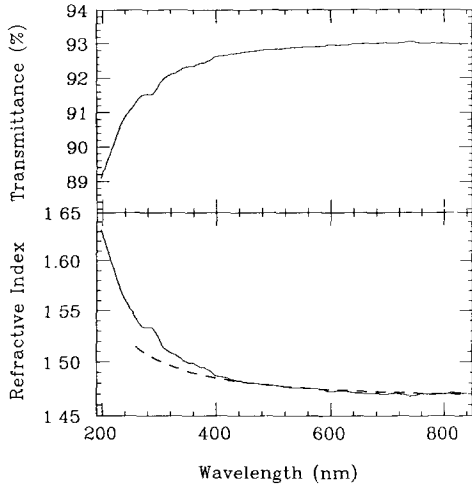
$$L \approx 1 - l/\lambda. \tag{6}$$

Fig. 1. Ideal transmittance and refractive index of a BaF₂ crystal are plotted as a function of wavelength. The refractive index from Malitson [2] is shown as a dashed line for a comparison.

Eq. (5) can be written as

$$T \approx T_s(1 - l/\lambda). \tag{7}$$

For a 2 mm thick crystal, which has a light attenuation length of > 200 cm and two parallel well polished surfaces, the l/λ is less than 0.001. We thus can use a spectrophotometer to measure its transmittance and approximately take this measured value as T_s. A crystal with a dimension of $40 \times 40 \times 2$ mm^2 was provided by Shanghai Institute of Ceramics (SIC) for this purpose. The crystal was polished at Lawrence Livermore National Laboratory by using diamond pitch wheel method. It is shown that this polishing technique provides almost perfect optical surface [3].

The transmittance of this 2 mm thick crystal was measured by using a Hitachi U-3210 UV/Visibla spectrophotometer equipped with double beam and double monochromator. The systematic uncertainty of the transmittance measurement is around 0.1%.

Fig. 1 shows the measured transmittance and the refractive index calculated by using eqs. (2) and (3). Also shown, as a dashed line, in fig. 1 is the refractive index of a BaF₂ crystal measured by Malitson [2]. It is clear that the agreement between this work and ref. [2] is very good at a wavelength range of longer than 400 nm. Below 400 nm, however, the refractive index calculated in this work is higher than Malitson's measurement [2]. This difference can be attributed to the intrinsic absorption of SIC sample in this wavelength region, as evidenced by the 290 nm absorption peak of SIC BaF₂ crystal [1] in fig. 1, and, to a lesser extent, to the difficulty in preparing a perfect surface at UV range.

The T_s value at 220 nm (for the fast component of BaF₂) is 90.06%, while the T_s value obtained by using Malitson's measurement [2] and eqs. (2) and (3) is 91.40%. Taking into account the uncertainty of surface property of BaF₂ in UV range, the real value of T_s relevant in calculating the light attenuation length of BaF₂ crystals may be somewhere between these two numbers.

3. Determination of λ

Eq. (5) can be written as

$$TR^2L^2 + (1 - R)^2 L - T = 0 \tag{8}$$

which can be solved with a solution of

$$L = \frac{\sqrt{4T_s^4 + T^2(1 - T_s^2)^2} - 2T_s^2}{T(1 - T_s)^2}, \tag{9}$$

where eq. (3) is used.

Using eq. (4), we thus have an accurate formula which can be used to calculate the light attenuation length (λ) by using measured transmittance (T):

$$\lambda = \frac{l}{\ln\left\{\left[T(1 - T_s)^2\right] / \left[\sqrt{4T_s^4 + T^2(1 - T_s^2)^2} - 2T_s^2\right]\right\}}. \tag{10}$$

Note, in case T_s close to 1, the eq. (10) can be approximated to eq. (1). Fig. 2 shows light attenuation length at 220 nm, as a function of the transmittance, calcu-
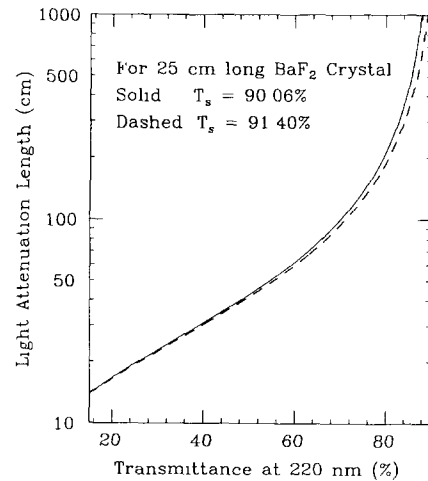

Fig. 2. Light attenuation length at 220 nm as a function of transmittance measured for a 25 cm long BaF₂ crystal. The solid line is for $T_s = 90.06\%$ from a thin sample measurement, while the dashed line is for $T_s = 91.40\%$ from an extrapolation of Malitson's measured refractive index [2].
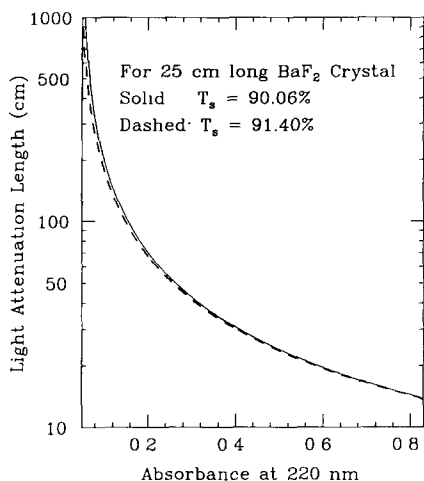
Fig. 3. Light attenuation length at 220 nm as a function of absorbance measured for a 25 cm long BaF$_2$ crystal. The solid line is for T_s = 90.06% from a thin sample measurement, while the dashed line is for T_s = 91.40% from an extrapolation of Malitson's measured refractive index [2].

lated by using eq. (10) for a 25 cm long BaF$_2$ crystal, assuming T_s = 90.06% (solid line) and 91.40% (dashed line).

Theoretically, absorbance (A), or optical density (D), are more adequate to describe quality of crystals, since they are linearly related to the color center density, or the cross section of light absorption. They are defined as:

$$A \text{ or } D = \log(1/T) = -\log T. \qquad (11)$$

By using approximate eq. (1), one has

$$\lambda = \frac{0.43 \, l}{A - \log(1/T_s)}. \qquad (12)$$

Fig. 3 shows the light attenuation length at 220 nm, as a function of absorbance, calculated by using eqs. (10) and (11) for a 25 cm long BaF$_2$ crystal, assuming T_s = 90.06% (solid line) and 91.40% (dashed line).

Note, both figs. 2 and 3 show light attenuation length of 25 cm long crystals. For a BaF$_2$ crystal with length of l cm, the corresponding light attenuation length should be multiplied by 0.04 l.

It is also interesting to note that the effect of different T_s values used in calculating the light attenuation length is very small, except in the region where the measured transmittance is close to the theoretical limit of T_s. As discussed in section 2, the real value of the light attenuation length of BaF$_2$ crystals is somewhat between these two curves.

Acknowledgement

References

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[2] I.H. Malitson, J. Opt. Soc. Am. 54 (5) (1964) 628.
[3] C. Wuest and B. Fuchs, GEM TN-92-163 (1992).