BEM 103

10-21 Class 7 : Risk

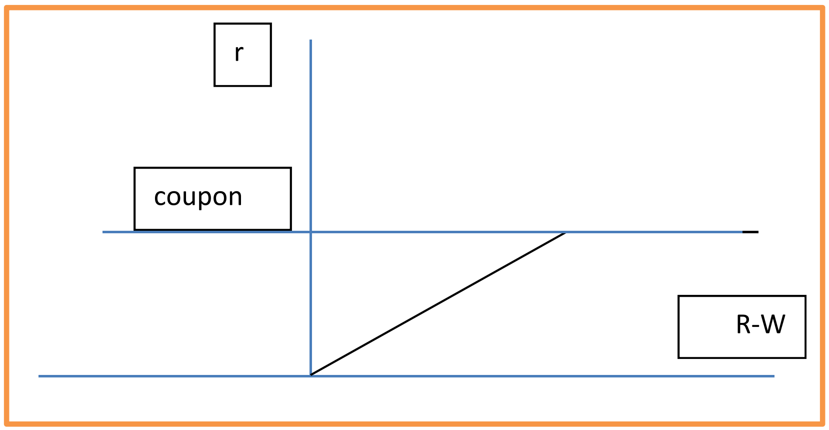
- **Each Security has uncertain payoffs; Down side VS Upside;**
- **Risk aversion: Expected utility and utility of expected value;**
- **Basic mean variance trade off in utility;**
- **Individual heterogeneity and sorting in markets.**

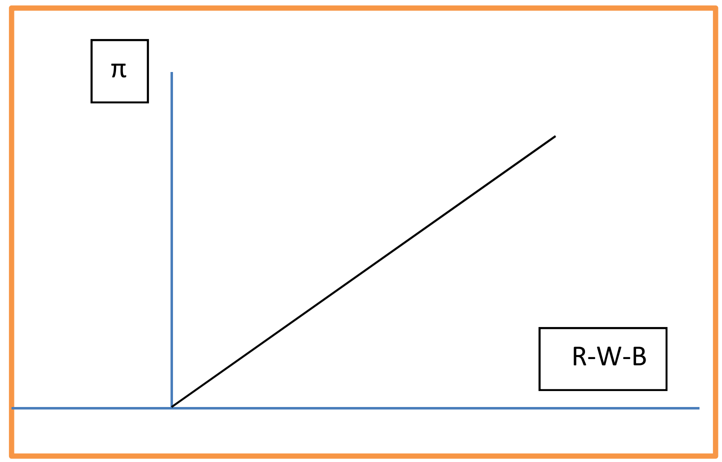
Return to Class 1

- Finance is the study and valuation of Risk
  - An asset's return = riskless return + price of risk
- Risk A la Donald Rumslfeld (Feb 2002)
  - There are known knowns; there are things we know that we know.
  - There are known unknowns; there are things that we now know we don't know.
  - But there are also unknown unknowns – there are things we do not know we don't know.
- Risk vs Uncertainty (Frank Knight 1921)
  - Risk involves issues where we do not know the exact outcome (e.g. future price), but we can accurately measure the odds (e.g. we know the distribution).
  - Uncertainty, involves issues we cannot know all the information we need in order to set accurate odds in the first place (e.g. we do not know the distribution).
- In practice
  - If we can forecast the outcome accurately then there is no risk
  - If we know the distribution then we can evaluate, manage, and price risk
  - If we do not…
- Practically hope there is some connection between the distribution of past outcomes and future outcomes

A Security has uncertain payoffs.

- Simple securities
  - Bonds because of inflation and default
  - Equity because of variability in profit rate and bankruptcy
- Both types of securities have both a 'continuous' and a 'non-linear' form of risk.





- Other securities do too but it may be more complicated
  - Contingent claims (Options, CDW, Insurance)…
  - Need to worry about the occurrence of the contingency and then what the value of the security will be when the contingency occurs.

Statistical Risk

- Take any security observe its prices for T periods
- Use whatever data you can get your hands on to forecast its price at time T+1
- If your best forecast has error then you have risk. The distribution of the residuals is the distribution of risk.
- But what of uncertainty?

Statistical Risk

- **Technically a security value is a state of the world and we want to attach a probability (p_i) to each state of the world (x_i).**
- **Note x_i is not a particular event—it's a value and p_i is the likelihood represented by the set of events such that your security i takes on value x_i**
- **From the states-of-the-world perspectives exactly what factors cause these likelihood is not important. What matters are prices and probabilities.**
- **Generally people do not like risk—they are risk averse--but you do not have to be risk averse to want to know about p and x. Because pricing risky securities under risk neutrality => P(X)=EV(X)**
- **Need probabilities and states of the world to compute expected value.**
- **Discrete**
  - E[X] = $\sum_{1}^{n} p_i * x_i$ and $\sum_{1}^{n} p_i = 1$
- **Continuous** $E[X] = \int_{\underline{x}}^{\overline{x}} p(x)x\,dx$

- **If we are risk neutral then we can just price that vs consumption or some alternative investment.**

Example

- Security X has two states x_1 =0 and x_2=-100. The probability of x_2 is p_2 =0.01 (so p_1 =0.99). What is its price?
- The expected value of this security
  - EV=0*0.99 -100*0.01=-1
  - NPV=-x-1/(1+r) =>x=-1(1+r)
- So a risk neutral investor would have to be paid 1/(1+r) to invest in that security.
- Note this is like a life insurance contract for 59 year old man (or a 65 year old women), today
- There is risk
  - Buyer : if you do not die you have paid X= 1/1+r and get nothing. If you die you get a 100.
  - Seller : if the person does not die you get the (1/1+r), if the person dies you have to pay the 100.
- Life insurance companies buy these contracts and people who invest in life insurance companies are implicitly investing in such ventures
- Why should the life insurance company be risk neutral?
- Can we measure these likelihood accurately? Or just Accurately enough?

Changing risk

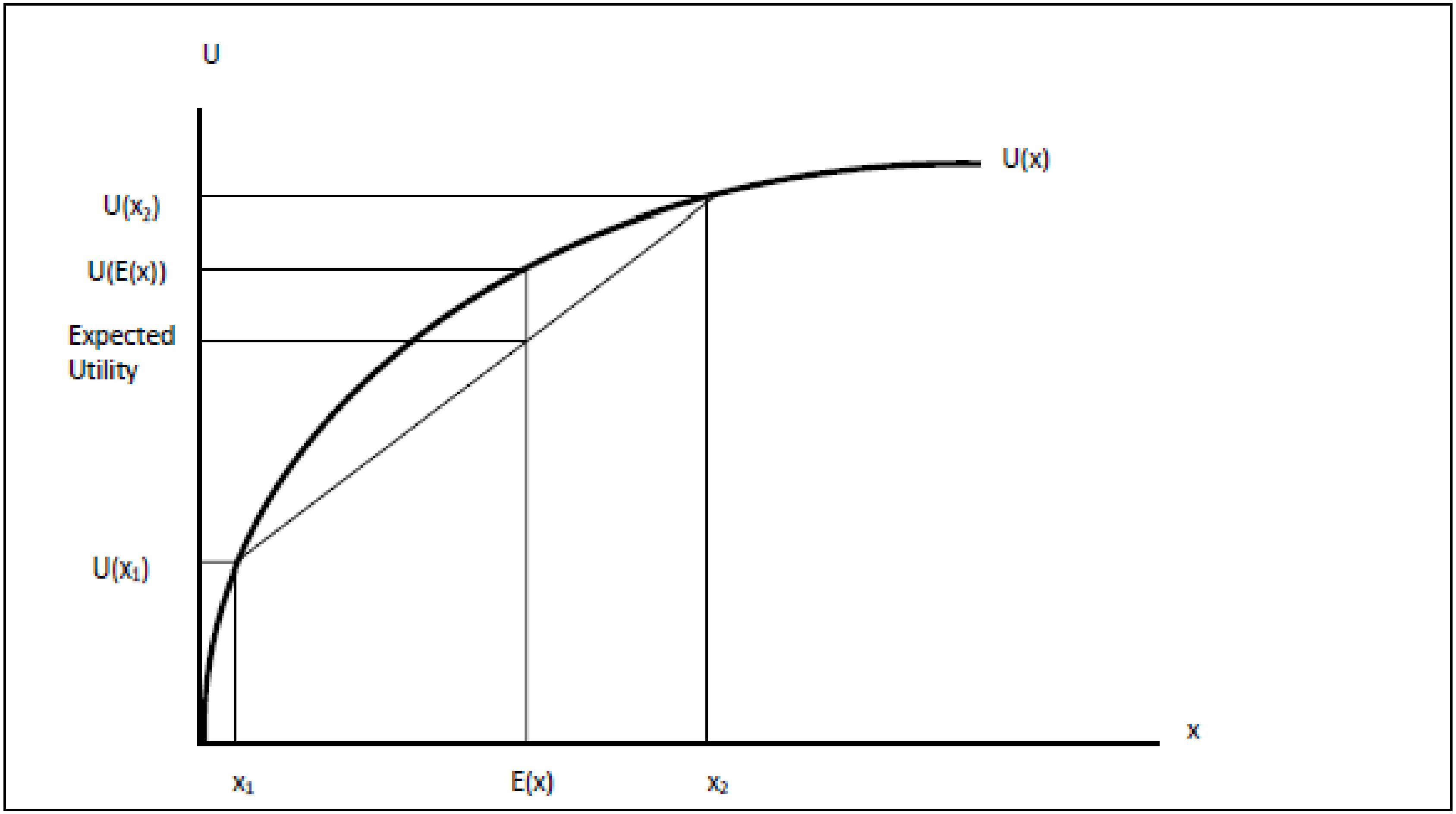
1% death probability:

- **1959-61 Age 50 for men and 59 for women**
- **1969-71 Age 50 for men and 59 for women**
- **1979-81 Age 53 for men and 61 for women**
- **1989-91 Age 56 for men and 62 for women**
- **1999-01 Age 58 for men and 63 for women**
- **2009-11 Age 59 for men and 65 for women**
- Is this good or bad for insurance companies?
- Is this good or bad for buyers of insurance
- is it good or bad for social security.
- In any case if everyone is risk neutral why are there such contracts?
- What about the market for lemons?

Risk aversion

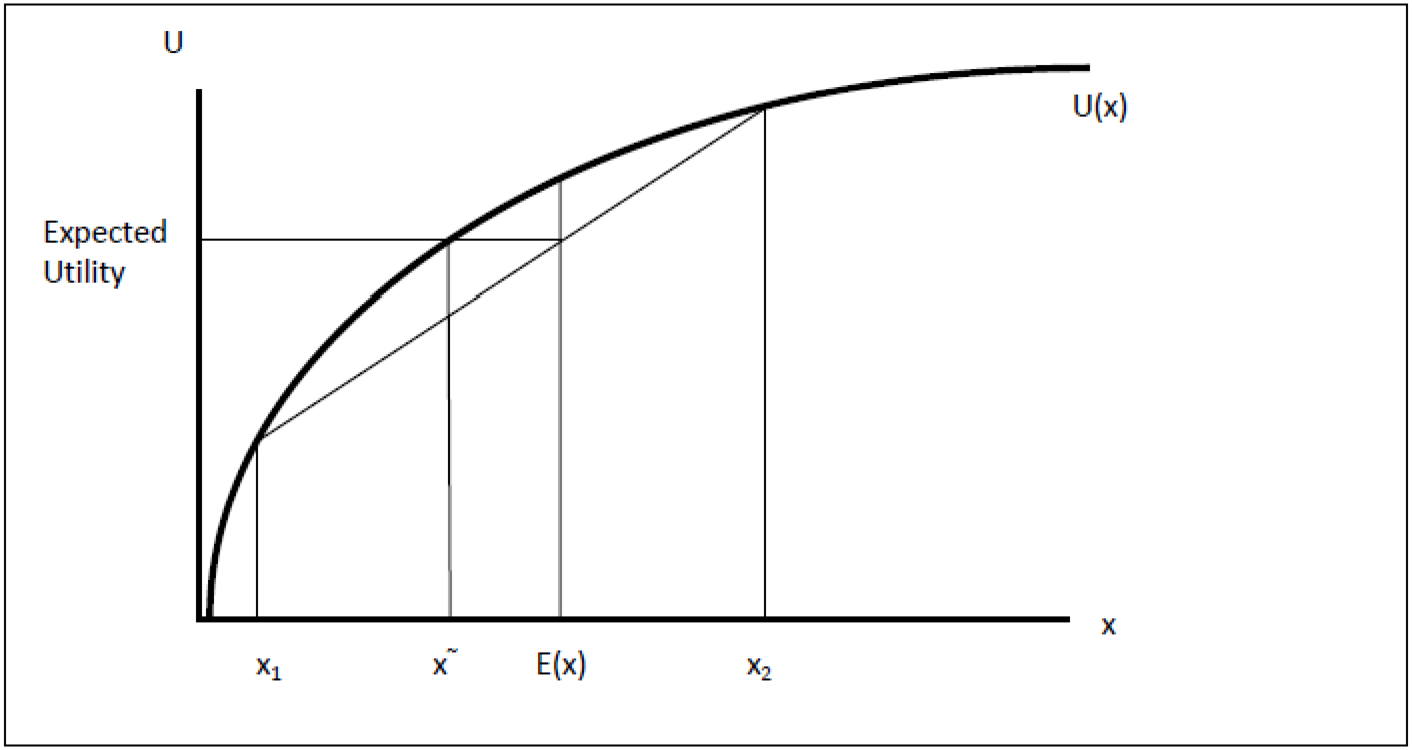
- About preferences—thus about utility function
- Or about the structure of payoffs
- Suppose there is a lottery that pays
  - x^h with probability p and x^l with probability 1-p.
  - Expected value is $px^h + (1-p)x^l$
  - Utility of the expected value $U(px^h + (1-p)x^l)$
  - Expected Utility is $pU(x^h)+(1-p)U(x^l)$
- Risk neutral
  - $U(px^h + (1-p)x^l) = pU(x^h)+(1-p)U(x^l)$
- Risk averse
  - $U(px^h + (1-p)x^l) > pU(x^h)+(1-p)U(x^l)$
- Risk Loving
  - $U(px^h + (1-p)x^l) < pU(x^h)+(1-p)U(x^l)$

Risk aversion in Graphic terms



- **Definition of risk aversion: Expected utility is less then utility of expected value.**
- **You prefer the sure thing to a fair gamble with the same value**
  - **It could be either local (given the lottery)**
  - **Or global (you always prefer a certain payoff over an uncertain one with the same expected value**
- **You have to be paid to bear risk**
- **Clean definition of risk loving: Expected utility is greater then utility of expected value.**
- **Risk aversion is generally the rule because people are stuck with what they get.**
- **But suppose you get to experiment: (over jobs, research topics, Vacation locations) should you be risk averse or risk loving?**
- **Why are financial assets lotteries?**

What induces you to bear risk?
Pricing risk when people are risk averse.



Even if people do not like risk, they will bear it and some time they will bear tremendous risk.
They will bear it in particular if they are paid.

Certainty Equivalent

- Lottery with expected value is $px^h + (1-p)x^l$
  - Utility of the expected value $U(px^h + (1-p)x^l)$
  - Expected Utility is $pU(x^h)\ ) + (1-p)U(x^l)$.
- What is the certain payoff that gives you the same utility?
  - Find X^C such that $U(X^C) = pU(x^h) + (1-p)U(x^l)$.
  - If risk averse $X^C < px^h + (1-p)x^l$

- **Definition**: certainty equivalent is the value of x that given with certainty leaves you indifferent to the gamble between x^h and x^l.

  - Certainty equivalent depends on utility (not just return) so it varies across individuals (just like discount rates).
  - Price of the security < certainty equivalent: you buy/hold the security.
  - Price of the security > certainty equivalent: you sell/or do not buy the security.
  - To know how to invest you need to know how patient you are and how risk averse you are.

From certainty Equivalent to Insurance

- If risk averse
- $U(px^h + (1-p)x^l) > pU(x^h)) + (1-p)U(x^l)$.
- $X^C < px^h + (1-p)x^l$
  - If $V(px^h + (1-p)x^l) = px^h + (1-p)x^l$
  - Then $V(px^h + (1-p)x^l) < X^C$
- So you always sell a security that pays its expected value.
- Implication.
- If you buy a security that is risky it must have a return that is higher than its expected value
- True or false?

Preference structures

- Risk neutral preferences are linear
- Risk averse preferences are concave
  - But how to make statements about who has more risk averse preferences?
  - Coefficient of Absolute Risk aversion $A(c) = -\frac{U''(c)}{U'(c)}$
  - CARA utilities $U(C) = 1 - e^{-\alpha c}$
    - ARA Here is constant
  - And so on and so forth, finance (unlike economics) like to take a short cut. And look at special case

Mean variance tradeoffs

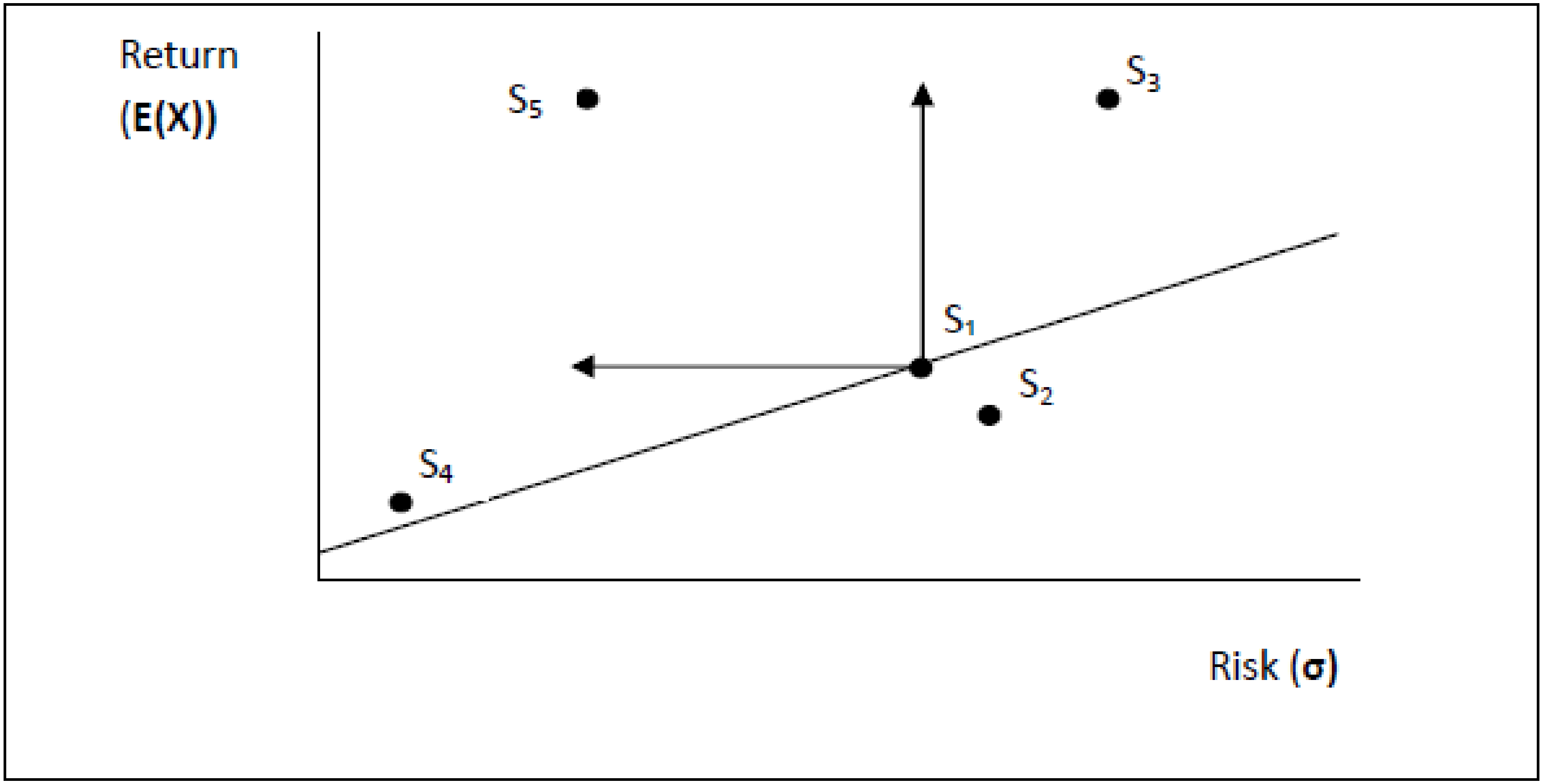
- Mean

$$E(x) = \sum_{1}^{n} p_i * x_i \quad \text{or} \quad E(x) = \int_{\underline{x}}^{\overline{x}} p(x)x\,dx$$

- Variance
- $$Var[x] = \sigma = \sum_{1}^{n} p_i * (x_i - E(x))^2 \quad \text{or} \quad Var[X] = \sigma = \int_{\underline{x}}^{\overline{x}} p(x)(x - E(x))^2\,dx$$

- Because utility is increasing in income we assume more mean is better.
- Because the agent is risk averse we assume more variance is bad.
- That means the objective function is to maximize a function of an investor like V(x)=E(x)-b(Var(x)). The more risk averse the larger b is.
- We generally assume that b is negative (but if you are risk loving then b will be positive).
- Max V(x)=E(x)-b(Var(x)) requires data.

Mean variance tradeoff

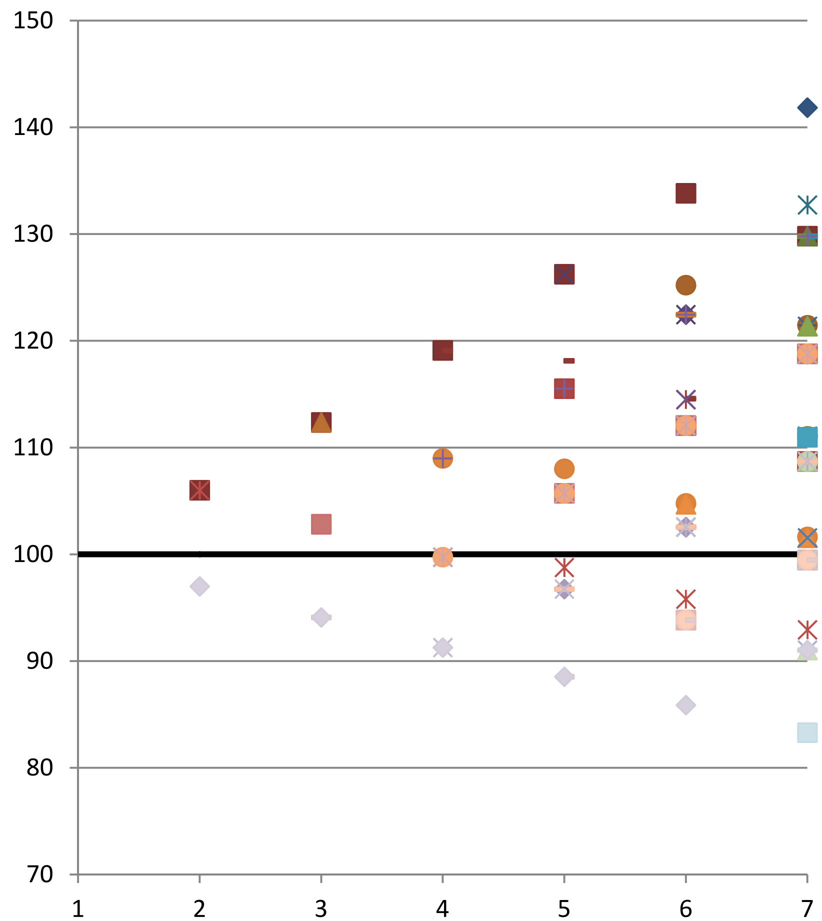


Each dot is a security with an expected return and variance.
You hold the S_1, Security but you like most of the other ones better.

Assessing risk under Indepedence

- You are a farmer. You own a tree that grows at rate r and thus yields (1.r)x fruit each year.
- The initial crop is 100 figs.
- Depending on whether it is a rainy or dry season the growth rate r is 0.06 or -0.03.
- The probability of a rainy season is 0.5. So the expected growth of your fig harvest is 1.5% per year.
- What happens over time

Growing the Tree

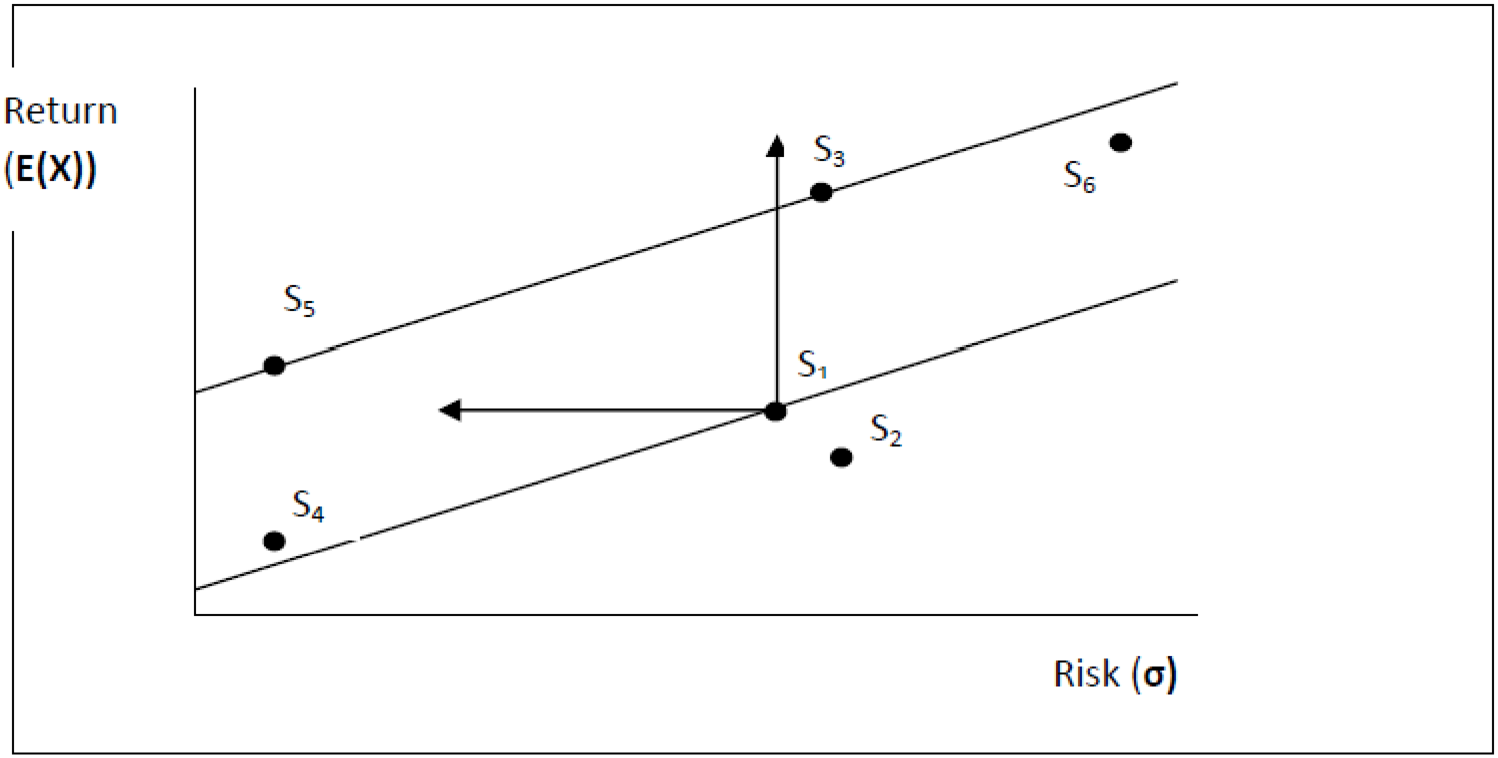


- So year 1 you get 100 figs
- Equally likely 106 or 97 figs.
- Variance is 40 (std dev 6.36 and coef of variation 6.3%)
- **Year 3**
- 25% probability you get 112 figs
- 50% probability you get 103 figs
- 25% probability you get 94 figs.
- **After 7 years** with 33% you get less than 100 figs (though the probability you get less than 90 is only 1.5%).
- On the other hand your expected return is 109 figs that year and with 33% you are getting more than 118 figs.
- The variance of the process after 7 years is large (the standard deviation is 12 for a coefficient of variation of 11%).
- That is the power of drift.

Drift vs independence

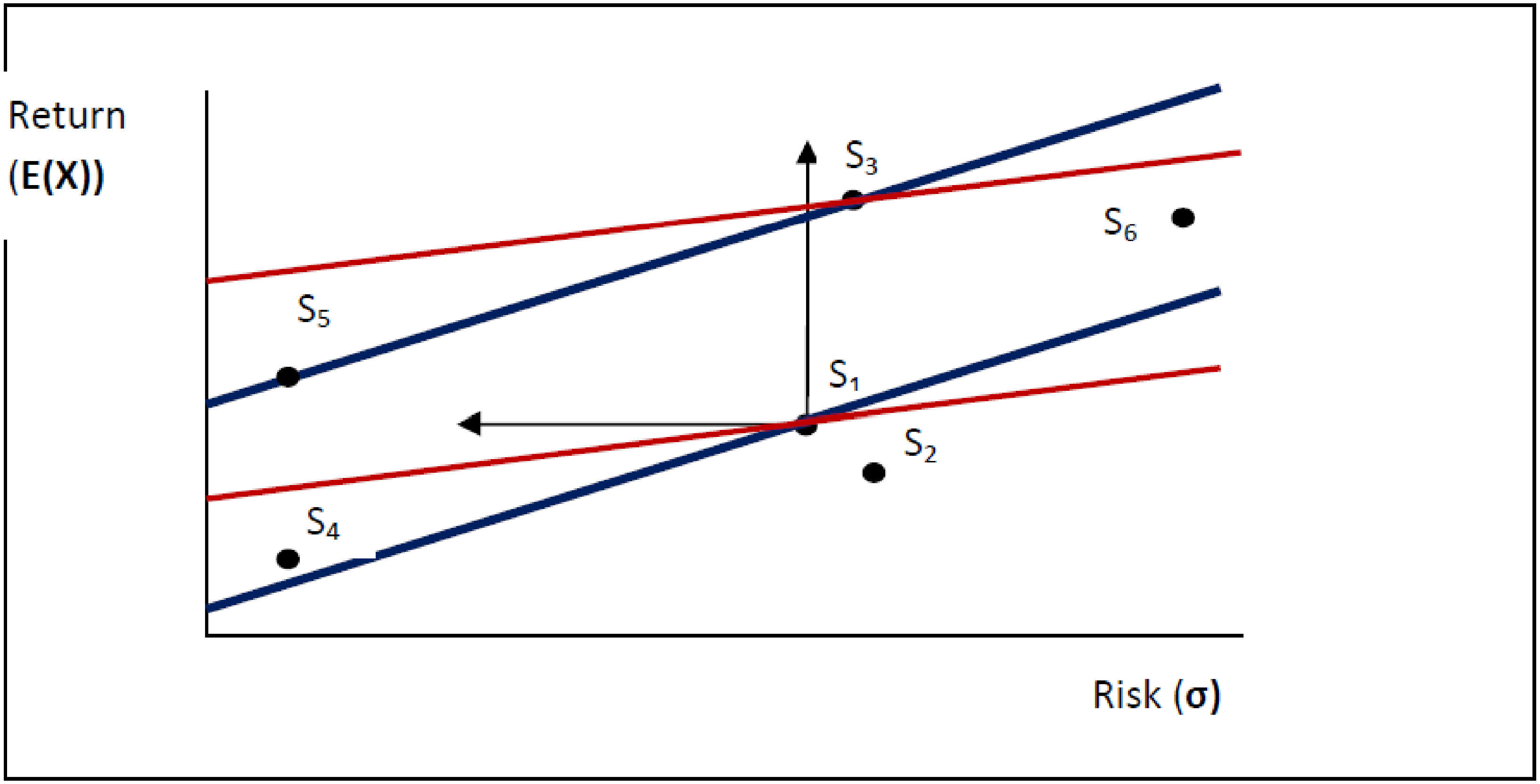
- What ever the size of the tree year 6
- you have an equal probability of rain vs drought, so the past is irrelevant to expected return henceforth.
- You do need to know the size of the tree and the likelihood of events (rain or drought)
- Suppose you had the worse outcome so you are getting 85 figs a year after 6 years.
  - Next year you are equally likely to go up to 91 (rain and growth at 6%) or down to 83.3 (dry and decline at 3%) mean expected growth 3% of an expected yield of 87.1.
  - That gives a variance of 29.86 (st dev 46 and coef of variation of 62.6)
  - But normalize for difference in initial conditions and the world is the same in each period.
- Reverse the logic. If the world is IID then you can infer the mean return and the mean adjusted variance from the data, you can then forecast the likely future states of the world.

Lottery choice



Individual is mean variance tradeoff
U(X)=E(X)-bVar(X). B is the slope of the indifference curves
$U(S_5)=U(S_3)>U(S_6)>U(S_4)>U(S_1)>U(S_2)$
So if individual has choice he will pick S_5 or S_3
But what are the prices of these lotteries?

Lotterv markets



- Two individuals, Blue has a higher mean-variance trade-off (is more risk averse) than Red.
- The 'price" of a lottery is the intercept of the indifference curve because that is the certainty equivalent. And you can see that red pays more for ANY risky asset than Blue.
- That is because they can only hold one risky asset.
- Suppose that both Red and Blue have some cash and Blue owns S_3 Is there a deal waiting.
- But you would not want to price single securities. Why?

Next time

- <u>10-23 Class 8: The Portfolio approach to risk</u>
- **More than one security out there and returns not perfectly correlated;**
- **Portfolios have better mean return profiles than individual stocks;**
- **Efficient frontier and the Sharpe value;**
- **Basic portfolio separation;**
- **Why not insurance contracts?**