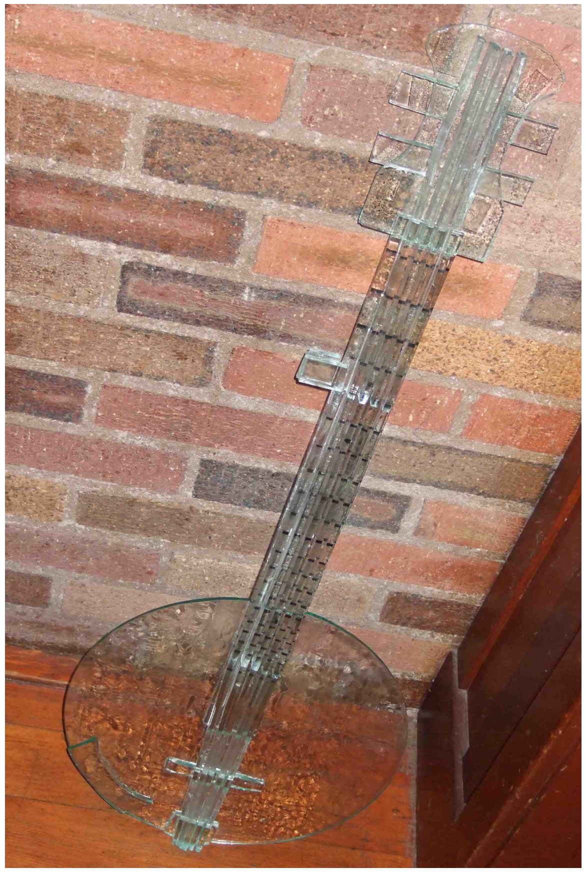

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The Open Back of the Open-Back Banjo

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...in which a simple question turned into a great adventure and even got answered. (Of course, you might already know the answer yourself.) In a triumph of elementary physics, six measured numbers receive a satisfactory account using two adjustable parameters.

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The Open Back of the Open-Back Banjo

I. THE RIM QUESTION

The question seemed straightforward. What is the impact of rim height on the sound of an open-back banjo?

FIG. 1. an open-back banjo's open back

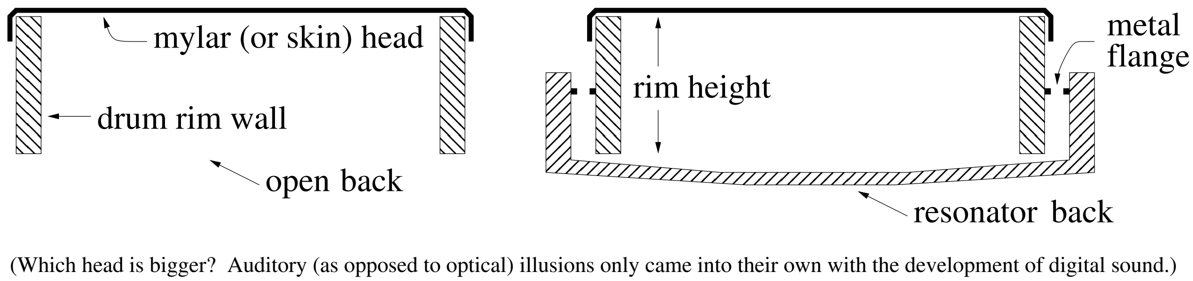


FIG. 2. schematic banjo pot cross sections

There are a great many choices in banjo design, construction, and set-up. For almost all of them, there is consensus among players and builders on the qualitative effect of possible choices. Just a few of the many are: string material and gauge; drum head material, thickness, and tension; neck wood and design; rim material and weight; tailpiece design and height; tone ring design and material. However, there is no universal ideal of banjo perfection. Virtually every design that has ever existed is still played with gusto, and new ones of those designs are still in production. Solo performers will often choose to feature several radically different instruments in the course of a show.

A one-foot high rim would be impossible to hold and play in the normal fashion. At the other extreme, a 1/4″ rim banjo could be constructed, e.g., using a pre-tensioned drum head made by Remo (www.remo.com). [In fact, Sam Farris makes a very light weight, full-scale, travel banjo based on that; it weighs less than three pounds and easily disassembles to fit into a carry-on suitcase (www.tranjo.com/txpictures.php).] But physics says that some of the sound coming off the back would leak around to the front and *cancel* some of the sound from the front. They're 180^o out of phase. This process is increasingly effective for lower frequencies because they are better at going around corners. (That's why woofers live in a box.) Somewhere between zero and twelve inches exists the real world of banjo rims. But the how and why of particular choices was not clear.

In discussing musical instruments, physicists generally seek grossly simplified models of restricted parts of what's going on. Sometimes the lessons are only qualitative. Nevertheless, some people appreciate gaining a sense of understanding what's going on. It's a bonanza when equations yield numbers that actually match measurements. (Very modest, limited success of that sort in the present context is reported in section X.) In contrast, the more results-oriented approach of acoustical and electrical engineers is often of more practical value.

The open-back rim presented a puzzle because it did not seem to fit into any of the simple physics pictures that had been applied to musical instruments. A handful of people have actually published serious banjo physics research. (See **REFERENCES**.) A quick survey of the more theoretically oriented of them confirmed that no one thought the answer was obvious. The voices of experience noted that theorists can spin all sorts of tales. Perhaps we shouldn't believe anything without the foundation of some good, careful measurements.

Thus began a wonderful and enlightening journey.

In retrospect, part of the physicists' confusion arose from picturing a banjo in a laboratory, clamped somehow on a bench and surrounded with poking and measuring apparatus.

II. THE MUSICIANS' ANSWER

Along the way, it became clear that many players and builders have long known the answer. Here are just two examples.

Several people, when the subject was raised, noted that David Holt, a professional performer (www.davidholt.com), plays a "deep pot" (i.e., high rim) open-back. ("Pot" refers to the whole drum assembly; "rim" is its wall.) Holt explains that years ago he noticed that he liked the rich and deep sound of some old minstrel banjos that had particularly high rims. He asked Greg Deering to make him one. Holt has happily played that banjo (or similar replacements) ever since, and Deering (www.deeringbanjos.com) offers a version as one of his standard, high-end models. Its rim height (including wood tone ring) measures 4 1/2″, in contrast to the 2 3/4″ or so typical of other open-backs.

Adam Hurt (www.adamhurt.com) is a young man who earns a living as a performer and teacher. In an interview with Craig Evans [on disc 3, volume 3, in a section titled "Extras" of Evans' North American Banjo Builders DVDs (www.northamericanbanjobuilders.com)], Hurt makes a point of explaining that, when he gives workshops, he likes to spend serious time discussing tone. He's not promoting a concept of "great tone." Rather, he emphasizes the enormous variety available to the player by varying how the instrument is held and how the strings are attacked. The key insight relevant to the question at hand is that there are very discernible variations in tone that come from varying the amount of space left between the open back of the banjo and the player's body.

III. THE EPIPHANIC EXPERIMENT

Preparation for some "good, careful measurements" took a couple of months. What was required was at least three banjos, as identical as possible in every way — except for their rim heights. I had learned early on that "similar" was not good enough. The effects of typical instrument-to-instrument variations in rim heights were less prominent than the effects of other qualitatively understood differences like scale length or break angle of strings over the bridge. That left it difficult to judge what was due to what when comparing "similar" instruments.

In the meantime, a very simple observation pointed to a very promising direction. It's something anyone can do and hear in a few seconds with an open-back banjo. Here's how: Hold the banjo as you would to play it. Damp the strings, and tap on the head repeatedly, four or five times a second. While continuing to tap, gradually angle the banjo away from your body, i.e., pivoting from the tail end and moving the tuning head away. Listen. [The file "tap-opening-up.mp3" in the accompanying on-line directory (see **REFERENCES**) is a recording of just that.] This is the simplest example of what Adam Hurt described.

The rim and head form a drum. As explained in several of Tom Rossing's books (e.g., *Science of Percussion Instruments*), drums can be crudely classified as being of definite or of indefinite pitch. And the banjo has the latter. The perception of definite pitch of a complex sound requires a preponderance of overtones with roughly integer frequency ratios. The banjo head produces a jumble. (Nevertheless, some people, with greater or lesser effort and a carefully chosen tap protocol, can reliably pick out particular frequencies.) For most people, a single head tap does not produce an identifiable pitch. However, the sequence of taps, as the banjo/body spacing opens up, includes a clearly rising pitch as well as a lot of other sound that doesn't change very much. Evidently, our brains process the sensory input in context. With hearing, as with vision, touch, and smell, we naturally focus attention on changing stimuli.

This is just like what can be heard with resonator-backed banjos (illustrated in Fig. 2), either as part of learning to identify the head tap tones or tuning the resonator-rim spacing. (See, e.g., Rae & Rossing; and Siminoff in **REFERENCES**.) And it puts the physics of the open-back banjo squarely in the camp of all other stringed instruments with hollow bodies, e.g., guitars, violins, harps... The player's body forms the banjo's back, leaving an easily adjustable size "sound-hole," i.e., the open space between body and rim. And rim height determines the enclosed air volume. The rim height and opening are parameters that enter the determination of the two lowest resonances of the body. The lowest of these resonances is the so-called "Helmholtz" resonance in which expanding and contracting enclosed air pushes some air in and out of the opening. The other low one is the lowest vibrational mode of the head. This motion is subject to "air loading." It, too, must push on air, inside as well as outside the enclosed volume. Physicists love this system because the two motions are intimately entwined and necessarily obey the Laws of Motion set down by Isaac Newton in 1687. The two seemingly distinct motions necessarily effect each other's frequency. Luthiers and violin makers, knowingly or not, rely on this physics because these resonances determine the lowest frequencies that can be turned into sound by the body. Without body resonances, the vibrating strings by themselves are essentially silent. Consequently, very close attention is paid to the volume of the air cavity and the area of the sound hole. And tone is strongly effected by their variation. Of the many resonances of the banjo pot (its body), these two are the simplest to describe in terms of physics and likely the most affected by rim height and air opening. However, as we'll see, the height and opening color the full spectrum.

The player's belly absorbs more sound than the wood of a resonator back. This makes open-back body resonances a bit weaker (quieter) than resonator banjos. But it also makes the open-back resonances broader in frequency. And broader in frequency makes them more effective at transducing string vibration into sound over the corresponding broader range of string frequencies. Actually, I suspect that most of the discerned differences between open-back and resonator banjos come from playing style differences and other typical construction details (e.g., tone rings) rather than just the nature of the back. (See section VIII, FIG. 8.)

What follows is an account of the acoustical engineering journey that culminated in recordings and measurements using three research grade, custom design banjos contributed by Greg Deering and Deering Banjos. One goal is to hear the acoustic range offered by an extreme range of rim heights. Another goal is to explore the simple model calculations and observe their degree of success in the hope that they lend some credence to the theoretical picture presented.

IV. THREE GOODTIMES: $2''$, $2\frac{3}{4}''$, & $5\frac{5}{8}''$

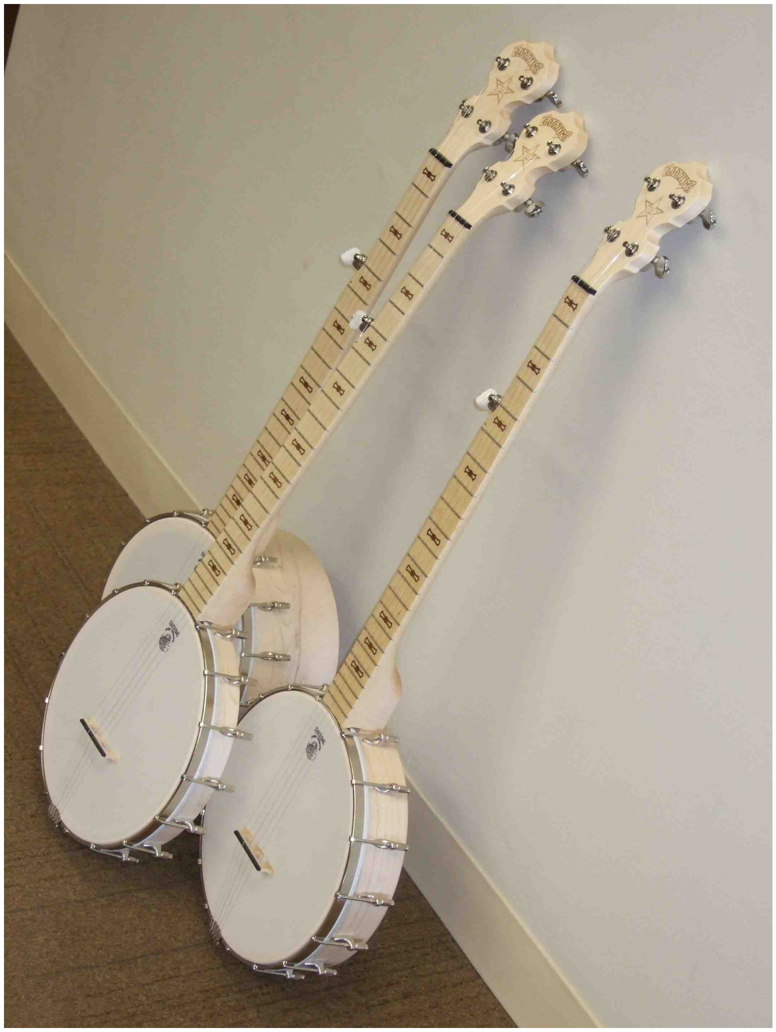
Three banjos as identical as possible except for rim height? I finally decided that my modest research budget could afford three Goodtime banjos. As best I understood, they are mass produced and hand finished, yielding instruments that are, indeed, as identical as CNC mills can produce out of wood **AND** are of substantial quality and integrity. I could cut one rim down, leave one the same, and glue a second rim blank onto the third to double its rim height. Goodtime banjos retail for about $400. Stewart-MacDonald (www.stewmac.com) sells rough rim blanks for about $90. But I thought I might be able to get an already turned Goodtime blank directly from Deering, whose factory is located just north of San Diego — a couple hours drive away.

(An aside on "Goodtimes": Goodtime banjos, originally a single model introduced in 1996 and now a whole line, are the economy instruments from Deering Banjos. Often described as entry-level instruments or for travel or knocking around, Goodtimes are many people's choice, irrespective of cost. That includes some professional performers, for both live and recorded performances.)

A letter to Greg Deering describing my project yielded an invitation to come and discuss things in person. For a banjo player, stepping into the Deering factory in Spring Valley, California is a breath taking, awe inspiring experience. At its outset in 1975, Deering Banjos was just Greg and his wife Janet. Today, they have about forty five employees and expect soon to pass the milestone of 100,000 banjos produced over their career. Greg has had a life-long passion for machine tools and industrial processes. While guiding me through the factory, he took enormous pride in showing off old machines he saved from the scrap heap and modified for his production. And he loves explaining the newer machines he invented and developed. Producing quality instruments at affordable prices has been his continuing goal. Of course, "affordable" is relative. You can spend much, much more on a Deering banjo than the cost of the base Goodtime if you so choose.

After I described the particulars of my quest, Greg offered to build the three banjos I had in mind and to donate them in the name of science. He noted right away that the cut-down version would require some minor changes in design (i.e., neck mounting and tailpiece) and that those changes should be implemented on the other two to keep the three "identical."

They were ready for pick-up a few weeks later.



My immediate reaction to their sound was straight out of Goldilocks and the Three Bears. The fat one sounded fat. (David Holt knows his stuff.) The thin one sounded thin. And the middle one sounded... just right. (Of course, these judgements are really only relative to sounds that were familiar. There certainly is no "right" with banjos.)

Listen to the files "banjo-A.mp3," "banjo-B.mp3," and "banjo-C.mp3," in the accompanying on-line directory. (See **REFERENCES**.) Better speakers than the typical lap-top built-ins will make the differences clearer but might not be necessary.

The issue is not whether you like the tune. The issue is not whether you forgive my playing or recording technique. It's not even what you think of the banjos. The issue is whether you can discern differences between the banjos. If so, then the challenge is to understand their origin and perhaps even quantify the relation of some of those differences in sound to differences in construction. The recordings were made under as identical conditions as I could arrange. As advertised, the banjos differed only in rim height: 2″, 2 3/4″, and 5 5/8″. The heads were adjusted to the same tensions as read on a Drum Dial (i.e., 88 — [Do you remember to zero yours?]). That tension produced different head tap tones on the different banjos — but those differences are part of what the physics analysis will try to address.

Which is which? For the acoustically challenged, banjo A is the 2″, banjo B is the 5 5/8″, and banjo C is the 2 3/4″.

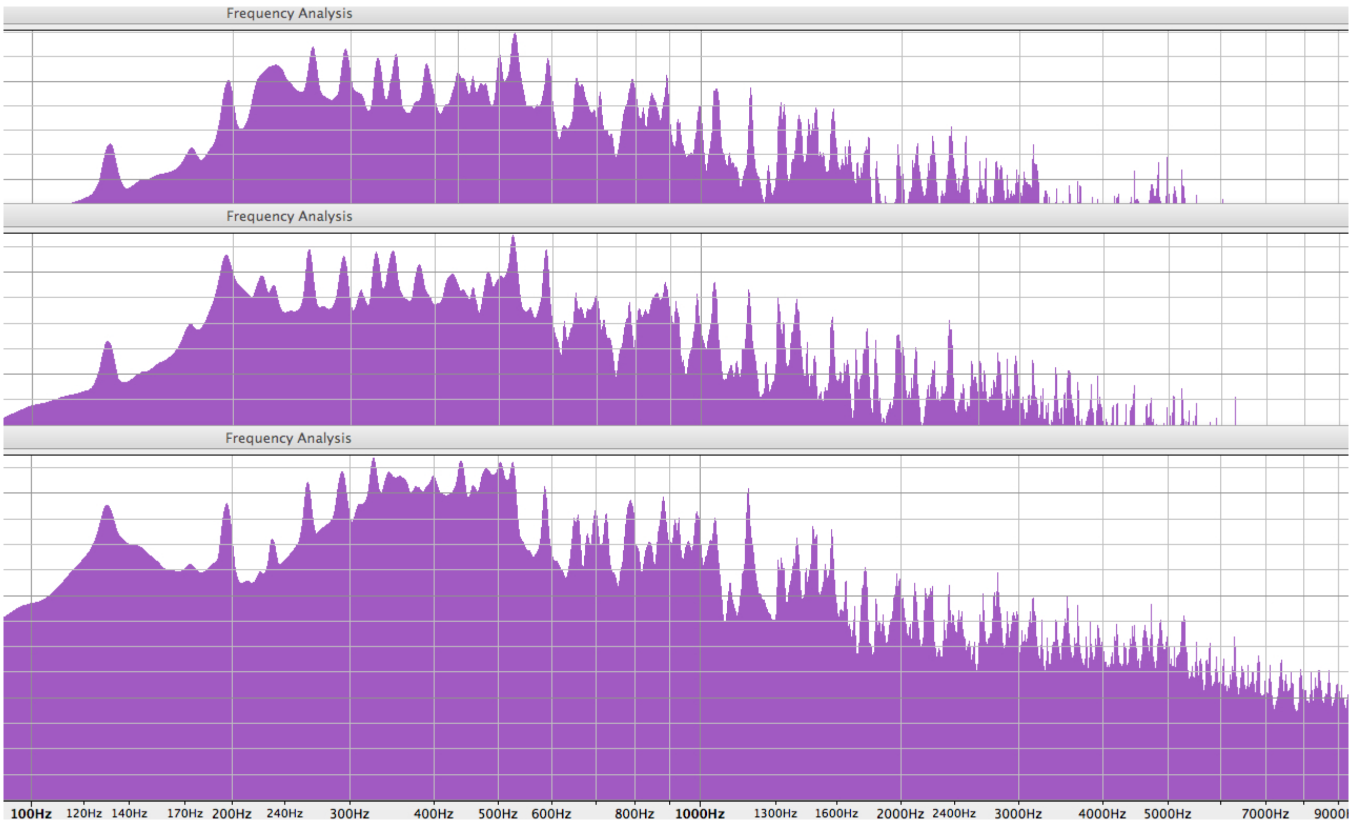


FIG. 3. Frequency spectra (from top to bottom) of the 2″ (A), 2 3/4″ (C), and 5 5/8″ (B) banjos: vertical scale in decibels (each line is 6 dB) *vs.* frequency on a log scale, 100 to 9000 Hz

Above, in FIG. 3, is a comparison of the frequency spectra of the three, entire, one-minute recordings as generated by the handy freeware package "Audacity."

The recordings are of the same notes, played essentially at the same volume for the same duration. They span the full range of the banjo, more or less representative of how often those notes are played. So the differences in spectra are one way to represent how the different banjos turn string motion into sound. Of interest are the relative heights of peaks of different frequencies as you go from one banjo to another. The lowest frequency peaks are at ~130 Hz (C3), the fundamental frequency of the 4^{th} string. ~525 (C5) is the fundamental frequency of the highest note played (and a harmonic of many others). Considering the relation of these peaks to the rest of the spectra is one way to compare the banjos. With the fat rim, the ~130 Hz peak is only about 12 dB below the strongest other peaks. That ratio is some 20 to 25 dB for the 2 3/4″ and over 25 dB for the 2″. (A 10 dB difference or ratio is a factor of 10 in volume or power.) But clearly many other features of the spectra are affected as well.

V. VISIT WITH RICK

Amateur recordings don't do the banjos justice. But professional recordings could make them sound like anything you choose.

Just as we recognize individual people by their voices, players recognize individual banjos by their voices. But I don't know anyone who can do either from a plotted spectrum.

What's needed is a first-hand evaluation. But one can't do it oneself. Playing a banjo, you definitely do not hear what others nearby hear. There are two obstacles. 1) The sound is radiated in a highly directional fashion — increasingly so with higher frequencies. 2) There is a lot of sound very nearby that cancels other nearby sound as it travels away from the instrument. (See **APPENDIX**. Playing between two bare walls — as in a stairwell — is a partial solution.)

So I took the three Goodtimes to see my friend, Rick. He is a far better player than I, has a better ear, and has a lot more Old-Time music experience and knowledge. His own favorite banjo is a really fine high-end old Vega Tubaphone pot with an expertly crafted replica neck. While I was unpacking, Rick reminded me that he and I belong to different tribes. He is of the people who insist on stuffing a rag (or sock or sponge) at the edge of the head, inside between it and the dowel stick or coordinator rod, nearest the neck joint. This is certainly *de rigeur* for mylar heads and steel strings. (This strategy preferentially damps the highest frequencies and produces a sound akin to skin heads and gut strings.) My tribe likes to hear the wild and savage ring of the banjo, as unfettered as can be. But there'd be no stuffing that day. I needed to hear the Goodtimes as they were.

Rick acquiesced. I liked the plink, sparkle, and snap as he played the 2″, but Rick, perhaps out of politeness, had no comment. When he moved on, he remarked briefly that the 2 3/4″ sounded more like a banjo. Yes, I thought; it's what we're used to. After I played the 5 5/8″ for him, Rick commented on the evident strength of the 4^{th} (lowest) string notes, both open (which I tend to hit a lot) or fretted. I had become somewhat familiar with the sound of the 5 5/8″ from playing it previously, but I was surprised that when we were both playing banjo and Rick had the 5 5/8″, I heard it louder than the banjo I was playing in my own lap. He also made better use of the 4^{th} string. We played a tune I learned from Mary Cox (www.maryzcox.com), which she plays with her husband, Bob, accompanying on guitar. When in my mind I could hear Bob Cox's bass run, there in real time Rick had something sounding quite similar going on the 5 5/8″.

The real surprise was Rick's reaction to the fat banjo. First off, he had immediately found a solution to the ergonomic challenge. I had struggled just a bit to reach around it and play in the same fashion as the others. Rick simply placed the fat pot in his lap, angling the upper edge back a bit towards him. He was comfortable. After a while he announced that he **really** liked it. No rag or sock needed for this one! He thought it was great as is. He was not only delighted with the tone; he also really liked the feel. The inherent volume allowed him to play with the gentlest of touch, which, in turn, facilitated a subtlety not commonly available. In the end, he didn't want to part with it that day.

VI. THE SYNTHETIC BELLY

A comparison of the head tap and Helmholtz frequencies to some physics expectations would require more precision and reproducibility than afforded by playing the banjos on my knee. Given the perspective that my belly was actually the back of the open-back banjo, what was needed was a synthetic belly whose location and "sound hole" could be controlled.

Acoustic engineering references suggested that sound reflection off of cork is similar to reflection off of people and closed-cell foam is one of the better sound absorbers, pound for pound. So I constructed an 11″ (the outer rim diameter) snap-on disk back out of those materials and added a layer of cotton shirt (FIG. 4.) The snap-on fixture height was adjustable from the outside with a wing-nut, and calibrated spacers could be held snugly between the rim and the back. This gave the banjos a flat disk back whose diameter matched the outside diameter of the rim with an adjustable rim-back air gap.

FIG. 4. The synthetic belly: 1″ closed-cell foam, 5/8″ cork, & cotton Hawaiian shirt

I compared the head tap sound spectrum of the normally-held mid-size Goodtime to a range of synthetic belly spacings. As expected, the lowest frequency peak of the tap sounds steadily decreased in frequency as the rim-belly spacing was decreased. 1/4″ spacing was not quite low enough, but something else happened by 1/8″ spacing. The increased friction of the air moving in and out through that narrower spacing created a qualitatively noisier and broader spectrum at the low end, i.e., around 200 Hz, which is the region in which I hoped to do some quantitative comparisons. Angling the back so that it touched the rim at the tailpiece end and rose to a separation of 3/8″ at the neck joint end gave a better approximation to the shape of the gap when the banjo is normally held and produced a much better match to the normal playing position. Below are those three tap spectra in FIG. 5.

VII. SINGLE STRING PLUCK SPECTRA

With the synthetic belly mounted with the angled air gap described above, I recorded the sound of single strings plucked at 1/10 of the scale length from the bridge, with the other

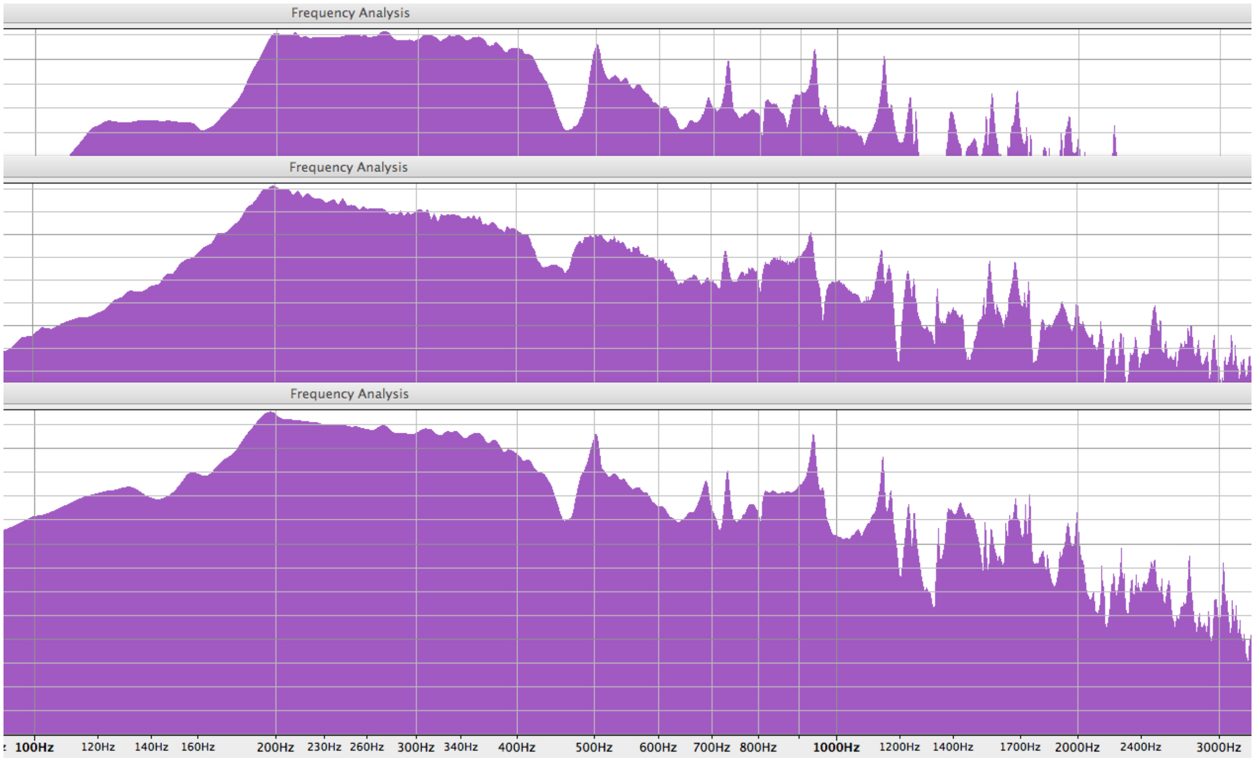


FIG. 5. Head tap spectra of the 2 3/4″ rim, strings damped: vertical scale in decibels (each line is 6 dB) *vs.* frequency on a log scale, 100 to 3000 Hz; the middle trace is the normally held, open-back banjo; the upper trace is a 1/8″ uniform spacing of the synthetic belly; the lower trace is the 0 to 3/8" angled spacing of the synthetic belly.

strings damped. That's a pluck position vaguely typical of "clawhammer" playing. FIG. 6 displays the results for the 4^{th} strings. (Using a linear frequency scale yields a reminder that most of the action is at integer multiples of the fundamental – as it has to be for a vibrating string.)

Two aspects are clear. 1) The peak corresponding to the fundamental frequency of 131 Hz (C3) is only appreciable for the fat rim. (On the other banjos we hear that pitch perfectly clearly even when that frequency's peak is inaudibly weak. It is an example of the "missing" or "virtual" fundamental, described in many places, my favorite being Rick Heller's *Why You Hear What You Hear.*) 2) The relative heights of the peaks up to 1049 (C6) are different for the different banjos, yielding distinctly different tone and timbre. The instruments certainly have different resonances, accessed with different strengths, but, to my knowledge, there's no single, simple physics story that goes with them.

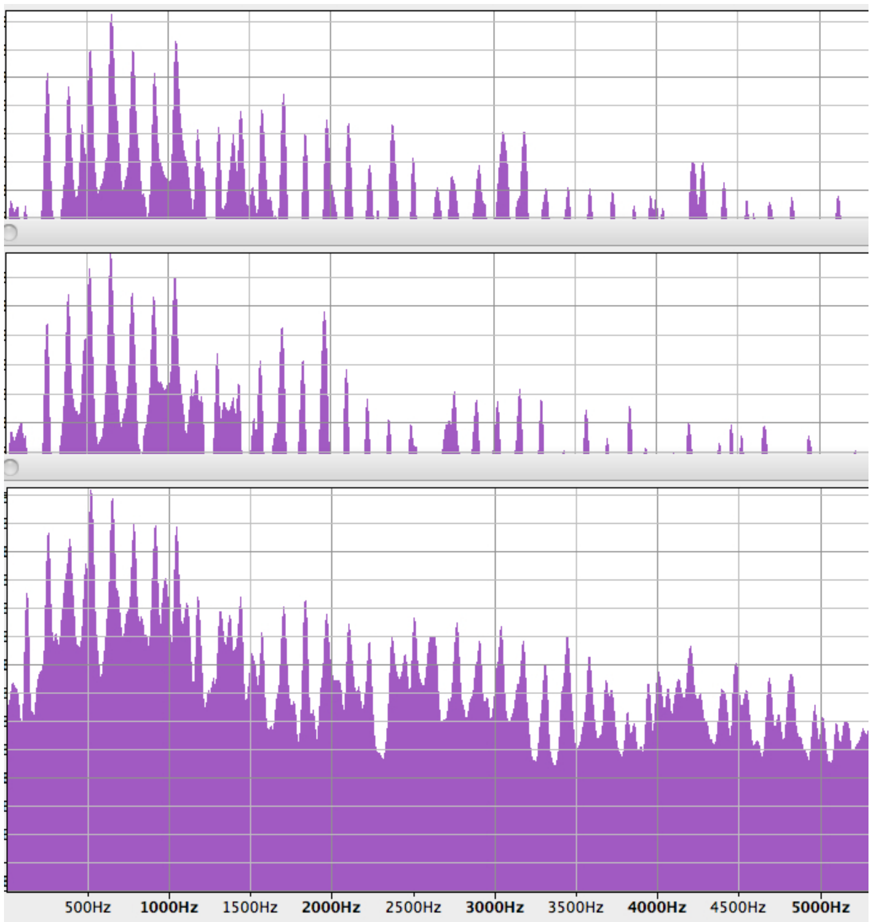


FIG. 6. 4th string spectra: power (6 dB per line) *vs.* frequency (linear scale 0 to 5000 Hz) for 2″ (top), 2 3/4″ (middle), & 5 5/8″ (bottom)

The same comparisons, now with the 1st string are shown in FIG. 7. All rims give a substantial peak at the fundamental frequency, 293 Hz (D4), and the relative strengths of successive peaks in the three cases are... just different.

VIII. DIFFERENT BACKS

The "synthetic belly" was introduced to facilitate controlled, reproducible measurements. But how good is it at mimicking normal open-back playing? I took pains to match the strongest and lowest head tap tone and also the general spectrum shape from 200 to 450 Hz. What about other frequencies? And, while we're at it, how do different backs compare? The real test is listening, but that yields subjective judgments and impressions that are difficult to communicate in words. This next round of computer analyzed recordings yields challenges

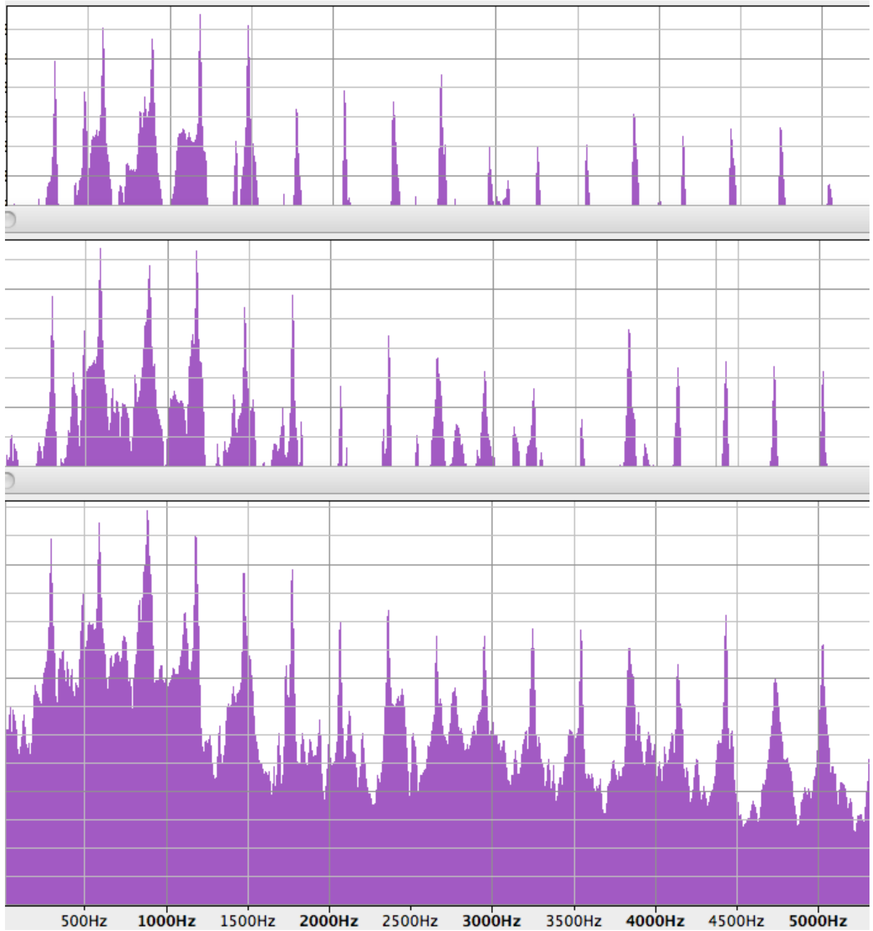


FIG. 7. 1st string spectra: power (6 dB per line) *vs.* frequency (linear scale 0 to 5000 Hz) for 2″ (top), 2 3/4″ (middle), & 5 5/8″ (bottom)

not encountered in the previously displayed comparisons. In those previous comparisons, pains were taken to keep everything fixed except for the one change under scrutiny. Now that's nigh well impossible because one of the cases to be compared is simply holding the banjo naturally, while another case requires holding the banjo away from anything. But here's a shot at it anyway.

The 4th string was plucked repeatedly on the 2 3/4″ Goodtime at the 1/10 scale position with other strings damped with five different backs: 1) completely open to the air; 2) as would be held for normal clawhammer playing; 3) with the synthetic belly at the previously chosen spacing; 4) with a flat birch plywood, snap-on disk resonator of the same diameter and spacing as the synthetic belly; and 5) with a standard resonator banjo back at standard spacing. [For 5) I used an old Goodtime 2 resonator back.]

The problem is that it was impossible to have the relationship between the banjo, the microphone, and the room be the same in all cases. And those differences can appear in what the microphone records. Open to the air meant putting the banjo on a stand. For all the other cases I held the banjo in my lap as for normal playing and moved only for the changing of the backs. The synthetic and birch-ply backs were mounted with air gaps that approximately matched the head tap tone of normal playing. But the resonator back was at its standard spacing. It also directs part of the sound in some directions quite differently from the other backs, even when the whole instrument is held in the same fashion.

FIG. 8 shows the five spectra for comparison. For reference, the lowest frequency peak, visible on all traces, is the 130 Hz (C3). The highest amplitude peak on all is the 654 Hz (E5). With a bit of imagination, you might conclude that the banjo that's least like the others is, indeed, the one with no back at all.

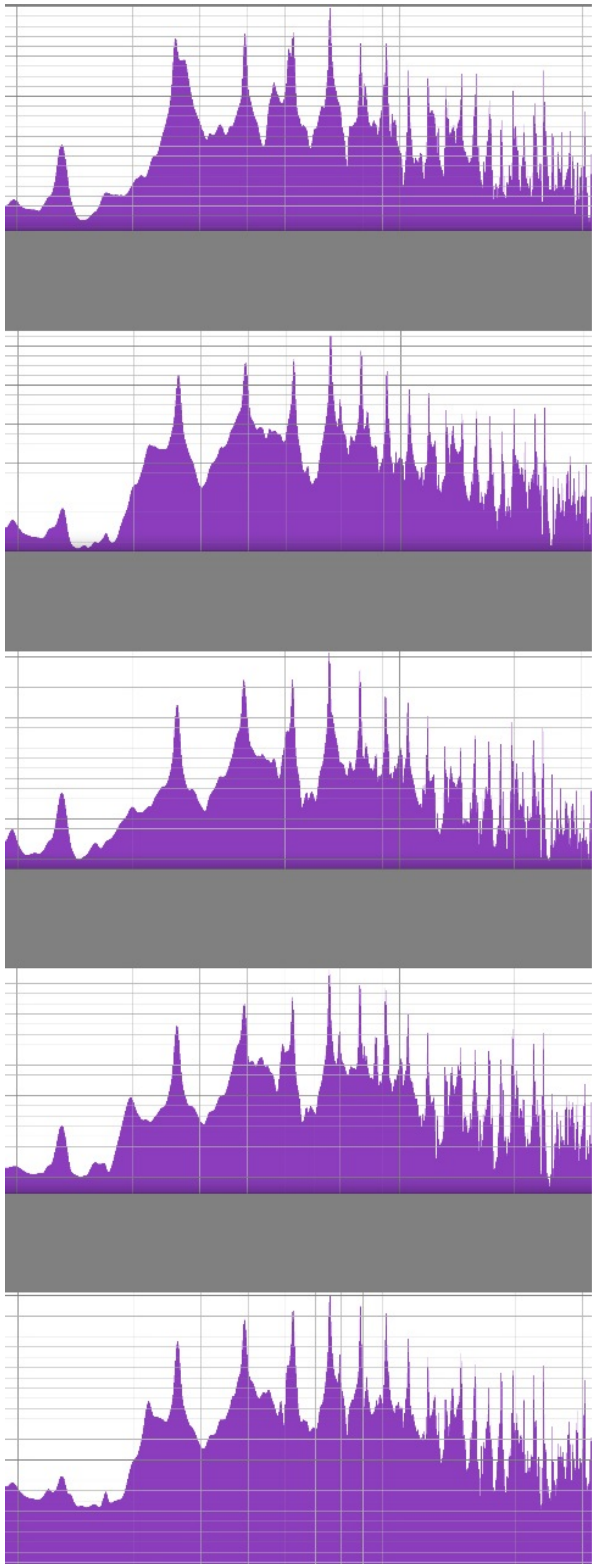


FIG. 8. 4^{th} string spectra for different backs: power (6 dB per line) *vs.* frequency (log scale 100 [first line] to 3000 Hz [last line]) for 2 3/4″: open-back open to the air (top), natural human belly (second), synthetic belly (third), birch-ply flat disk (fourth) & typical resonator back (bottom)

IX. SUSTAIN

The 5 5/8″ rim gives somewhat more sustain than the others, as judged both by the recordings and from direct listening. However, the time development of single, plucked notes with any of the rims is so complex that I will leave its discussion for further study in the future. The physics issues are clear: For a single plucked string, the various resonances and instrument partials decay at different rates, with highest frequencies decaying the fastest. Furthermore, for a given frequency there are at least three very close-by modes. The string has two modes in the transverse plane, and to make any sound at all, these must be coupling to at least one nearby vibrational mode of the pot. These modes have different decay rates and strengths of producing actual sound, and energy sloshes back and forth between them with shifts so slow that the shifts can be heard in real time.

X. QUANTIFYING THE HELMHOLTZ AND AIR-LOADING FREQUENCIES

If the Helmholtz resonance story has any validity, it could be subjected to quantitative measurement and test. This might not yield any further information of practical use regarding matching instrument design to desired tone, but it can serve to support a picture and way of thinking that might at least be a guide to design and playing. Three was the minimum number of banjos needed to do such an analysis, and it seemed inappropriate to start with more without knowing at the outset the likelihood of success. In this section, I take the measured Helmholtz and air-loaded drum lowest frequencies of the three Goodtimes (that's six measured numbers) and show how simple physics relates them. Those equations have three parameters that I am unable to determine from first principles but must fit, using the six measured frequencies as input. That uses up three, leaving the three others constrained by the equations. In fact, one of the three equation parameters initially deduced from the measured six frequencies turns out to be very close to what one would, indeed, naively estimate from first principles. So, a more positive spin on the success of the simple physics would be to claim that six measured frequencies can be well accounted for by equations with only two adjustable parameters. — That's not bad.

The interaction of the Helmholtz resonance of the acoustic guitar with the lowest flexural mode of the sound board (or body top) is described in many places (e.g., §9.5 of *The Physics* *of Musical Instruments*, N. Fletcher & T. Rossing, Springer (2010).). The banjo pot has a similar story, but some aspects are harder to access (e.g., involving burying an open back in the sand) while others open new possibilities, e.g., dramatic changes in the sound hole size.

The ideal Helmhotz resonator can be pictured as a large, air-filled bottle with a small, cylindrical neck, as sketched in FIG. 9.

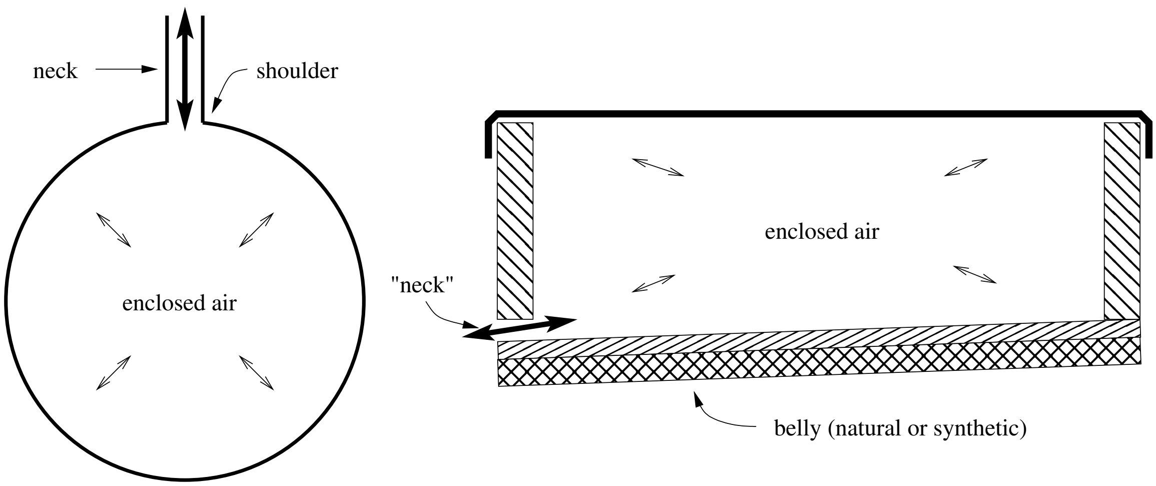


FIG. 9.

The air inside can expand and contract periodically, and it pushes the air in the neck back and forth. Because the bottle volume is hugely greater than the neck volume, a tiny fractional expansion of the air in the bottle pushes the neck air a significant distance. This whole system is to be thought of as akin to an ideal mass on an ideal spring. The "mass" is the mass of the air in the neck that is pushed back and forth. The "spring" is the springiness of the air in the bottle under compression and rarefaction. The dimensions of the bottle can be measured with a ruler. The springiness of air and its density are well known and enter into the calculation of the speed of sound. Using these, the frequency of oscillation can be predicted as an exercise in elementary physics.

There are several caveats. Although the bottle shape is irrelevant, its dimensions must be small compared to the resulting wavelength of the produced sound. The neck air volume must be relatively small *versus* the whole bottle. The only neck geometry that enters is its contact area with the bottle body. Neck shape is irrelevant — unless it becomes too skinny or too jagged. Those are cases where the motions of air through the neck create significant turbulence and friction. Those are forces not included in the simplest model. They are also forces that you want to minimize in a musical instrument: turbulence because it makes hissing noise and friction because it eats up energy that could otherwise have gone into sound. You can hear some of this in the head taps as an open-back banjo is angled away from the body — as in section III. The earliest notes aren't extra low. They're muffled.

A bottle with a well-defined neck and shoulders satisfies those criteria. With a ruler and knowledge of the speed of sound, you can get a reasonable number for its Helmholtz frequency, which is the note produced by blowing across the opening.

Ocarinas, violins, guitars, and banjos (with human, cork, or wooden backs) are said to have important Helmholtz resonances. However, calculation of their frequencies faces one big obstacle: the neck geometry. One can measure the contact area; that's just the area of the sound hole. But you also need to assess the mass of the "neck" air that's being pushed back and forth. The wall of the instrument is thin compared to the dimensions of the sound hole (thought of as an area). Surely, more air is being pushed back and forth than just the air contained in the volume defined by the sound hole area times the wall thickness. But the involvement of outside air is ambiguous and very hard to estimate. Imagining an open-back banjo as literally having an open back was daunting for serious physicists. Thought of as a "bottle," its neck was as wide as the bottle itself, *and* it was open to the ambient, outside air.

There is a similar but much simpler problem that has long been discussed in the physics of musical instruments: the pitch of a cylindrical pipe (ends open, closed, or one of each) that is long compared to its diameter. Using only the pipe length and the speed of sound, simple arguments yield remarkably accurate predictions for the observed pitches. The diameter or cross sectional shape of the pipe is irrelevant. And this works for PVC tubing as well as for first class instruments (as long as they have straight, cylindrical bores).

Under close scrutiny, it turns out that the pipe diameter does matter — just a bit. You can hear it yourself if you pay attention. For a given piece of pipe, the pitch heard (by blowing across one open end) is supposed to be an octave higher if the other end is open, as compared to closing the other end. But the open-open pitch is audibly just a bit flat compared to the octave lower open-closed. This is known as the "end correction." For an open end, the tube length is observed to be effectively longer than its actual length by 0.3 to 0.5 times the tube diameter (for cylinders). The open-open tube has two open ends. The open-closed has only one. In earlier generations, great physicists tried to calculate this effect and didn't really succeed. It can't be done with pencil and paper. I suspect that numerical work with modern high-speed computers could produce some plausible results, but it would have to address some thorny issues that are clear upon reflection. The end correction will depend on the shape of the tube, the thickness of the tube wall, loudness of the produced sound, and the means by which the sound is produced (e.g., blowing across the end *vs.* moving a piston carefully at the other end).

Out of a huge number of possible measurements and a large number performed, I present here just two sets of three, chosen as being the ones with the clearest, quantitative interpretation. For the three Goodtimes, I determined the frequency of the lowest peak in the head tap tone with 1) the synthetic belly set at the angled configuration described earlier and 2) with the back wide open to the ambient air and interpret these as the Helmholtz and lowest drum head modes, respectively.

Strictly speaking, these modes are coupled to each other because they both involve forcing some of the same air back and forth. Coupled oscillators exhibit resonant frequencies that are combinations of the abstract, separate oscillators, and the exact combination reflects the strength of the coupling. (Physicists love this stuff.)

The wide open back shouldn't care much about the springiness of the air. Aside from the drum head itself, the air only pushes on other air. The mass of that air is relevant (and unknown *a priori*), but it doesn't contribute much to the springiness because the wavelength of sound at those frequencies is much longer than the banjo rim height. Hence, I interpret the wide-open case as the air-loaded lowest vibrational mode of the drum head.

I interpret the angle-back case as the Helmholtz oscillator. The opening is almost as small as it can be without choking off the in-out air motion. This makes the frequency about as low as it can be and, therefore, as far from the drum head mode as possible.

As to the effective "length" of the Helmholtz "neck," it's certainly about the same for each banjo (with the backs carefully placed with the same spacing). Putting the frequency measurements into the physics equations, one can deduce an effective neck length. In the end, it came out to what one would most reasonably guess, .i.e, a bit longer than the thickness of the rim wall. So the calculation could be presented in the opposite order, inputting that guess at the outset and having one more parameter-worth of predictive power.

In principle, one could damp the head and excite the Helmholtz vibration without any cross talk with the head vibration. I tried blowing across the rim-back gap with compressed air (like a whistle), but that gave very noisy spectra. Nevertheless, their lowest features were at the same frequencies that appear more clearly from the head taps. Another method I tried was to place a very small speaker and microphone inside the pot and drive the speaker with a signal generator. With the head damped, the lowest resonance should be the one of interest. Again, the spectra were consistent with the taps but much noisier. It's likely that spending more on the hardware and taking greater care with the set-up would yield cleaner results. On the other hand, listening to and analyzing taps sounds can be done with a typical computer, with the addition of an external microphone being optional.

The numbers are rough, but here's what I got. With the synthetic belly angled at 3/8″ by the neck down to zero at the tail, the three Goodtimes had resonances at 240, 218, and 161 Hz for the 2″, 2 3/4″, and 5 5/8″ respectively. The oscillation frequency f of a mass m on a spring with stiffness or springiness k is $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Because the sound hole "neck" area and volume are the same for all three, the sloshing mass of the air of the "neck" is the same for all three banjos. Because the pot inner diameters are the same, their volumes are in ratios of their rim heights. And, since the springiness k of the enclosed air is inversely proportional to that volume, the ratio of the frequencies are predicted to be equal to the the inverse of the square root of the ratios of the rim heights:

$$\frac{240}{218} \approx 1.10 \text{ while } \sqrt{\frac{2\frac{3}{4}}{2}} \approx 1.17 \qquad \text{and} \qquad \frac{218}{161} \approx 1.35 \text{ while } \sqrt{\frac{5\frac{5}{8}}{2\frac{3}{4}}} \approx 1.43$$

The effective length of the sound hole "neck" cancels out in taking these ratios. However, one can ask what that effective length would have to be for, say, the 2 3/4″ rim to give a frequency of 218 Hz, given the angled back spacing and the rim diameter.

A version of the standard Helmholtz resonance formula is

$$f = \frac{v_s}{4\pi^2}\sqrt{\frac{A}{VL}}$$

where f is the frequency, v_s is the speed of sound, A is the "neck" area contacting the large volume V, and L is the "neck" length. Solving for L and plugging in the dimensions of the banjo and the angled back spacing, I find

$$L \approx 0.64''$$

L is the sum of the rim wall thickness plus a stand-in for however much of the outside air is being moved enough that it must be counted. The measured Goodtime rim wall thickness is about 0.59″. (This is a triumph — definitely less than 0.64″, given the accuracy of the measurements, but not much less.)

I measured head tap frequencies with the rims open wide to the surrounding air: 282, 277, and 244 Hz. The pitch decreases with rim height because more air has to be moved as the drum head vibrates. Within the pot, the air has to move along if it is pushed by the head or adjoining air. Understanding the motion of the air outside the pot in any detail is a challenging problem because the air has the option of moving sideways, i.e., out of the way, rather than just moving further along the length of the forced column. But that distinction suggests a simple model that allows a comparison of the three Goodtimes because they are identical except for the length of that constrained diameter column.

Again, this situation is idealized as a single oscillator with frequency f given by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The air outside the banjo certainly contributes to m, but, even if it can't be calculated or even reasonably estimated, it is essentially the same for all three banjos. Call that common mass M. That air may also contribute somewhat to k, although, for this lowest frequency mode, most of k comes from the tensioned drum head itself. The biggest difference between the banjos is the air inside the pot, whose masses, m_i (for $i = 2''$, 2 3/4$''$, 5 5/8$''$), can be estimated from the dimensions. That air or, more precisely, the differences between those air columns do not contribute significantly to the springiness, k, because the differences in rim heights are much less than the wavelength of sound at the relevant frequencies. That is to say: the air is not doing much compressing inside the pot for the motion that corresponds to the lowest frequency up-and-down mode of the drum head. This leaves us with a prediction for the three Goodtime head tap frequencies:

$$f_i = \frac{1}{2\pi}\sqrt{\frac{k}{M+m_i}}\ .$$

The m_i are easy to calculate, but k and M are unknown. They can be fit from the measured three frequencies, leaving the explanatory power of just one, single more parameter — not much, but better than nothing. I performed a fit to this equation, giving all three frequency measurements equal statistical weight. That's displayed in FIG. 10.

A straight line with two adjustable parameters would come closer to the three points than the fit shown in FIG. 10. Furthermore, the 5 to 15 cm range is nearly all that is practical for an open-back banjo. So the virtue, if any, of the fit I present is not its extrapolation outside that range, which would take it away from the straight line. Rather, I believe we should just take comfort in the fact that a reasonable simplification of the actual physics yields an equation compatible with observations.

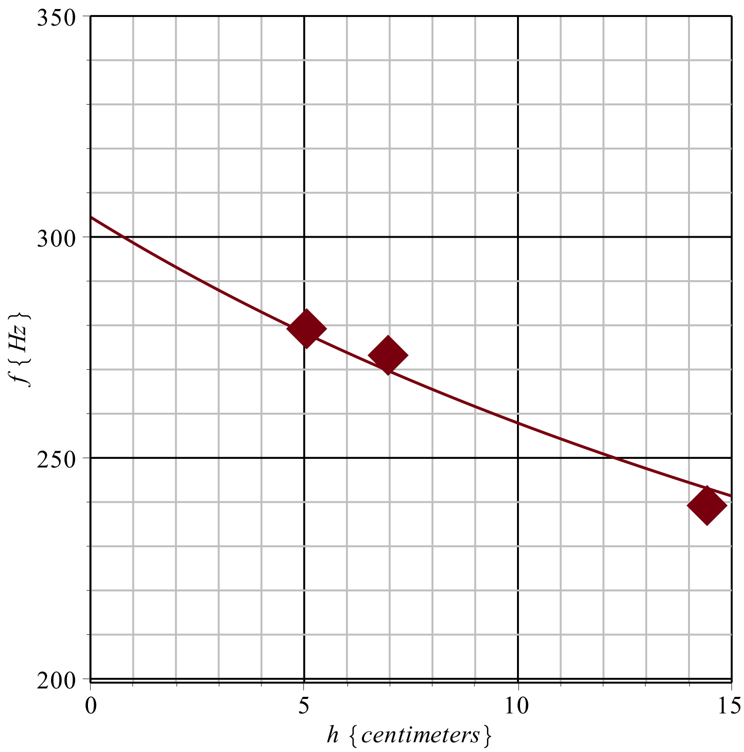


FIG. 10. Two parameter fit to the three observed open back tap tones

XI. LESSONS

The most important lesson here is that the player of an open-back banjo is an active and integral part of the instrument. Adam Hurt (quoted earlier in section II) and certainly many other alert players already knew that. But from that perspective, the height of the rim and its role are clear and totally analogous to large dimensions of the body of any stringed instrument. I focused on the Helmholtz resonance and air loading of the head because those are the easiest to model and quantify. But there can be no doubt that the motion of the air inside the pot as it responds to and, in turn, affects the motion of the head depends on its dimensions. Air motion of a given frequency inside the pot is different for different rim heights. Exploration of the practical range of possible rim heights revealed significant changes in timbre for notes over the whole playable range.

The open back of the open-back banjo and stringed instrument sound holes, in general, are not just places where the sound can come out — even if that's what seems to be going on if you put your ear right there. Sound comes out there, but it also goes back in. And the sound that comes out combines with sound radiated by the rest of the instrument. This involves a lot of wave physics. And a crucial characteristic of waves is that they go up and down (or back and forth). So the combination of sounds can involve cancellations as well as positive additions.

P.S.: There's More Than One Way to Skin a Cat

The boomiest, bassiest banjo I own has a rim that's only 2 1/4″ high. It's a fretless made by Eric Prust (www.chloesgarden.com/Banjos%20on%20Web/Banjos.htm). On the other hand, it's 13″ across and has a skin head that's kept pretty loose. I measured the pitch of the head tap tone when held normally against the body to be about 140 Hz. I tune it at least a fifth below standard, and it's wonderful. The "enclosed" volume, i.e., inside the pot, is the same as if an 11″ diameter rim were about 3 1/8″ in height. However, this is likely not the end of the story. "Air loading" of the head presumably scales as the cube of the diameter, not the square, and that would put the lowest head frequencies yet lower than scaling just from head area...

P.P.S.: If you'd like to write, send an e-mail (see page 1) with banjo in the subject line.

ACKNOWLEDGMENTS

Greg Deering gave his enthusiastic support to this project, not just in word but in deed. He and his crew did a much better job of construction than I could have — and then donated the banjos in the name of Science. I sent out inquiries to people who I thought might have some advice. That I got responses at all was somewhat amazing and certainly encouraging. So thanks goes to Joe Dickey, Rick Heller, David Holt, Jim Rae, and Tom Rossing. And there's no way to adequately thank my wife, Maria, for her enthusiasm for all things banjo and her encouragement, comments, suggestions, and support in this endeavor.

REFERENCES

Joe Dickey, http:www.joedickey.com → PHYSICS → LAY LANGUAGE PAPER ON THE DYNAMICS OF THE BANJO: *The Banjo: the Model Instrument*; *The structural dynamics of the American five-string banjo*, J. Acoust. Soc. Am., **114**(5) 11/2003, p 2958

Jim Rae & Tom Rossing, *How a Banjo Works* (in three parts), Banjo Newsletter; v XXXII, no 1, 11/2004; no 3, 1/2005; no 5, 3/2005; *Banjo, The Science of String Instruments*, ch 5, Springer (2010)

Laurie Stephey & Thomas Moore, *Experimantal investigation of an American five-string banjo*, J. Acoust. Soc. Am. **124** (5) 11/2008, p3276

Rick Heller, *Why You Hear What You Hear*, Princeton U Press (2013)

Roger Siminoff, *How to Set Up the Best Sounding Banjo*, Hal Leonard (1999) and http:www.siminoff.net

The on-line directory of sound files: I'll be setting this up at a later date – when I've written up at least a couple other of the banjo investigations now in progress.

APPENDIX: Measurement techniques, hardware, & notes

I used a middle-aged MacBook Pro and a Samson CO1U USB microphone. I suspect that the built-in Mac microphone would have worked for some but been marginal for other of the variations of interest. The spectrum Fast Fourier Transforms were done with Audacity, matched to the microphone A-to-D bit rate, and FFT sample sizes were chosen to highlight the issues at hand. (A signal generator and oscilloscope salvaged from the garbage [at Caltech we have interesting garbage] and a variety of small sound devices bought from Radio Shack were used along the way but don't really figure in the material as presented here.)

I enjoyed contemplating how the manufacturer's specs on the linearity of a microphone might be checked, but the actual task was beyond me for now.

Room sound (e.g., reflections off the walls, floor, ceiling, furniture) colors the sound you hear and record at all frequencies, not just at the resonances of the room. Those reflections are coherent with the initial sound and arrive at the microphone long before a given note dies out. (So the relative phase between the echo at the mic and the direct sound is crucial.) Without access to an anechoic recording studio, I did try recording in the middle of an outdoor playing field, with the microphone on the grass and the nearest wall-like structure some 200 yards away. Some low resonances observed in my office recordings disappeared and some of the relative peak heights of the spectrum shifted. But I judged it not worth the while.

The frequencies I observed are well-determined by the measuring apparatus. However, they're not really the things of interest. They depend on string tunings and head tension. It's the relative amplitudes or strengths of those frequency components of the sound that I wished to study. Those are, indeed, sensitive to the room and recording conditions. However, I took great care that sounds that were to be compared directly were recorded under as identical circumstances as I could provide. Not a single piece of furniture or equipment (save the banjos themselves) was moved during the process. I positioned myself in the same way, and played the instruments in the same way. So, while the sound in another location would not exactly match the sound in my office, the differences within my office are reasonably attributed to differences in the banjos.

To determine the single string spectra, I marked the string with a felt tip pen at 1/10 the scale length from the bridge and plucked repeatedly with a dental pick thirty or forty times, with the other strings damped. The spectrum I display is the spectrum of the entire run. Actually, analyzing individual plucks gives very similar plots. The head taps were done with a felt piano hammer near the center of the head (with strings damped). The relative strength of excitation of the lowest frequency vibrational modes is much greater this way than when a metal finger pick strikes the head near the rim.