Banjo Bridge Mutes

David Politzer*

California Institute of Technology

(Dated: March 8, 2015)

Adding extra weight to the bridge of a banjo (or violin) is a common strategy for muting the sound. The result is, indeed, a quieter instrument, but this method of muting also increases sustain and yields a more mellow tone. Some examples and measurements with a variable-mass mute are presented. Two very simple models of pieces of the physics offer some understanding of the prominent features — at least, on average. However, the details of loudness and sustain prove to be complex and subtle, both in terms of physics and perception.

*politzer@caltech.edu; `http://www.its.caltech.edu/~politzer`; 452-48 Caltech, Pasadena CA 91125

Banjo Bridge Mutes

I. MIKE'S BANJO MUTE®

Mutes designed to make a banjo (or violin) quieter by clamping extra mass onto the bridge also have a substantial effect on sustain and tone. Do-it-yourself-ers use a clothes pin or bend a padded strip of metal into a hairpin shape. And a variety of designs are available commercially.

A very effective mute is available from http://www.mikesbanjomute.com. It attaches quickly to most standard-design bridges, and it does the job, providing a reduction of about 20 dB in the loudness of a plucked note's attack. This mute adds substantial mass to the bridge — an extra 43 gm to a typical 2.5 gm bridge.

FIG. 1. Mike's Banjo Mute® installed, along with three alternate weight bars

But inertial mutes do more. They produce longer sustain of individual notes, and they suppresses high frequency overtones. To explore these phenomena, I fabricated a variety of bars of different weights that could be used in different combinations on Mike's Mute. They appear in FIG. 1. The brass bar that comes with Mike's mute is about 34 gm, and it clamps onto a 9 gm aluminum bracket. The alternative bars shown are 4 gm, 11 gm, and 39 gm.

On-line at http://www.its.caltech.edu/∼politzer/mute-demo.mp3 (or if you're reading this on a Web-enabled device, by clicking in this box, if one is visible), I play first unmuted, then with the 4 gm mute bar (a 13 gm mute), and finally with Mike's standard mute bar (a 43 gm mute). Every effort was made to pluck just as hard for each time through. In fact, it is a challenge to resist plucking harder to compensate for how quiet the banjo becomes.

The simplest account of the changes in the sound follow from Newton's Second Law, $\mathbf{F} = M\mathbf{a}$, and can be understood qualitatively without recourse to further equations or calculations. Increasing the mass M of the mute while keeping the string force $\mathbf{F}$ fixed gives the mute less acceleration $\mathbf{a}$. So the bridge move less. That means the head moves less. With the head moving less, less energy is transferred from the strings to the head in a given time. The produced sound is not as loud, and the string vibration lasts longer. Lasting longer is what we hear as "sustain."

If we ask what is the relevant measure of the mute mass M, i.e., to what should it be compared for it to be considered "large" or "small," the answer depends on the frequency of vibration. For the case of a string vibrating at a particular frequency (as explicit analysis will show below) M is to be compared to the mass of the string in the adjoining section that goes up and down. i.e., the mass between nodes. Hence, a given mute M is increasingly effective in direct proportion to the frequency of interest — because the string wavelength (and mass between nodes) is inversely proportional to that frequency.

Also, every resonant frequency of the combined mute-bridge-head system decreases as the mute mass increases. This also shifts the sound producing response of that system to string vibration to slightly lower frequencies.

And, finally, the longer the vibration energy is kept in the string and not turned into sound, the longer friction can act on it. Most friction effects are more effective on dissipating higher frequencies than low. So this is another means for the mute to mellow the tone.

FIG. 2 displays a comparison of the frequency spectra of the the first and third selections in the mute-demo.mp3, i.e., with and without the standard Mike's Banjo Mute®. Note that the spectra are evaluated for the full duration of the tune. The vertical axis of the graph reports the total energy delivered per frequency interval during the entire twenty seconds. Also, the frequency resolution is deliberately chosen to give a general picture of the sound, rather than what happened at particular resonant frequencies. Three features are evident. The mute makes things quieter. The mute's suppression of loudness increases

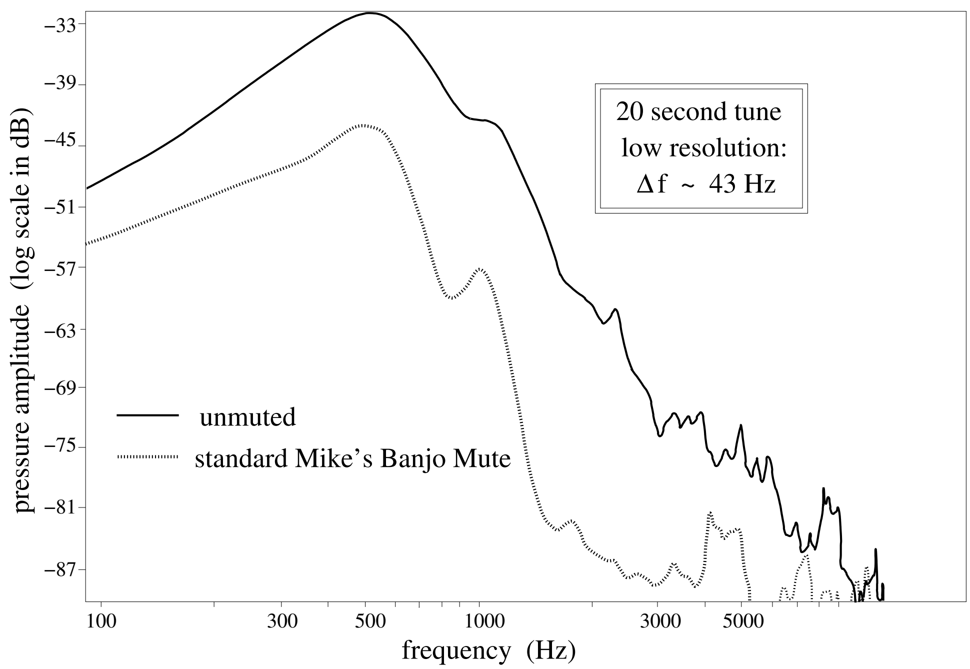

FIG. 2. spectrum for the full 20 second recording

with increasing frequency. And, somewhat more subtly, prominent features of the spectrum are systematically slightly shifted to lower frequencies by the mute.

Perception of a pluck's loudness really has to do with the peak volume reached during the note rather than the average over its duration. For the samples in the mute-demo.mp3, the standard Mike's Banjo Mute® delivers about a 20 dB reduction in the attack sound pressure of individual notes. The two curves in FIG. 2 are closer than that because the muted, quieter notes each have longer sustain than their unmuted, louder counterparts. Their energy is delivered over a longer time.

The next section presents more specific results of recordings of head taps and single string plucks. Then I present two very simplified physics models that can be easily solved. They capture the qualitative effects, on average, of varying the mute mass. However, more careful measurements challenge us to think more deeply about what we hear and about the more complex physics that must be involved.

II. MEASURES OF HEAD SOUNDS AND SINGLE PLUCKS

The sounds of head taps are often used in the ritual of banjo set-up as indicators of head tension. This is an important aspect of achieving a desired final sound. For this part

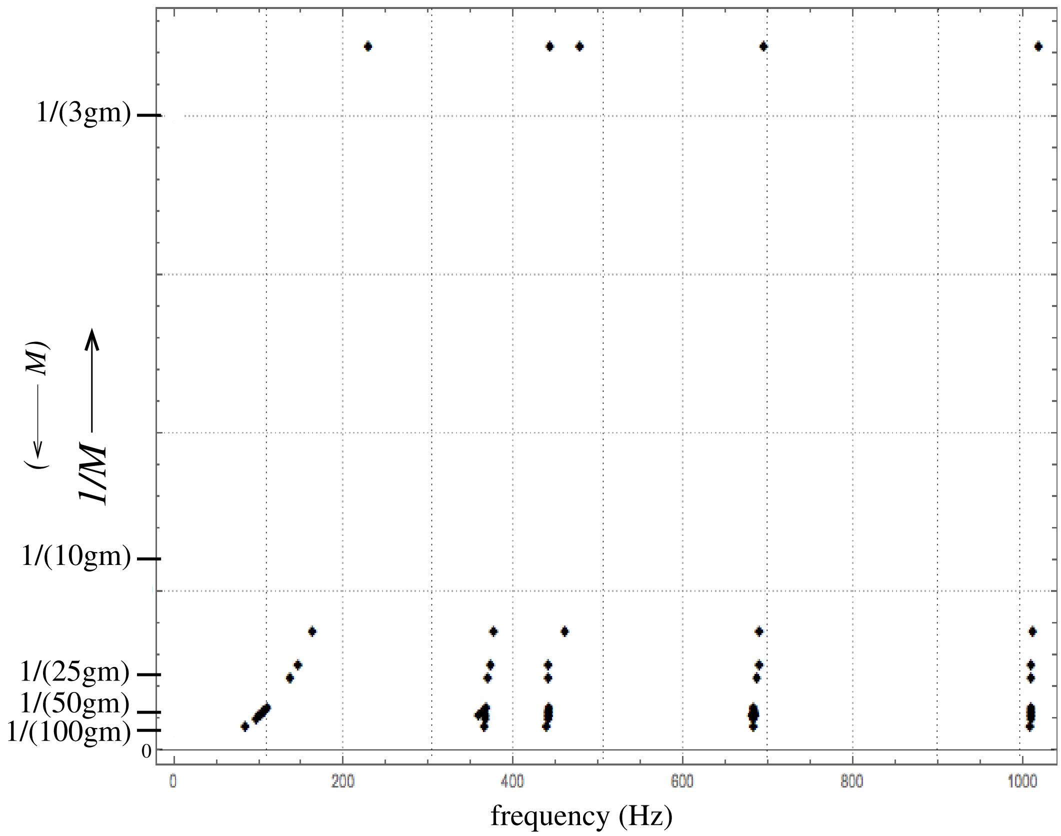

FIG. 3. no mute and 9 different mutes for the lowest 5 prominent head resonances

of the investigation, I left the head and strings on a real banjo unchanged and varied the mass of the bridge-mute combination. The frequencies of the five lowest, easily identified head resonances with ten different bridge+mute masses are plotted in FIG. 3. The peculiar format, i.e., plotting the inverse of the combined bridge+mute mass M versus frequency, was chosen to make clear the relationship to the model calculation presented in section III, i.e., to FIG. 9. The actual experimental method, which is far less ambiguous and more accurate than recording simple head tap sounds, is described in appendix B.

The next three graphs concern the sound of a single plucked, open string (tuned to D(294)), comparing various mass mutes to the unmuted bridge. The other strings are damped. FIG. 4 shows the frequency spectrum evaluated for a very short interval following the pluck. The time interval was chosen to be precisely the time that it took for the pluck disturbance to traverse the full length of the string three times. The three traversals take close to 1/100 of a second. This short time was chosen in the hope of making a convincing

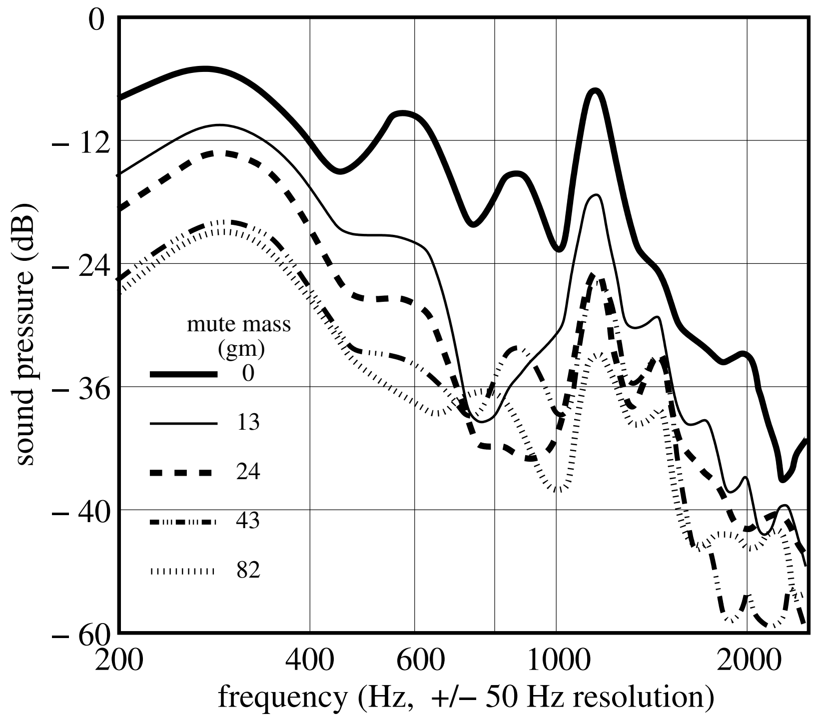

FIG. 4. spectra for the first 1/100 second, four mute masses and no mute

comparison to the modeling in section III. However, it is dramatically apparent that there are some features that are not simple functions of the mute mass. In particular, the region of roughly 700 to 900 Hz does not show a steady trend with increasing mass. Such features are reproducible with a given banjo, bridge, and set-up, but can change when those aspects are changed. The shortcomings of the simple models of section III and the likely origin of these complex features are discussed in section IV. On the other hand, for much of the analyzed sound, increasing the mass does decrease the amplitude and, to some extant, that decrease is greater at higher frequencies. (Appendix A has some technical comments on spectrum evaluations.)

FIG.s 5 and 6 show the results of single plucks on an open D(294) string (others damped) — the same five plucks but displayed for different time intervals. The curves show the maximum sound pressure (log scale) *versus* time. (Only the easily visible beats [periodic time variations] are plotted; where there were much faster peak-to-peak variations, only the highest peaks were used to construct the curves.) The three second interval displayed in FIG. 5 is too long to be very relevant to actual playing. However, it helps make contact with physics ideas presented in sections III and IV. In particular, in addition to the mutes making the initial pluck sounds quieter, there is at least some evidence that the mute mass makes the sound decay *rate* typically somewhat slower. The times in FIG. 6 are of more practical significance for banjo music because a long succession of notes may be just a tenth of a second apart, with some additional notes inserted between them. Two tenths of a second is a long time in banjo playing.

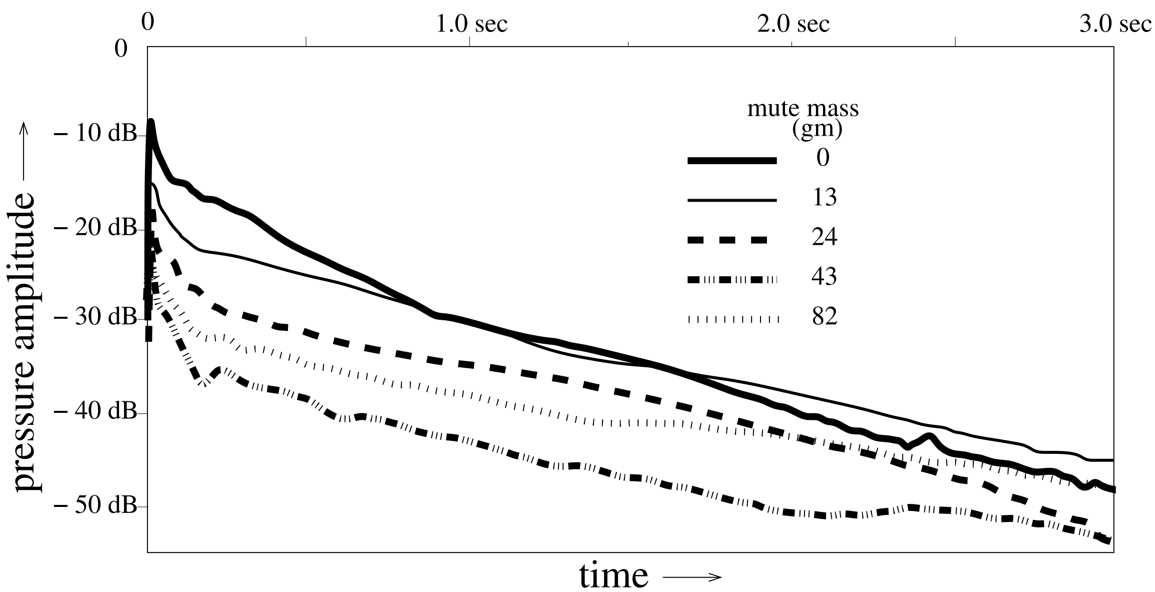

FIG. 5. comparison of single string plucks for various mute masses

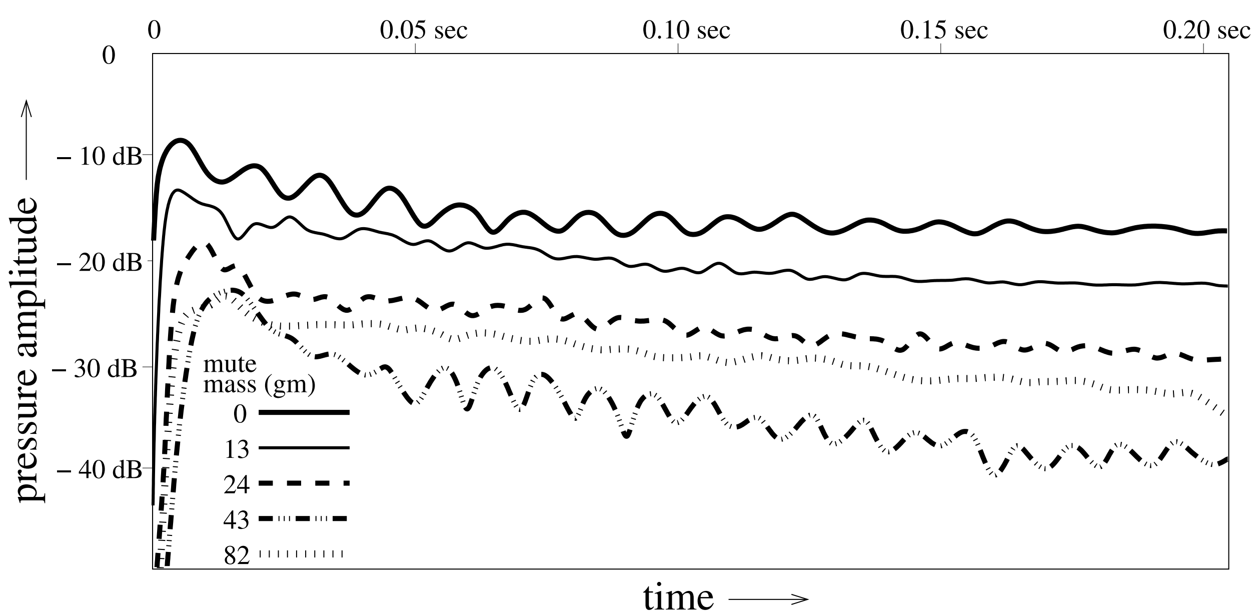

FIG. 6. the initial 0.20 seconds

The measurements presented in FIG.s 4, 5, and 6 were made on a banjo that was modified slightly to make the comparison with the model of section III somewhat closer. What was altered was the tone ring, a ring that sits on the top edge of the wooden rim, over which the head is stretched. The original banjo had a tone ring made of $1/4''$ diameter brass rod. That was replaced with rubber, and the head tension was reduced somewhat from typical (e.g., reduced to 80 on a DrumDial instead of 88). As a consequence, the head tap tone decay rate was about twice as fast as with the original brass. The instrument still sounded unmistakably like a banjo, albeit somewhat more primitive.[1]

In section IV, the single string sound is decomposed into narrow frequency bands, i.e., around the fundamental and around its harmonics. That reveals at least some of the things that combine to give the total sound such complex and varied structure.

III. THE MODELS

Banjo physics pioneer Joe Dickey put together a mathematical model that contained the barest essential sequence of elements that connect plucked strings to actual sound in the air.[2] He called it "the one-dimensional banjo." It includes many major simplifications and approximations. Nevertheless, it ends up with a reasonable representation of prominent features of banjo design and sound. However, much of the math is, by any measure, pretty sophisticated, and the final evaluation requires a serious amount of computer-performed numerical calculations. The goal of the present work is to extract just a few aspects of that model that, by themselves, can be understood with simple physics and paper and pencil. These, then, shed some light on how at least one aspect, the mass of the bridge, contributes to the sound.

FIG. 7 is a cartoon of the deconstruction of a real banjo that leads to two highly simplified models that will be analyzed in detail. One model (on the bottom, left) is relevant to the mute mass dependence of the head resonances. Here, the qualitative agreement with measured sounds is quite satisfying. The second model (bottom, right) is relevant to how string energy gets to the head, as a function of frequency and mute mass. It fleshes out the qualitative description given in section I. At best, the agreement with actual banjo performance only appears when we average over many features. Sections III and IV raise at least some of the details that are missing from the model.

In the first step of FIG. 7, the real banjo is replaced by one with a single string of finite

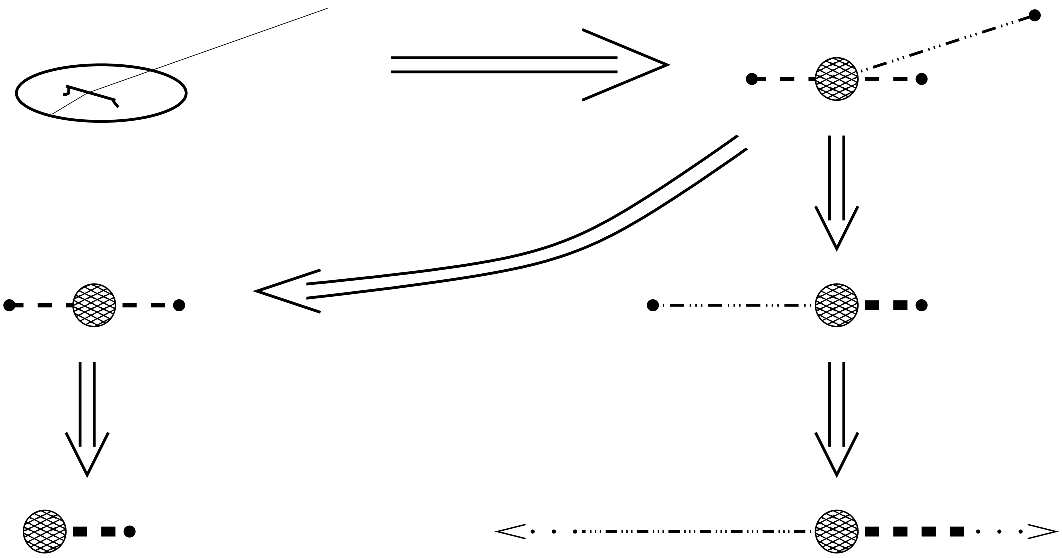

FIG. 7. cartoon of the origin of the two models to be analyzed.

length. The string (the dot-dashed line) is under tension, fixed at one end, and attached to the bridge at the other. The bridge (now a cross-hatched circle) is just a point mass. Its motion is imagined to be simply up and down relative to the head. The head, which acts to push or pull the bridge back to its equilibrium position while it is vibrating, is replaced by strings (dashed lines) on each side.

The vertical motion of the point-like "bridge" is the model's stand-in for the production of sound. As seen by the bridge, the two "head" strings are not exactly the same as a real drum head, but they play the same role. Their tension and deflection counterbalance the equilibrium down-pressure of the strings. When the bridge moves, it launches waves outward, and these are reflected back by the rim. The parameters of these head-stand-in strings (tension, length, and mass density) could be adjusted to be roughly comparable to a real drum head. Of course, the actual nature of wave propagation in the head replacement will not be identical to a real head. So the sequence of actual head resonances is replaced by the much simpler sequence of the ideal string. That simplification makes the math something one can handle by hand (instead of requiring a computer).

Continue in FIG. 7, down and to the left, and focus just on the bridge and head as a single system. How does varying the bridge mass effect the resonant frequencies of that system? This can be answered qualitatively in words. If there were head vibrational modes for which the bridge didn't move at all, then those modes would be unaffected by the bridge mass. However, all banjo sound is connected to bridge motion. So this situation is not of interest. In general, increasing the moving bridge's mass will decrease the frequency of the corresponding mode — just as it would for a single mass on a spring. As we'll see, the frequency of the lowest mode gets lower and lower as M increases. However, higher mode frequencies approach values corresponding to motions for which the bridge hardly moves (at least relative to the motion of the rest of the head).

In the left, center sketch, there are resonant "head" modes in which the two strings have opposite motions and the bridge doesn't move. In the modes of interest, the two sides move up and down identically and, so, can be represented by a single, double-the-tension string. This is represented in the bottom left sketch, reproduced as FIG. 8.

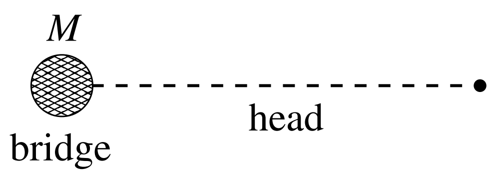

FIG. 8. a cartoon model for the bridge-head system resonances

The picture analyzed here is rather similar to Dickey's "one dimensional banjo".[2] He considers the bridge as a point mass in the center of the head. That generates "radial" modes, described by Bessel functions. In the model at hand, the head "string" represents radial motion from the center to the rim. It just has a less accurate version of the resonant frequencies than the ideal drum head model. A real banjo also has bridge rocking motions and azimuthal modes. These are very important contributors to the rich, high frequency components of the sound.[3] But the role of the inertial bridge mute mass there is quite similar, as long as the extra weight is a bar across the length of the top of the bridge (as it is for Mike's Banjo Mute®). The relevant physics equations for the rocking motion are the rotational analogues of those discussed here. In particular, the mute provides an increased moment of inertia to counter the applied torques. The qualitative consequences on sound are the same.

Setting up the model's equations and solving them could be assigned as homework in a second-year college physics or engineering course. Whether that is daunting or trivial depends on one's day job.[4] The problem is to find the transverse vibrational resonant frequencies of a taut string that is fixed at one end and attached to a mass M at the other.

M moves only in the transverse direction. Representing the real head by an ideal string makes this a calculation that can be done by hand, without recourse to reference books or computers, but it retains the important qualitative features.

The answer can be expressed simply in terms of the bridge mass M, the total mass of the finite length string m_o, and the fundamental (i.e., lowest) resonant frequency of the string with one end fixed and the other end free to move (i.e., corresponding to $M = 0$). (This is half of the fundamental frequency of the same string with both ends fixed, but with the same tension and density.) Let x be frequency in units of the fixed-end-free-end fundamental. Then the series of resonant frequencies are the solutions to the transcendental equation

$$x \tan x = \frac{m_o}{M} \; .$$

The qualitative behavior is clear from a sketch. (A computer makes easy work of producing a careful drawing.). Also for the lowest mode if $M \gg m_o$, then $x \tan x \approx x^2$. (For $M \gg m_o$,

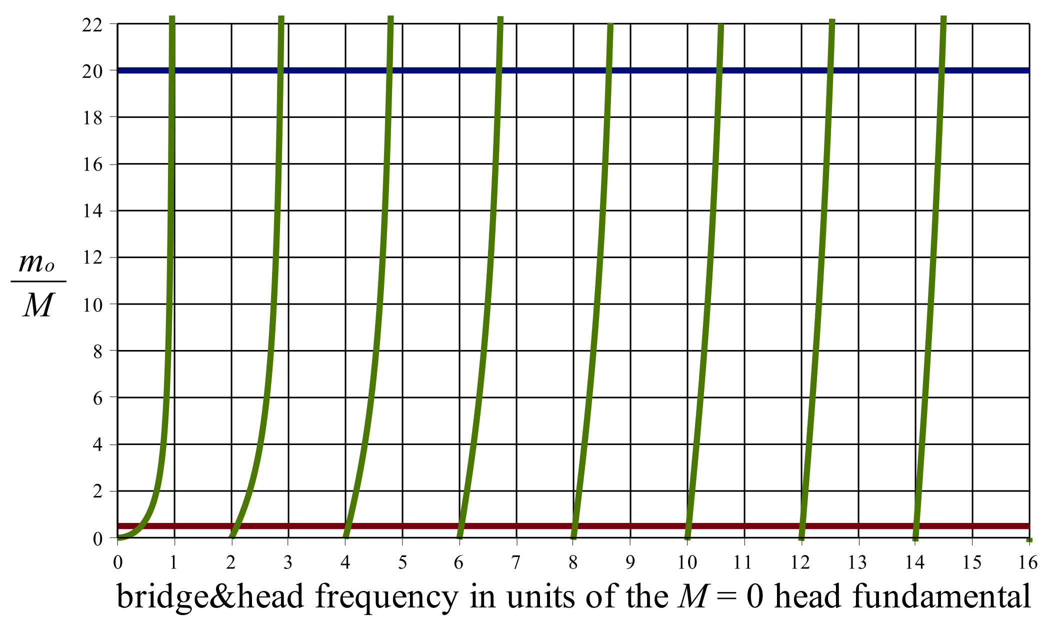

FIG. 9.

one could also expand the tangent function about its zeros, where it is approximately linear with slope equal 1.) The solution to the transcendental equation is illustrated in FIG. 9. The resonant frequencies are the intersections of the curves (which plot x tan x) with some particular value of m_o/M. Two examples given are $m_o/M = 0.5$ and 20.

For $M \to 0$, we get the odd-integer multiples of the fundamental frequency, which coincide with the free-end solutions. However, for any specific large value of m_o/M, the high frequency modes above a certain point will approach the even-integer multiples. For $M \gg m_o$, we get the even-integer multiples, corresponding to the fixed-end solutions and a lowest mode with frequency $\sqrt{(m_o/M)}$ (i.e., in the same units).

Of course, the simplicity of the result (and the work needed to get it) reflect the simplicity of the ideal string as compared to even the ideal drum head. But the payoff is the qualitative understanding offered. All resonant frequencies are shifted down by the large M but only as far down as a nearby, would-be resonance for which M hardly moves. The lowest head resonance is the only one that goes to zero frequency as $M \to 0$. These shifts are relevant to the produced sound because the degree to which a particular string vibration is converted to sound depends on the string coupling to the body (i.e., bridge and head), and it's the body that moves the air. And that coupling depends sensitively on how close the body frequencies are to the string frequency in question.

Compare FIG. 9 to FIG. 3.

To get some insight into how the string vibrations drive the motion of the bridge and head there are two further steps when moving down on the right side of the FIG. 7 cartoon. First, relative to restricted up and down motion, the two "head" strings are again no different from a single string, appropriately parameterized. And the direction in the picture of the strings has no meaning relative to the head vertical motion. That takes us to the middle cartoon on the right. The next step is more extreme. Imagine that the string and the "head" extend away from the bridge indefinitely far. That means we are only considering part of the vibration process. In particular, as better illustrated in FIG. 10, we consider a string wave incident on the bridge from the left. It applies an up-and-down force to the bridge.

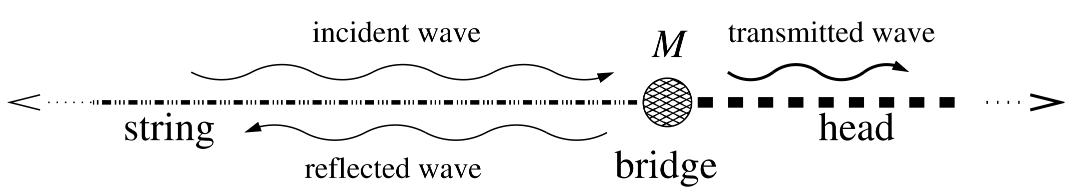

FIG. 10.

The consequent bridge motion launches a reflected wave back to the left on the string and a "transmitted" wave to the right on the head.

On a real banjo, the strings and head have fairly well-fixed outer ends and edges (nut and rim, respectively). The reflected and transmitted waves reflect back toward the bridge, and the process repeats. It is somewhat more complicated now. Furthermore, the reflections repeat again and again, at least until they run out of steam, their energy going to sound and friction. The repeated reflections build up resonances, and this process enhances the sonic response of the instrument at particular frequencies. What happens at or very near those resonant frequencies is strongly effected by the multiple reflections. In practice, this is a very dramatic effect for the strings and somewhat less so for the head (because it has a great many overlapping resonances with much shorter lifetimes). That there be at least several dozen relevant reflections along the string before it dies out is essential to making the instrument musical. Repeated reflections refine the frequency interval corresponding to a given note. Without them, a string pluck would sound like a click or a thud.

Nevertheless, the physics represented by FIG. 10 tells a bit about how the bridge mass effects the sound. This is summarized by a reflection coefficient R and a transmission coefficient T. These describe the relative strength of the reflected and transmitted waves.

In general, disturbances of the string travel in both directions. We imagine them as sums of sinusoidal waves of various frequencies, and we focus initially on an individual frequency f. We consider a string wave of amplitude $A_{\rm in}$, incident (right-moving) onto the bridge. The bridge is characterized as a mass M. That string wave will reflect off of the bridge, producing a left-moving wave with the same f but of amplitude $A_{\rm ref}$ (and different phase). The bridge motion produces an outgoing "transmitted" wave in the head of the same frequency f and its own amplitude $A_{\rm trans}$ (and phase).

The reflection coefficient R is defined as the ratio

$$R \equiv A_{\rm ref}/A_{\rm in}.$$

In this situation, R is less than 1. Its value is determined by the motion of the bridge M, which is also subject to the forces from the head. R has to be very close to 1 to make music. Otherwise, the sustain would be **very** short and the total resulting pitch ill-defined. On the other hand, if R is too close to 1, friction in the string would dissipate its vibrational energy before a substantial amount had a chance to get to the head and make sound.

The sound is made by the wave transmitted to the head, whose amplitude is $A_{\rm trans}$. The energy in the head due to that wave is proportional to A_{trans}^2, and so is the rate that energy is radiated by the head. Energy transferred per unit of time is called "power." And human perception of loudness is roughly a logarithmic measure of power.

So the other very important quantity is the transmission coefficient T defined as the ratio

$$T \equiv A_{\text{trans}}/A_{\text{in}}.$$

This model is actually a set of coupled, linear differential equations. The string and head behaviors are all lumped into two parameters, the impedances Z_{string} and Z_{head}. This is possible because all that is really needed here is how some external object (external to the string or head) interacts with them by connecting at some point. The Z's are ratios of the applied vertical force to the vertical velocity it produces, and they are determined by the physical parameters of the wave-carrying media. (For example, for an ideal string $Z_{\text{string}} = \sqrt{\text{tension} \times \text{mass per unit length.}}$) Roughly speaking, Z tells you how hard you have to push to get something vibrating.

The important output of the model is how R and T depend on the physical parameters M, Z_{string}, and Z_{head} and the frequency f. This is the model's answer to how bridge mass determines the efficiency of turning various string frequencies and harmonics into head motion and sound. Also, the reflection coefficient R gives a measure of sustain. As R gets closer to 1, plucked notes have longer sustain.

Again, it is a second-year undergraduate physics homework calculation. Here are just the results. There are factors of 2π that disappear if we use "angular" frequency ω, defined by $\omega = 2\pi \times f$. Then

$$R^2 = \frac{(Z_{\text{string}} - Z_{\text{head}})^2 + \omega^2 M^2}{(Z_{\text{string}} + Z_{\text{head}})^2 + \omega^2 M^2}$$

and

$$T^2 = \frac{4 Z_{\text{string}}^2}{(Z_{\text{string}} + Z_{\text{head}})^2 + \omega^2 M^2} \ .$$

Examining some simple limiting cases serves as a partial check on the calculation. For example, as $M \to \infty$, $R^2 \to 1$ and $T^2 \to 0$. That's just because $M \to \infty$ implies that the bridge doesn't budge. If $M = 0$ and $Z_{\text{string}} = Z_{\text{head}}$, then $R^2 = 0$ and $T^2 = 1$: no reflection, and the wave just keeps on going. If $Z_{\text{head}} \to \infty$, then $R^2 \to 1$ and $T^2 \to 0$ — because the head doesn't move. If $Z_{\text{head}} = 0$, ... etc.

Another check is provided by the fact that R^2 and T^2 are related by conservation of energy in this dissipation-less, idealized example. The energy carried in by the incident wave is carried out by the reflected and transmitted waves. So the sum of the rates of the latter two must match the incident rate. The power or energy transfer rate of a traveling wave is its velocity times its energy density. The energy density of the vibrating string is its mass density times ω^2 times its amplitude-squared (or A^2). Putting these together, one concludes that the power carried by a traveling wave on a string is ZA^2. Hence,

$$Z_{\text{string}} = Z_{\text{string}}R^2 + Z_{\text{head}}T^2 \ ,$$

which is satisfied by the formulas quoted above.

The message of the complete formulas is clear. Increasing ωM decreases T^2 and drives R^2 closer to 1. At higher M and at higher ω (or frequency f), decreasing T^2 means less volume, and R^2 closer to 1 means more sustain.

Further insight into the formulas comes from an alternate expression for Z, which is the string and vibration quantity to be compared to ωM. For an ideal string, $Z = \omega m_\omega/\pi$, where m_ω is the mass of the string between two successive nodes (i.e. half of the wavelength) when the string vibrates with (angular) frequency ω. So M can be considered large or small depending on how its size compares to m_ω/π.

With enough different values of M and ω, I thought it would be possible to make sound measurements that could support this model and these calculations, i.e., fit the relevant parameters and confirm the functional forms. Sections III and IV address some of what happened and possibly why.

There is a third, not particularly harder calculation suggested by the middle cartoon on the right in FIG. 7, i.e., the resonant frequencies and "normal modes" (i.e., the spatial shapes of those resonances) of the entire string-bridge-head system. However, it does not shed much new light relevant to the present focus, at least not without a further, far more sophisticated analysis. For parameter values appropriate to a plucked instrument, for a first approximation we think of the string as only weakly coupled to the bridge and sound board. The large R and small T allow string resonances to build up to musically satisfactory proportions. From that perspective, various string vibrational modes can be thought of as sinusoidal driving forces acting on the bridge, at least for the duration of the vibration. Of course, the reflections back to the bridge from the edge of the head (or sound board) influence the subsequent motions. A thorough analysis would have to include the energy transfer to sound and friction and the specific initial pluck conditions. This is a fine acoustical science problem, interesting from a physics perspective and potentially enlightening from a player's or listener's perspective. Furthermore, I do not know to what extent it has been already be carried out. I have not seen it prominently discussed in any standard acoustics reference, and I suspect that there's little that can be concluded just from paper and pencil.

IV. SINGLE PLUCK TIME AND FREQUENCY MEASUREMENTS

One can imagine a very simple and direct connection of the reflection and transmission coefficients R and T discussed above to the sound amplitude of the various harmonic components of a single pluck as a function of time. It is very credible elementary physics. However, in the end, we'll see that it's just not up to the task at hand. Too many features relevant to a single pluck are not included.

The very elementary picture goes as follows. Any single transverse direction motion of the ideal, flexible string held taut between fixed ends is the superposition of sinusoidal shapes with frequencies that are integer multiples of the lowest, "fundamental" frequency f_o $(= \omega_o/2\pi)$. The are called "normal modes." A pluck results in a disturbance that retains its shape as it travels at fixed speed up and down the string. The time of a single round trip is the period of the fundamental oscillation, $2\pi/\omega_o$. The string energy is distributed among the normal modes, and they behave independently of each other in the simplest version of the physics. If each mode's energy loss is governed by the transmission to an adjoining system, i.e., the head or soundboard, and that transmission is a small fraction of the total energy in the mode, then the fractional energy loss per time $2\pi/\omega_o$ would be approximately $T^2 Z_{\mathrm{head}}/Z_{\mathrm{string}}$. Consequently, the time dependence of the string energy and also the transmitted power would be a decaying exponential, $\exp(-\alpha t)$, where $\alpha = [T^2 Z_{\mathrm{head}}/Z_{\mathrm{string}}]/[2\pi/\omega_o]$. Equivalently, the energy and power half-life would be $\alpha^{-1}\log_e 2$. And, if the mute mass M is large enough to be the controlling factor in the determination of T, then $T \approx 2Z_{\mathrm{string}}/\omega M$. If the energy or power were displayed on a log plot (e.g., in decibels) *versus* time, it would appear as a downward sloping straight line. The slope would be proportional to $1/\omega^2 M^2$ (for large M) — or at least get smaller with increasing M and more dramatically so for higher ω. (Of course, sound radiation is not the only place that the initial vibrational energy ends up.)

Because R and T depend on frequency, it is natural to look at the time dependence of

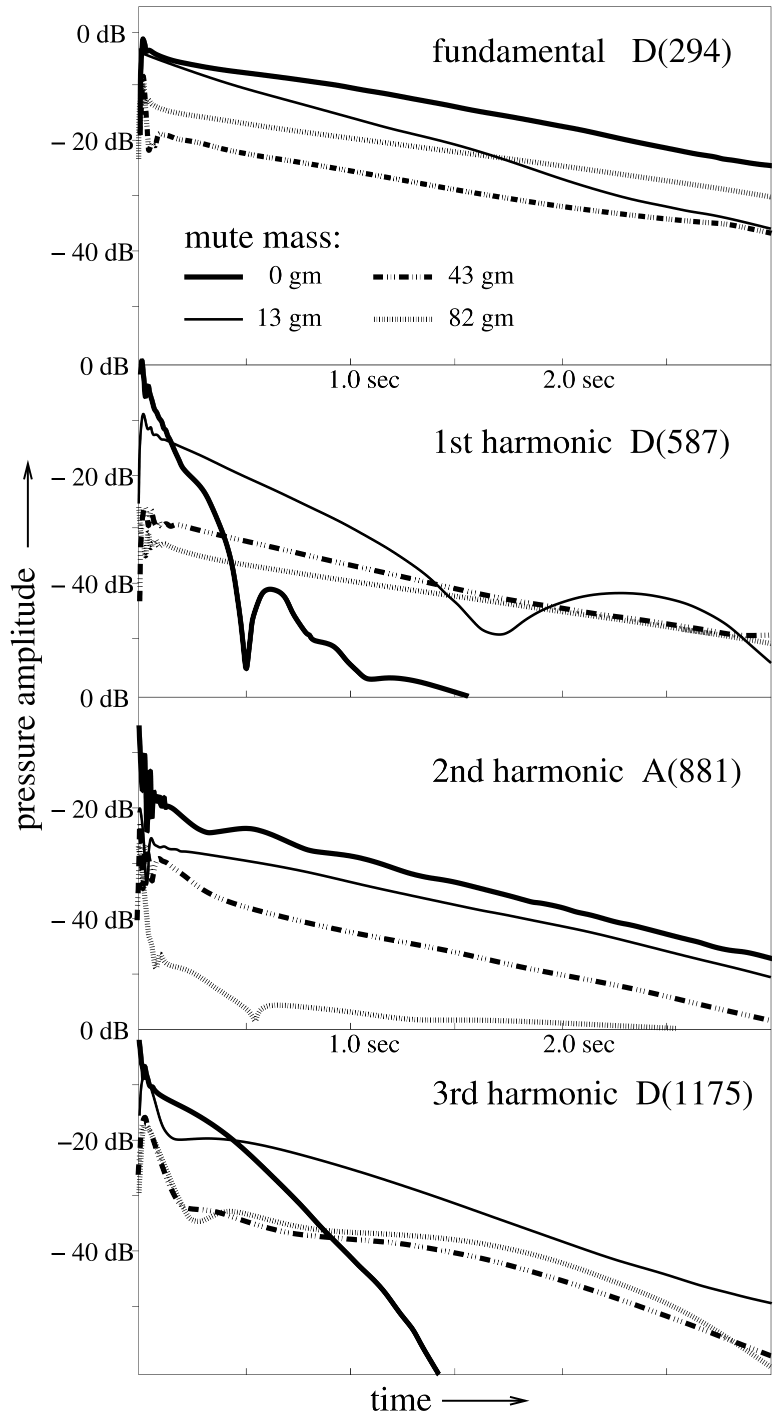

FIG. 11. first 3 seconds of the sound of a plucked D(294), with various mass mutes, filtered into narrow bands centered on D(294), D(587), A(880), and D(1175)

particular frequencies. As shown in FIG. 11, I chose frequency bands around the fundamental (i.e., D(294)) and the next three harmonics (D(557), A(880), and D(1175)) for the particular plucks analyzed in FIG.s 5 and 6. The recorded pluck sound was filtered into relatively wide bands around those frequencies with widths ± one whole note.[5] However, the band cut-offs were relatively sharp — about 10 dB per 2% change in frequency, an issue further discussed in appendix A. (A musical half-step is about a 6% change in frequency.)

The time dependence of the recorded sound of actual plucks is apparently not simple, even if we restrict attention to a single band centered on one of the string's harmonics. For the most part, a heavier mute makes the initial sound quieter. A crude measure of sustain is simply the long-time persistence of the sound. This is reflected in the average downward slope of the curves in FIG. 11. Again, for the most part, heavier mutes produce a more lingering sound. And there is some evidence that the mutes' effectiveness is greater for the higher frequencies.

However, the curves are not simple decaying exponentials (i.e., straight, downward sloping lines). The actual behavior is characteristic of systems where there is more than one kind of motion associated with the chosen frequency band. The string itself has two such motions, e.g., up-and-down and side-to-side. The head typically has at least two separate motions that are very nearby in frequency. The mathematics of the case of only two separate motions can be worked out by hand.[6] With more motions, it becomes analytically intractable. But the lesson of the two motion case is clear. For systems of coupled, damped oscillators of nearly the same frequencies, there are slight frequency differences that manifest as beats (slow variations of loud and soft) and multiple possible exponential decay rates. Some or all of these may be present in any one particular case, and they depend *very* sensitively on the relative sizes of the various coupling and damping parameters. In particular, changing the mute may change the coupling and damping parameters by a tiny amount; yet that may manifest as a dramatic change in the sound. And that's what makes FIG. 11 so complex.

V. RELATION TO WHAT WE HEAR

The neural signals generated by our sense organs are processed in parallel in a great variety of different ways before any interpretation reaches our consciousness. Comparisons are made to the immediate past, the recent past, and the distant past. In a sense, we even look to the future. Receptors are continually readjusted to enhance their effectiveness to sense anticipated stimuli. The anticipation is based on experience and expectations. In addition, our Darwinian "goals" as creatures are quite different from the goals of scientists observing natural phenomena. One net result of all this is that the relation of our perceptions to physical conditions is often surprisingly complex. With vision, the relation of color, brightness, and even scene to the characteristics of light focused on our retinas is not trivial. With sound, perceived pitch and frequency spectrum are related — but not always that simply. Loudness and sound wave power are another pair with complex relationships. Sustain has to do with the evolution of loudness in time. So perhaps it is foolish to expect a simple relation to exponential decay times of power amplitudes.

The bridge mutes studied in this investigation make the banjo quieter but also give it more sustain and make it more mellow. That's what we hear. However, if we look closely at individual pluck sounds, there are other things that have significant impact as the mute is changed.

The simple calculations concerning the mutes appear to account for what happens on average, i.e., averaging over different notes, slightly different mutes, different harmonics, etc. I actually suspect that there is a mathematical way to make this averaging precise. In mathematical physics, averaging over time or frequencies might be doable by what are known as Kramers-Kronig or dispersion relations. Averaging over slightly different mute, bridge, and other banjo parameters might be representable by some sort of statistical ensemble. I just don't have it quite right yet for the present context. It's a good problem, though.

APPENDICES

A. Comments on some technical issues

1. the banjo used

The banjo used in this investigation was a 1999 Deering Goodtime, modified to take a 1/4″ rolled brass tone ring. As noted in section II, the tone ring was replaced by a rubber ring for the single pluck analyses.

2. uniformity of plucks

Frequency measurements and comparisons can be made without too much attention to how hard you pluck. Of course, place on the string dramatically effects the frequency composition of the initial vibrations, but that is easy to control. The frequency content of the produced sound is not too sensitive to the initial pluck force as long as it is reasonably gentle, i.e., within a civilized (mostly linear) range. The plots of sound amplitude *versus* time or frequency simply shift up or down by a constant amount if the amplitude is on a logarithmic scale (such as decibels). So rapidity of fall-off is also something that can be compared between plucks that might have different strengths.

But comparisons of absolute loudness require standardized plucks. After a variety of trials of different techniques, I settled on a plastic fork. One can estimate the reproducibility of the plucks by recording a few sequences with the same banjo set-up, and comparing the microphone record. I could get to ±1 decibel in a given session and ±2 decibels session-to-session with the following procedure: Hold the fork below the base of the tines; steady the hand in a set reference position against the neck; slide the last tine across the fret board to catch, pull, and ultimately release the string.

3. *Audacity®*

Audacity® is a very powerful, user-friendly, open-source, audio freeware package. (Send them a contribution if you find it useful!) I matched the bit rate to a Samson U01C USB microphone (48,000 per sec). When plucking strings are the nut end, the first sound that appeared barely above the background noise presumably came from the very fast longitudinal waves. Then came the transverse wave, producing sound with rise times that clearly depended on the mute mass.

Audacity® can do Fast Fourier Transforms with a selection of possible window function shapes and lengths. However, the code is written to require a certain amount of data before calculating a spectrum. To get FIG. 4, even three pulse traversals of the length of the string (i.e., $\sim 1/100$ second) was too short. So I made multiple copies of the first $\sim 1/100$ second and strung them together. That is equivalent to imposing periodic boundary conditions on the Fourier integrals. This induces artificial enhancements of multiples of 100 Hz in the spectrum, but at least it's the same analysis for each mute mass.

Audacity® also allows the user to write frequency filters as specifically detailed as one chooses. A well-understood phenomenon is that "ringing" can appear in the filtered time sequence when a sharp-edged filter is applied to a function with rapid time dependence. Although often termed an "artifact," this ringing is a genuine property of the sharp filtering process. Filters can be designed to reduce its appearance. However, reasonable choices and proper interpretation require a thorough understanding of the form of the original signal and the purpose of the filtering. In the present context, the unfiltered signal exhibited ringing on many scales — longer than, at, and shorter than anything attributable to the sharp filters themselves. So it did not seem unreasonable to leave refinement and interpretation of the filter effects to future consideration.

B. Head-bridge-mute spectrum measurements

FIG. 3 shows the result of sound measurements on a real banjo, in particular, the frequencies of five prominent head-bridge-mute resonances as they depend on bridge+mute mass M. Head and strings were at normal tension. The strings were muted with felt. A signal generator was used to drive three piezo-electric disks (RadioShack buzzers) in parallel (10 volts peak-to-peak), one under each foot of a three foot bridge.

The only further banjo modification was to move the bridge to the center of the head. The purpose was to enhance the driving of the low, radial modes. That was also the rationale for three disks in phase rather than just one on an outer foot, which would also excite rocking bridge modes and azimuthal head modes. The generator did a sinusoidal logarithmic sweep from 70 to 1200 Hz. And the sound was recorded by a microphone placed $3''$ above the bridge — again, to be most sensitive to low lying radial modes which have large up and down motion of the center of the head. The back was left open, which reduces the impact

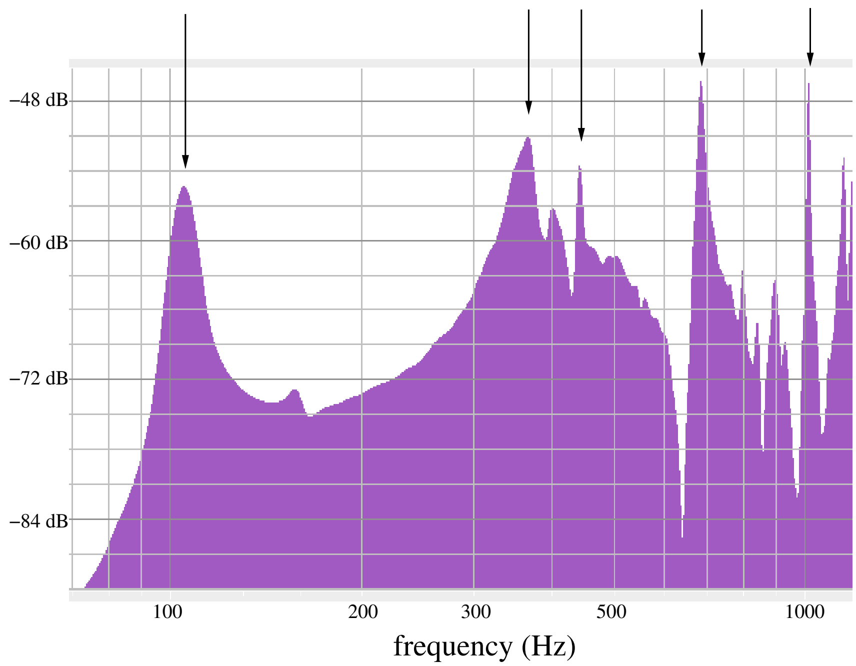

FIG. 12. five prominent head/bridge resonances with Mike's Banjo Mute®: 70 to 1200 Hz sweep

of air vibration in the pot on the motion of the head. The lowest values of M at the top of FIG. 3 are for the 2.5 gm bridge by itself. The high M points are for mutes with nine different combinations of bars such that the total M of bridge plus mute ranged from 16 to 85 gm. The odd format for presenting this data was chosen to emphasize the similarity to the forms predicted in FIG. 9.

As an example, FIG. 12 shows the sound amplitude for a sweep with Mike's Banjo Mute® installed. Indicated with arrows are the five prominent peaks plotted in FIG. 3. They were easy to identify and follow as they evolved from one M value to the next.

REFERENCES

1. The original rationale for the rubber ring was to bring the instrument somewhat closer to the idealization used in the second model in section III. That model does not include reflections off the rim of the head back toward the bridge. The rubber ring certainly reduces those reflections. But, in the end, the measured sound was not particularly closer to the naive predictions that it had been with a brass tone ring.

2. Joe Dickey, "The structural dynamics of the American five-string banjo" J. Acoust. Soc. Am **114** (5) 2958 (November 2003)

3. Laurie Stephey and Thomas Moore, "Experimental investigation of an American five-string banjo" J. Acoust. Soc. Am. **124** (5) 3276 (November 2008)

4. This was frontier physics in the 17th and 18th Centuries and already appeared in reference books in the 19th Century, e.g., Rayleigh's *Theory of Sound.* Nowadays, references on physics of musical instruments abound, but the basics are in intermediate mechanics texts as well as in books specifically on vibrations and waves. In the latter category, my favorite to teach from was *Waves* by Frank Crawford. Each has its own style, emphasis, and order of presentation, and in none is it paeticularly easy to pick up the analysis in the middle.

5. It is actually quite simple, e.g., using the sound software package Audacity®, to write one's own narrow band pass filters and analyze a particular frequency component as a function of time in the sound of a pluck.

6. David Politzer, "The plucked string: an example of non-normal dynamics," Am. J. Phys. *(to be published)* (2015); www.its.caltech.edu/∼politzer