10-28 Class 9: The portfolio approach and the Price of securities

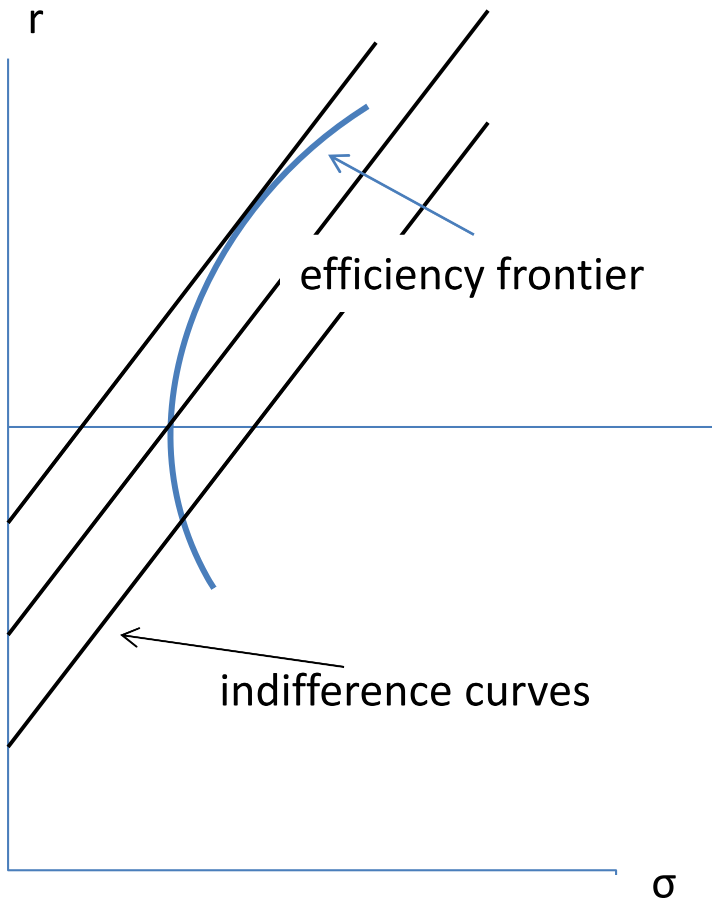
- Problems with the portfolio approach
- CAPM;
- Rise of the passive investor;
- What happens when new information comes in;
- Another no trade result?

The efficient frontier

- As long
  - Investors only care about some statistic related to the mean and variance
  - Not say some percentile value of expected returns or about higher moments of the distribution
- Then set of portfolios that are on the efficient frontier is invariant to assumptions
  - It does not matter
    - Min Var(x) conditional on E(X)≥ R
    - min G(Var(x) conditional on E(X)≥ R
    - Produce the same thing
- Finding the efficient portfolio's does not depend on methodology

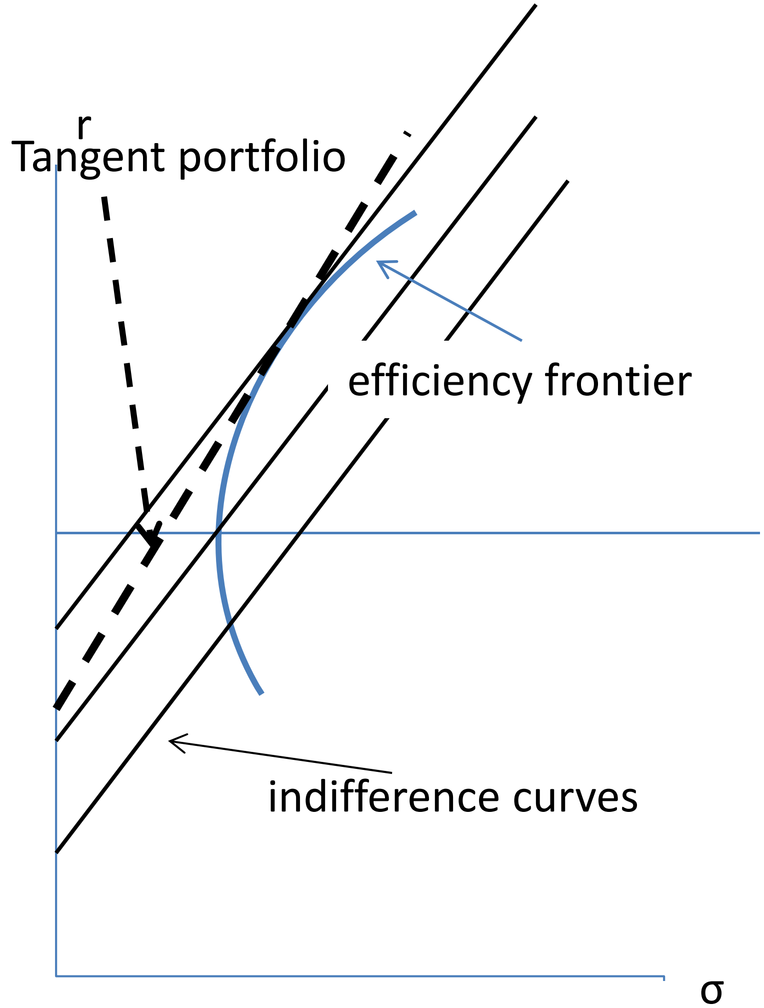
Choice on the efficient frontier

- Theory
  - efficiency frontier and un-dominated portfolios are universal
  - Each individual has specific indifference curves that trade of r and σ
  - Choice involve finding a tangency condition
  - If individuals are rational then can solve this problem
- Practice?
  - Give people the efficiency frontier (menu of efficient portfolios) and let them chose



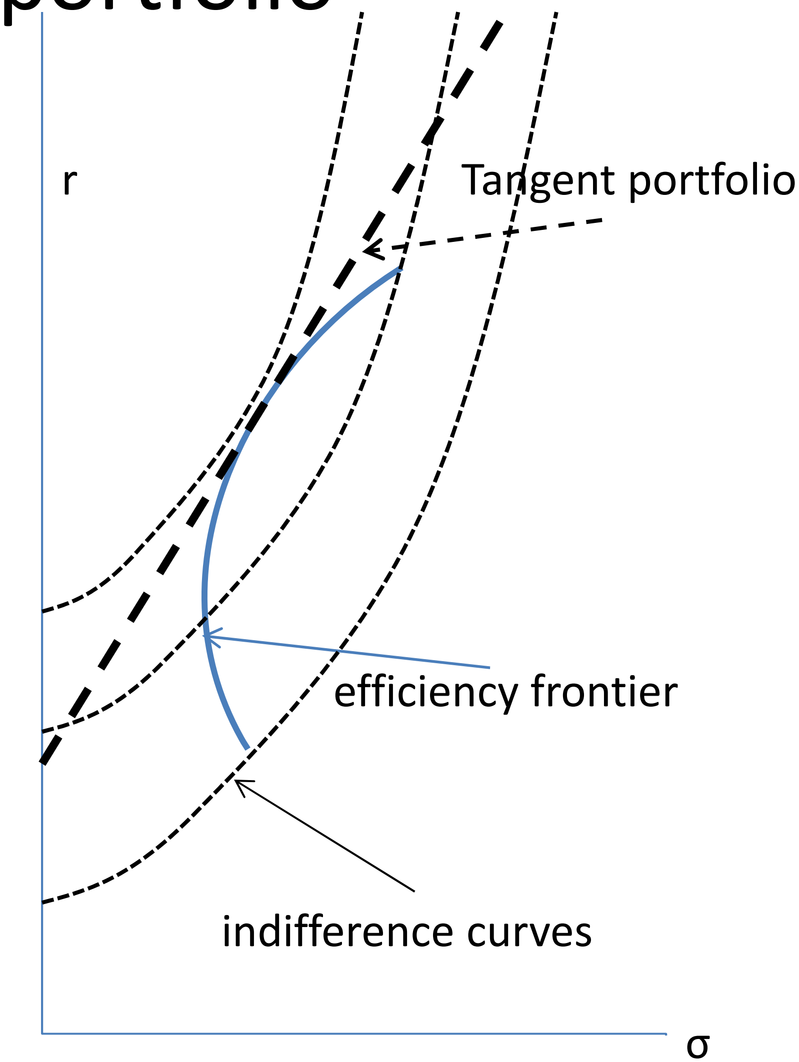
The efficient frontier and riskless assets

- Add in the riskless asset and the Tangent portfolio
- If individuals have linear indifference curves (r- σ trade-offs).
- Only corner solutions (all riskless assets or all tangent portfolio) until the riskless asset becomes irrelevant.



Choice on the Tangent portfolio

- Back to mean variance U(X)-E(X)-bVar(x)
- ⇔ $U(x)=r-b\sigma^2$
- Indifference curve is now
- $r=b\sigma^2$ => parabola
- So now
  - with the riskless asset and the Tangent portfolio
  - Can have interior solutions.
- So this is why we assume $r=b\sigma^2$ - utility in an r/σ space



Problem

- In this context we kind of throw a lot out into individual choice.
- Maybe ok (that is why we might have a variety of funds)
- Problem, if everyone does this it can't leave prices unchanged… and there may be securities that no one wants to hold (see Ford in the 5 asset case).
- So we need to think about market equilibrium conditional on efficient portfolios

Solution-**CAPITAL ASSET PRICING MODEL**

- Investment market is defined by
  - One riskless security
  - A bunch of risky securities.
- Question
  - what are the prices of the risky securities?
  - what do portfolios look like?
- Hard to answer if thee is a lot heterogeneity (people all have different preferences)
- So Start with a world where everyone is strictly identical.
  - Many people to establish prices via competition
  - In effect this is also an assumption of no private information

Assumptions

- In any equilibrium everyone will have the same portfolio.
  - They face the same prices
  - And have the same preferences
  - so they make the same choices
- And everyone must own the whole market.
  - Because if some player does not want a security then no-one wants it.
  - This also implies no one goes short.

Equilibrium Implications

- (1) The weight of each security in the portfolio (w_i) has to be the ratio of the total value of that security divided by the total value of all securities. ($w_i = v_i/\Sigma V_j$)
- (2) The market portfolio has to be on the efficient frontier
  - it minimizes variance conditional on the market return.
- (3) The market portfolio has to be the optimal portfolio on the efficiency frontier.
  - (Best trade-off between risk and return conditional on the preferences of all our identical individuals).
  - At the market portfolio the tangent of the frontier is the marginal cost of risk.

Implication 4

- (4) The line from the riskless asset to the market portfolio is tangent to the efficiency frontier at the market portfolio.
  - If there is heterogeneity then all optimal portfolios are on that tangent.
  - So having solved the large number of identical investors' problem we have also solved it for many different types of investors

How does this happen?

- Taking the securities as given the only way to make sure all these conditions hold is to adjust prices.
- How to set prices so all the assets are in the portfolio?
- The return to a CAPM portfolio is

$$r_p = \left(1 - \sum_{i=1}^{n} w_i\right) r_F + \sum_{i=1}^{n} w_i r_i = r_F + \sum_{i=1}^{n} w_i (r_i - r_F)$$

- The marginal contribution to the portfolio of asset i is it's return above the risk free rate:

$$\frac{\partial r_p}{\partial w_i} = (r_i - r_F)$$

How does this happen (2) ?

- From returns

$$r_p = \left(1 - \sum_{i=1}^{n} w_i\right) r_F + \sum_{i=1}^{n} w_i r_i = r_F + \sum_{i=1}^{n} w_i (r_i - r_F) \qquad \frac{\partial r_p}{\partial w_i} = (r_i - r_F)$$

- The variance of the portfolio is the weighted variance of the risky portfolio

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \, \sigma_{ij}$$

- $\sigma_F = 0$ so riskless asset has zero correlation with the other assets -> σ_P^2 is just the market variance times the square of the market weight in the portfolio)
- The terms that concern asset (i) are those in the i^{th} row and i^{th} column of the variance covariance matrix.

- From returns

$$r_p = \left(1 - \sum_{i=1}^{n} w_i\right) r_F + \sum_{i=1}^{n} w_i r_i = r_F + \sum_{i=1}^{n} w_i (r_i - r_F) \qquad \frac{\partial r_p}{\partial w_i} = (r_i - r_F)$$

- The variance of the portfolio is the weighted variance of the risky portfolio

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \, \sigma_{ij}$$

- We can take the derivative to get the marginal contribution of asset i to portfolio variance

$$\frac{\partial \sigma_P^2}{\partial w_i} = 2 w_i \sigma_i^2 + 2 \sum_{j \neq i}^{n} w_j \, \sigma_{ij} = 2 \sum_{j=1}^{n} w_j \sigma_{ij} = 2\sigma_{ip}$$

- That is just twice covariance between the portfolio and the asset.

Volatility

- If the marginal contribution of asset i to portfolio variance is twice covariance between the portfolio and the asset. $\frac{\partial \sigma_P^2}{\partial w_i} = 2\sigma_{ip}$
- Then the marginal contribution of asset I to portfolio volatility is $\frac{\partial \sigma_p}{\partial w_i} = \sigma_{ip}/\sigma_p$

- Definition: the marginal risk return ratio (RRR) of asset I in a portfolio is the ratio of the marginal return to the marginal risk

$$\text{RRRi} = \frac{\frac{\partial r_p}{\partial w_i}}{\frac{\partial \sigma_P^2}{\partial w_i}} = \frac{(r_i - r_F)}{\sigma_{ip}/\sigma_p}$$

- That tells us what happens to risk and return when we increase the weight of asset i.
- The bigger the number the more desirable it is to hold that asset
- Condition of equilibrium: RRR_i= RRR_j For all i,j—within the risky portfolio.

Equilibrium

- Condition of equilibrium: $RRR_i = RRR_j$ For all i,j—within the risky portfolio.
- $$\mathrm{RRR_i} = \frac{(r_i - r_F)}{\sigma_{iM}/\sigma_M} = \mathrm{RRR_T} = \frac{(r_M - r_F)}{\sigma_M}$$
- But that implies

$$(r_i - r_F) = \frac{\sigma_{iM}}{\sigma_M^2}(r_M - r_F) = \beta_{iM}(r_M - r_F)$$

- So the Capital Asset Pricing Model implies

$$\beta_{iM} = \frac{\sigma_{iM}}{\sigma_M^2}$$

- Where β_{iM} is a measure of asset i' s systematic risk in the market portfolio and $(r_M - r_F)$ is the price of systematic risk.

Side bar

- Definition
- **Systematic (Un-diversifiable risk)**: that part of the variance of an asset that is perfectly correlated with aggregate variation (so you can't offset it)
- **Diversifiable risk**: that part of the variance of an asset that can be offset by the volatility of another asset.

Security returns

- Security has a two part return. A return equal to the return to the risk free asset and a risk premium.
- The risk premium of a security is simply its systematic risk time the price of systematic risk.
- Once you know the covariance of an asset with the market portfolio you can compute its β_{iM} and then compute its return $r_i = r_F + \beta_{iM}(r_M - r_F)$ and you have priced the security!
- The more systematic risk it has (higher β_{iM}) the higher its return. That is the more the market has to pay you to hold it
- The less the systematic risk it has the lower the return
- If it has a negative β_{iM} ($\sigma_{iM}<0$) then it you are willing to 'pay' to hold it). By that we mean earn an expected return less than the riskless return.

Practice

- Let the one year T-bill be the riskless asset, (that gives you r_F).
- Let the market be the S&P 500, (that gives you $r_{M,}$ σ_M)
- Then given an asset you can estimate

$$(r_i - r_F) = \alpha_i + \beta_{iM}(r_M - r_F) + \varepsilon_i$$

- Assume (1) ε_i is IID normal with mean 0 and Cov(r_M, ε_i)=0. The you can estimate β_{iM} , σ_I, and α_i, Stdev(ε_i)
- If CAPM holds
- α_i is zero (so when it is not you face a pricing anomaly and you may try to take advantage of that, in particular if it positive)
- β_{iM} gives you market sensitivity.
- α_i (is non-systematic risk (and if you are a long term investor you should not care because it has mean zero). If you are a short term investor you might worry if sigma is large.

Implications of CAPM for investors

- Gives simple rules for making decisions—use a broad market portfolio of risky assets and then adjust risk by varying the quantity of riskless or very low risk assets. Because the market adjusts the weights you do not need to worry about choosing the risk portfolios.
- It has economic logic (prices are form as part of an equilibrium that takes in a risk return logic)
- Rise of index funds and the passive investors
- Prices take care of everything. If there is new information, don't rebalance because it all in the prices anyway.

10-30 Class 10: Market efficiency

- Investors and Alpha and Beta
- Problems with CAPM
- Variation in the price of risk
- Variation in alpha.
- Your portfolios