Basic stats

	25%	Median	75%	
Mid term	18.3	19.6	22.5	Max 24.5/25
Class so far	64%	73%	81%	(55 pt possible)

Misses are pretty much evenly distributed across parts of test.

11-11 Class 13 Options and Hedging

Why Hedge;
Why options;
Pricing options,
the one period problem;
Replicating portfolio,

Options

- **Definition**
- **Call Option:** The right to buy an item at a pre-specified price up to some date.
  - Example the right to buy 100 shares of HSBC common stock for $55 on or before 5 pm Nov 13 2013.

- **Put Option:** The right to sell an item at a pre-specified price up to some date.
  - Example the right to sell 100 shares of HSBC common stock for $55 on or before 5 pm Nov 13 2013.
- Like futures options are contracts where things happen in the future
- Unlike futures they are not firm on both sides (one side decides).

Option types

- American options: you can exercise them whenever you want after the contract is signed.
- European options: you can only exercise them at the expiry date
  - Ex the right to buy 100 shares of HSBC common stock for $55 on or Nov 1 at 5 pm Nov 13 2013. (price 0.65 on Nov 11)
- **Option is defined by:** the asset you have contracted over:(100 HSBC shares of common stock); The strike price: $55; Expiration date: Nov 13 2013, 5 pm; Type: European or American, and Put vs Call.
- Like other financial assets there some publicly traded options (see Chicago Board of trade), and some are over the counter.

Why options?

- Why not futures?
- Consider the case of an airline
- On the revenue side It sells tickets up to a year in advance.
- On the cost side
  - it does not know what fuel prices will be (it does know its other costs) and it can't change prices on tickets its already sold.
  - The same applies if you think about the routes it has decided to fly.
- So solution
  - Go long in the jet fuel market
  - In fact buy futures at the same rate as you are selling tickets.
- Makes sense because you know that you will be short in a very predictable way.

Why options?

- Now consider another of the airlines decisions.
- If business is good it wants to expand, if business is bad it will not want to do so.
  - Expansion means buying or leasing more planes
- Its business is likely correlated with that of other airlines, so when business is good the price of planes is high (or delivery times long)
  - It does not want to enter into forward contracts because it does not want the planes if business is bad.
  - What it want is an option.

Beyond airlines

- Futures and forwards
  - Solve price uncertainty when you know you will want to go long or short in the future
- Options
  - Not ideal if you want to deal with price volatility but you also face quantity issues.
  - They also affect incentives

NBC-Universal

- in December 3, 2009, after months of rumors, a deal was formally announced in which Comcast would buy a stake in NBC Universal from GE.[Under the agreement, NBC Universal is 51% owned by Comcast and 49% by GE. Comcast paid $6.5 billion cash to GE.

- Under the terms of the deal, Comcast reserves the right to buy out GE's share at certain times, and GE reserves the right to force the sale of their stake within the first seven years.

- February 12, 2013 announcement that Comcast intended to complete the $16.7 billion purchase early, all at once.
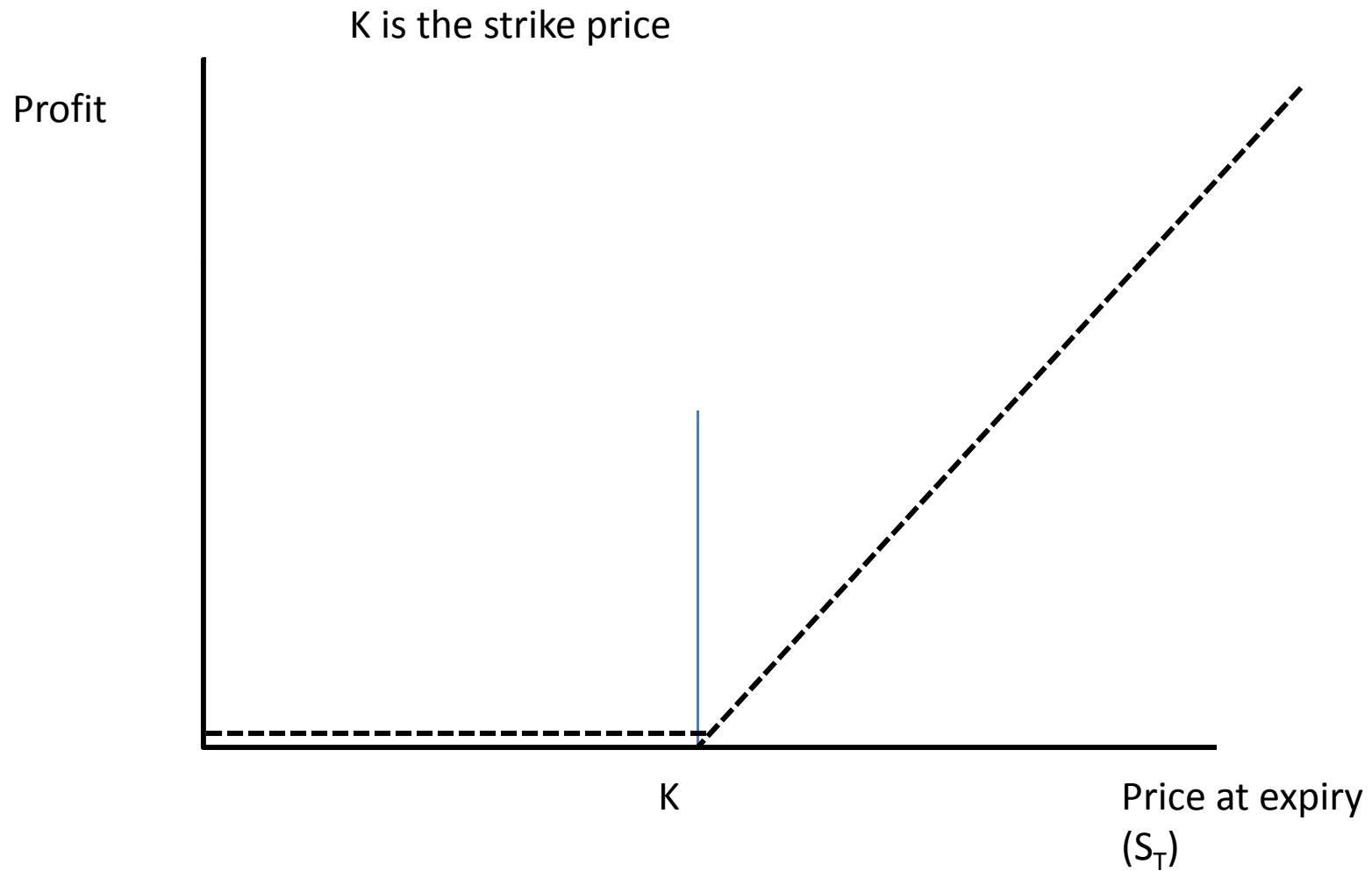
Why both reserved rights

- The merger is supposed to create synergies
  - Both sides stand to gain, but its risky
- On the GE side
  - They are handing over control so they want to ability to get out if they do not like what is happening.
  - They want to induce Comcast to make the venture profitable
- On the Comcast side
  - They want control
  - They want a large part of the upside
  - But they want protection.
- If Comcast does a bad job has to buy GE's remaining stake (expensive)
- If it does a good job then it wants the whole firm and does not want GE to be able to hold things up. So find mechanism to split profits.
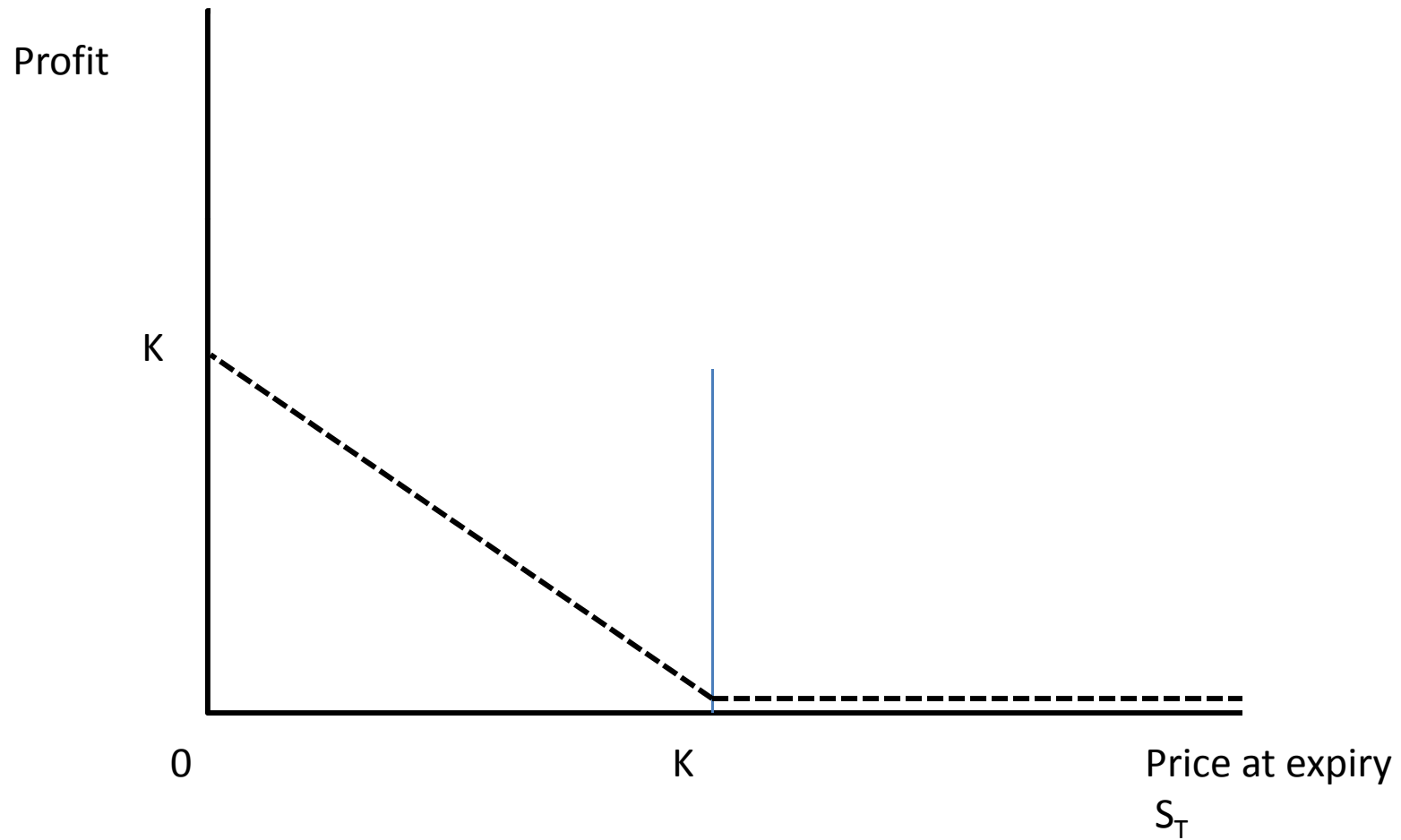
Back to the world of simple options

Ex the right to buy 100 shares of HSBC common stock for $55 on or before 5 pm Nov 1 2013.

Price Nov 13	Call Option			Put option	
	Return if exercise (P-55)	Decision		Return if exercise (55-P)	Decision
51	-4	Let Expire		4	Exercise
52	-3	Let Expire		3	Exercise
53	-2	Let Expire		2	Exercise
54	-1	Let Expire		1	Exercise
55	0	Let Expire		0	Let Expire
56	1	Exercise		-1	Let Expire
57	2	Exercise		-2	Let Expire
58	3	Exercise		-3	Let Expire
59	4	Exercise		-4	Let Expire

Call option

K is the strike price

Profit

K

Price at expiry
(S$_T$)

Put option

Profit

K

0
K

Price at expiry
S_T

Another zero sum game

- Leaving aside the transaction technology
- Put and call options at the same strike price are never both exercised
- The counter party's return to a call option are essentially the losses the holder of a put option avoids by not exercising his option.
- The holder of an option can never make a loss (after buying it). So the counter party cannot make a profit.
- So options must have a positive initial price
  – To compensate the counter party for the expected loss
  – That price is going to be different from either the futures price or the spot price of the asset.

Value of option

- Gross payoff is the difference between the strike price (K) and the price at expiry (S_T)
- If $K<S_T$ then call option valuable if $K>S_T$ then put is valuable
- Net Pay off has to include the price at which you buy the option (C), and the income foregone
- For a call option
  - $\Pi(K)=(S_T-K)-C(1+r)^T$
- For a put option
  - $\Pi(K)=(K-S_T)-C(1+r)^T$

Equity claims as option (and leverage)

- Firm has $50 in assets
- Its liabilities are $25 in equity and $25 in bonds.
- The bonds have face value of $50.
- So now consider the payoffs next period

| Firm value	Payments to bonds	Payments to Equity	Value of a call on the firm at 50
20	20	0	0
30	30	0	0
40	40	0	0
50	50	0	0
60	50	10	10
70	50	20	20
80	50	30	30
90	50	40	40

Prices

- The bounds approach (assume K=S)

- C>0 (the option is not free)

- C<S

  - Could buy stock or buy call at strike price K

  - At expiry call earns $K-S_T$ if ($S_T>K$), 0 otherwise
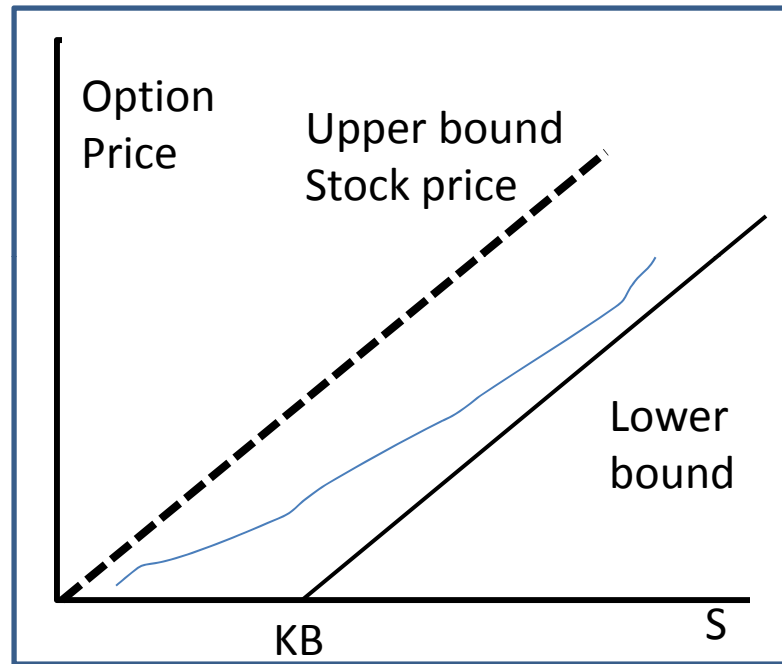
  - At expiry stock is worth S_T
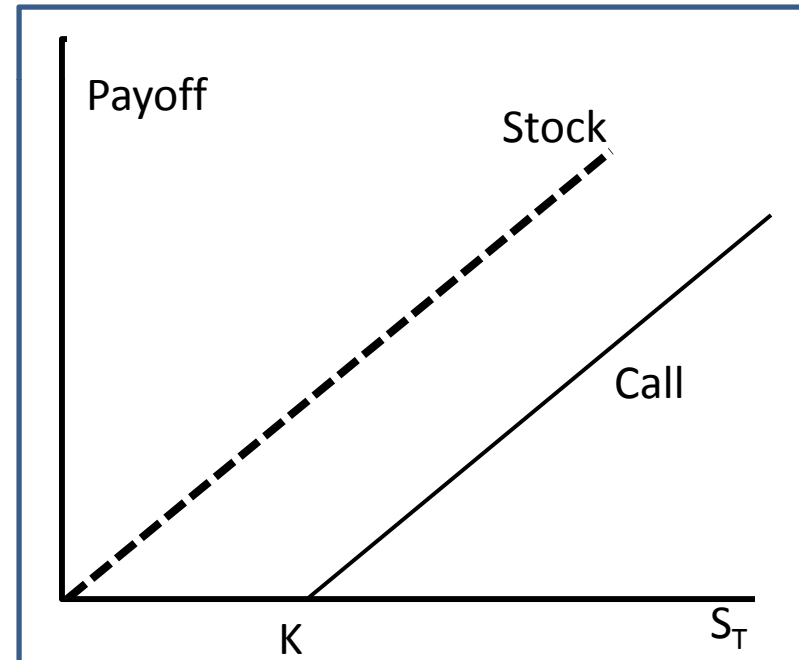
Bounds approach

- C>S-KB
  - Portfolio (borrow KB where B is the discounted present value of a bond that pays 1 at time T)
  - At T you sell the stock and pay off the bond earn S_T-K
  - If S_T>K then the call and the portfolio pay the same (S_T-K)
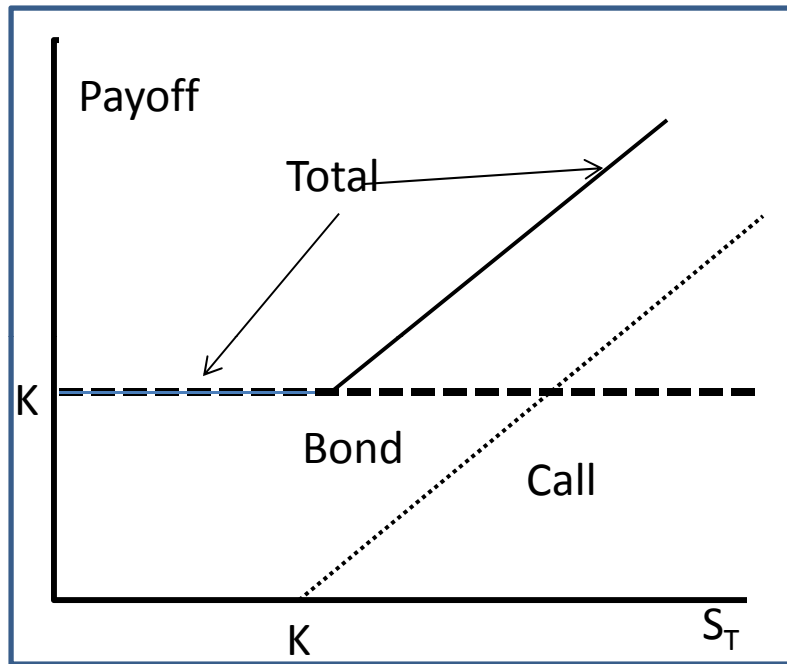 - If S_T<K, the Call is worth 0, the portfolio has a negative return

Payoff

K S_T
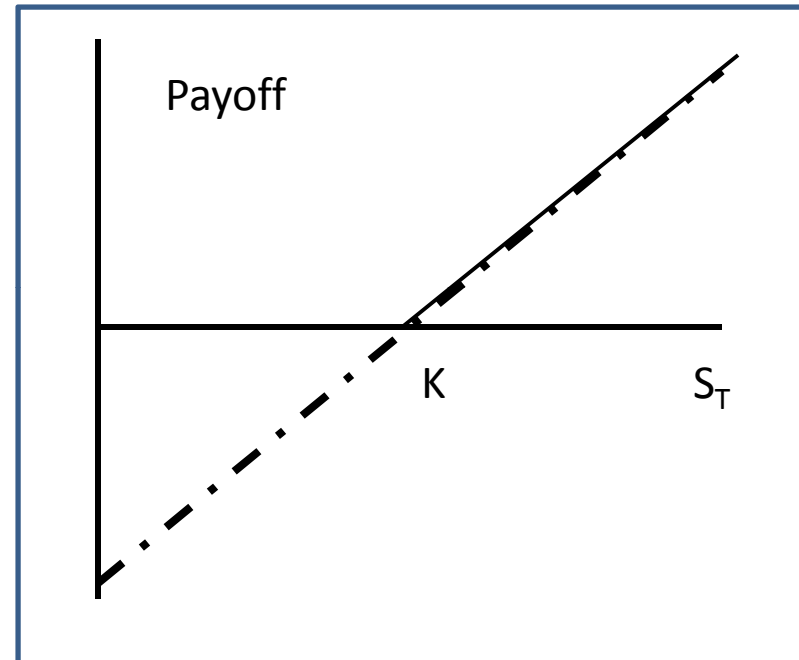
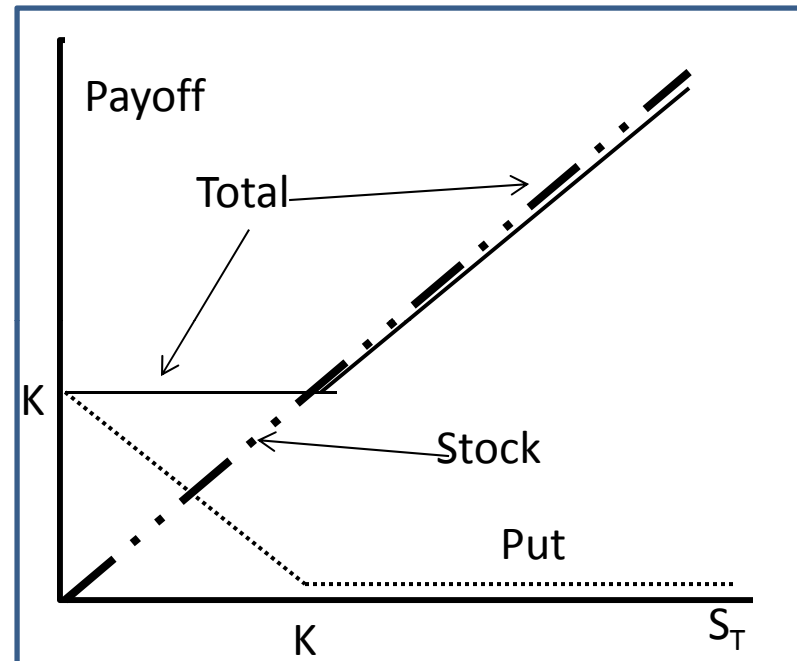- (1)&(3) => C≥ max[S=KB, 0]
- Add (2)=>S>C≥ max[S=KB, 0]

- Implications?
- As interval gets larger the bounds get quite weak

Put Call parity

- Here we have a call with strike price of K and bond with face value of K

- Here we have a put with strike price of K and one share of stock (S=k)

Put to Call Parity

- Bond+Call payoff

  $C+K/(1+r)^T$

- Stock +put payoff

  $P+S$

- From Graph its clear that payoff are the same

  $C+K/(1+r)^T=P+S$

QUIZ 7

One period binomial model

<center>Notation</center>

- Current stock price S_o
  - With probability (q) $S_1 = S_o + \Delta$
  - With probability (1-q) $S_1 = S_o - \Delta$
- Call option has strike price K= S_o
  - With probability (q) $C_1 = \Delta$
  - With probability (1-q) $C_1 = 0$
- Bond has return r.
- Can invest either in stock or bond or in the call

Replicating portfolio

- Consider a portfolio of a stock and b bonds
- Such that

$$1. \quad a(S_o+\Delta)+b(1+r)= \Delta$$
$$2. \quad a(S_o-\Delta)+b(1+r)= 0$$

- From (2) $b=-a(S_o-\Delta)/(1+r)$
- Replace in (1)

  - $a(S_o+\Delta)-a((S_o-\Delta)/(1+r))(1+r)= \Delta$
    - $a(S_o+\Delta)-a(S_o-\Delta)= \Delta$
    - $a2\Delta = \Delta \quad a= 1/2$

- a is the number of shares needed to replicate one call option. In this two state, symmetric case a=0.5
- Price of the option is $aS_o+b=$

$$0.5S_o-0.5(S_o-\Delta)/(1+r) = 0.5(rS_o+\Delta)/(1+r)$$

- Notice that there is no q in here.

Replicating portfolio

- Consider a portfolio of a stock and b bonds such that

    1. $a(S_o+\Delta)+b(1+r)= \Delta$

    2. $a(S_o-\Gamma)+b(1+r)= 0$

- From (2) $b=-a(S_o- \Gamma)/(1+r)$
- Replace in (1)

    - $a(S_o+\Delta)-a((S_o- \Gamma)/(1+r))(1+r)= \Delta$

        - $a(S_o+\Delta)-a(S_o- \Gamma)= \Delta$

    - $a(\Delta+ \Gamma) = \Delta \qquad a= \Delta/(\Delta+ \Gamma)$

- Notice the option delta a does not depend on probabilities (q) but on expected values of the upside and the down side.

- PV of the option has to be the cost of the replicating portfolio

- $C=aS_o-b$ and thus the price of the option ARBITRAGE (again)

Example

- Current stock price 55
- Δ=20
- Call option has strike price K= 55
- Bond has return 0.05
- a= ½
- b=-a(S_o-Δ)/(1+r)=-0.5(55-20)/1.05=-17.5/1.05=16.16
- Cost of portfolio=27.5-16.16=11.34
- We have just priced a binomial option

More than 2 states

- Let the current stock price be S_0
  - And the strike price is K= S_0.
  - But now there are 5 states in period 2
  - $S_0-2\Delta$, $S_0-\Delta$, S_0, $S_0+\Delta$, $S_0+2\Delta$
- So now the call option pays either Δ or 2Δ in either of the two up states($S_0+\Delta$, $S_0+2\Delta$) but nothing in the other three states
- To build replicating portfolio
  - I need the expected value of S_1 conditional onthe two up states($S_0+\Delta$, $S_0+2\Delta$) and the three down states ($S_0-2\Delta$, $S_0-\Delta$, S_0)
  - So I need probabilities. E.g. (q_1, q_2, q_3, q_4, q_5)
  - $E(C|S_1>S_0)=(q_4\Delta+2\Delta q_5)/(q_4+q_5) = \Delta_u$
  - $E(C|S_1\leq S_0)=0$
  - $E(S_1|S_1>S_0)=S+(q_4\Delta+2\Delta q_5)/(q_4+q_5)=S+\Delta_u$
  - $E(S_1|S_1\leq S_0)=S-(q_1 2\Delta+q_3\Delta)/(q_1+q_2+q_3)= S-\Delta_d$

More than two states

1. $a(S_o + \Delta_u) + b(1+r) = \Delta_u$
2. $a(S_o - \Delta_d) + b(1+r) = 0$

- From (2) $b = -a(S_o - \Delta_d)/(1+r)$
- Replace in (1)
    - $a(S_o + \Delta_u) - a((S_o - \Delta_d)/(1+r))(1+r) = \Delta_u$
        - $a(S_o + \Delta_u) - a(S_o - \Delta_d) = \Delta_u$
    - $a(\Delta_u + \Delta_d) = \Delta_u \qquad a = \Delta_u/(\Delta_u + \Delta_d)$
- So states don't matter so much as long as you can attribute probabilities (risk).

More time periods

- One period
  - With probability (q) $S_1 = S_o + \Delta$
  - With probability (1-q) $S_1 = S_o - \Delta$
- If IID over time

probability	q^2	2q(1-q)	$(1-q)^2$
S_2	$S_o + 2\Delta$	S_o	$S_o - 2\Delta$

- Two period $K = S_0$

From table $\Delta_u = 2\Delta$, $\Delta_d = 2\Delta q$

1. $a(S_o + \Delta_u) + b(1+r) = \Delta_u$
2. $a(S_o - \Delta_d) + b(1+r) = 0$

More time

- Approach above
  - 1 period
  - The two periods backward induction and IID
- Need a more general approach
  - Keep the IID assumption
  - But become more specific about time process
- Black-Sholes