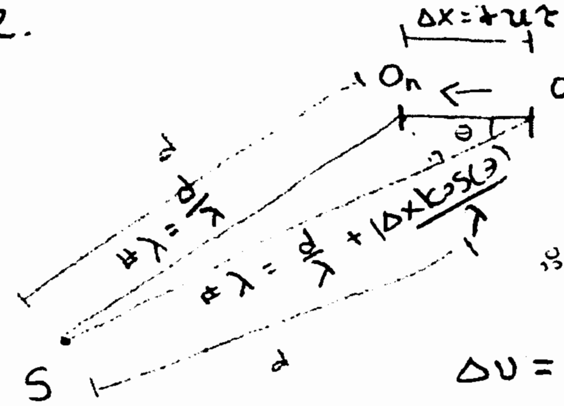
1

8-12.

(a)



$\therefore$ an observer moving from O_o to O_n with speed v in time τ will get $\frac{|\Delta x|\cos(\theta)}{\lambda}$

$\therefore \boxed{\Delta \# \lambda = \frac{|\Delta x|\cos(\theta)}{\lambda}}$ //

$$\Delta \nu = \frac{\Delta^{\#}\lambda}{\tau} = \frac{|\Delta x|\cos(\theta)}{\tau\lambda} = \frac{u\cos(\theta)}{\lambda}$$

/5

$$\therefore \nu' = \nu_0 + \frac{u\cos(\theta)}{\lambda}$$ $\frac{1}{\lambda} = \frac{\nu_0}{V}$ frequency (source) / velocity

$$= \nu_0\left[1 + \frac{u\cos(\theta)}{V}\right] \text{//}$$

(b)

$$\nu - \nu' = \frac{\nu_0}{1 - \frac{u\cos(\theta)}{V}} - \nu_0 - \frac{\nu_0 u\cos(\theta)}{V}$$

/5

$$= \nu_0 \frac{1 - 1 + \frac{u\cos(\theta)}{V} - \frac{u\cos(\theta)}{V} + \frac{u^2\cos^2(\theta)}{V^2}}{1 - \frac{u\cos(\theta)}{V}}$$

$$= \frac{u^2\cos^2(\theta)}{V^2\left(1 - \frac{u\cos(\theta)}{V}\right)} = \frac{u^2\cos^2(\theta)}{V(V - u\cos(\theta))}$$

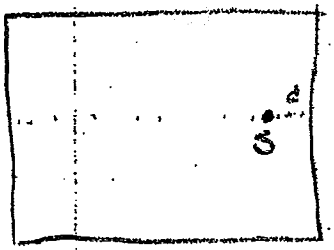
if $u\cos(\theta) << V$

observer moving slow wrt speed of wave then

$$\boxed{\nu - \nu' \approx \nu_0 \frac{u^2\cos^2(\theta)}{V^2}} \text{//}$$

8-14

a) Consider 1-D case wave acting a straight line



Wave Eqn for that line must be

$$y(x,t) = f_I\left(t - \frac{x}{v_1}\right) + f_R\left(t + \frac{x}{v_1}\right)$$

at the wall the wave eqn $= 0$ for all time, if we let $x=0$ at wall without loss of generality

$$y(0,t) = f_I(t) + f_R(t) = 0$$

$$\Rightarrow f_I(t) = -f_R(t)$$

Then we can say

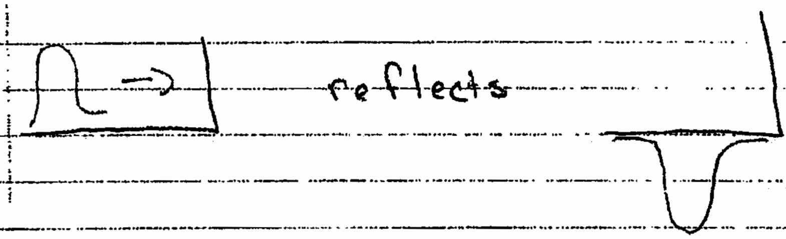
$$y(x,t) = f_I(x - v_1 t) - f_I(x + v_1 t)$$

or that $\quad g_1 = \dfrac{v_2 - v_1}{v_2 + v_1} \quad v_2 \to 0 \quad = -f_I$

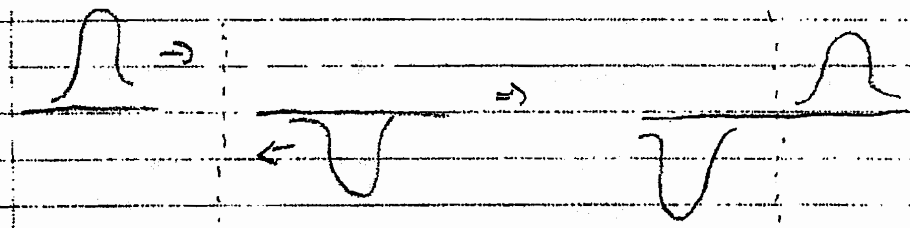
So reflected wave is just $-f_I(x + v_1 t)$

if we assume a sinusodal wave, then we can produce this wave with a source behind the wall that 180° out of phase that is twice the distance from origenal source.

ex

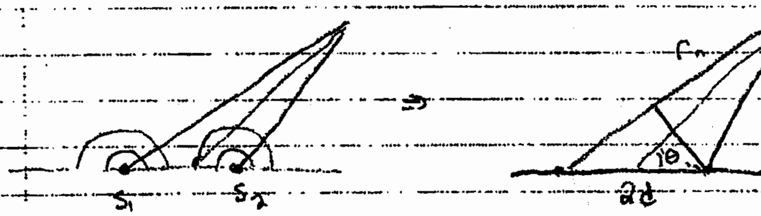


now imagine no wall and two pulse



so this gives us a second source, and with sinusodal waves you can achieve (−) sign with the 2nd wave being 180° out of phase.

This holds true for 1-D, now carry this to two-D, since the 2-D must reduce to previous part for that line.



$$r_n - r_0 = 2d\sin\theta$$

$2d\sin\theta$ is the extra distane the 1 wave must travel extra ⇒ so from part a

so $y(x,t) = A_0\left[\cos\omega\left(t - \frac{r_0}{v}\right) + \cos\omega\left(t - \frac{r_0}{v} - \pi\right)\right]$

π : from out of phase (brought (−) inside cosine) double angle formulas

$\cos a + \cos b$

$= 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

so

$$y(x,t) = 2A_0 \cos(\omega t)\cos\left(\frac{\omega}{2v}(r_2 - r_1) - \pi\right)$$

and if $\lambda = 2\pi v/\omega = v/\nu$

$$y = 2A_0 \cos\omega t \cos\left[\frac{\pi(r_n - r_1)}{\lambda} - \pi/2\right]$$

$$\Rightarrow y = 2A_0 \cos\omega t \sin\left[\frac{\pi(r_n - r_1)}{\lambda}\right]$$

this differs from book in cos vs. sin

so nodal lines occur at $r_n - r_1 = n\lambda$

interference maximum occur at $r_n - r_1 = (n + 1/2)\lambda$

$r_n - r_1 = 2d\sin\Theta$

so destructive interference

$$\sin\Theta_n = \frac{n\lambda}{2d}$$

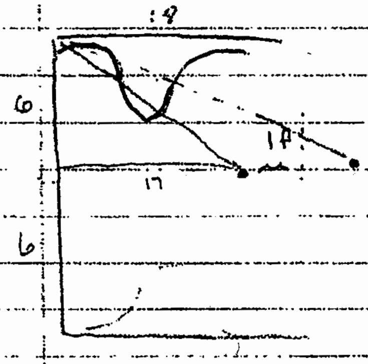
maxima $$\sin\Theta_n = \frac{(n+1/2)\lambda}{2d}$$

so interference pattern due to reflection

Question ambiguous:

1/3 i) Two peaks (set up # 1)

$r_n - r_o = \lambda(n + 1/2)$ $c = 340 m/s$

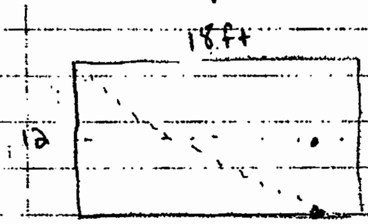


$r_o \cong 18$ $r_n \cong 19.9 ft$

$\frac{\lambda}{2} = 1.9 ft \cong .58 m$

$\frac{c}{\lambda/2} = \nu \cong 1200 Hz$

~~set up # 2~~



$r_o = 20.8$

$r_n = 22.5$

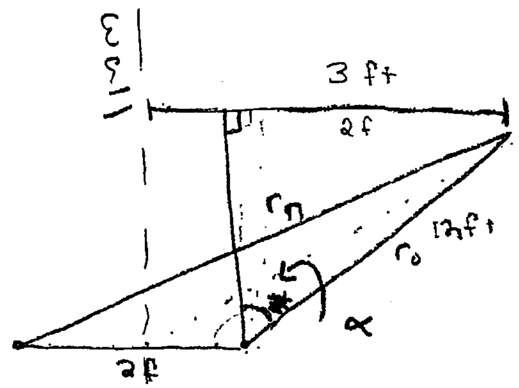
$r_o - r_n \cong 1.7 ft$

In this case we want two peaks or two nodal lines (so can see exactly 2 interference fringes).

$\Rightarrow$ $r_n - r_o = 2\lambda$ if $c = 340 m/s$

$\lambda = .85 ft \cong .26 m$

so $\nu = 1307 \cong 1300$

ii) 

1/3

$r_n - r_0 = \lambda n$ (from part a: destructive interference)

$r_n - r_0 = n\lambda$

but $\alpha = \sin^{-1}(2/12) \cong 9.6°$

so $r_n = [12^2 + 2^2 - 2(12)(2)\cos($[redacted]$)]^{1/2}$ $\frac{\pi}{2} + \alpha$

$r_n \approx 12.5$

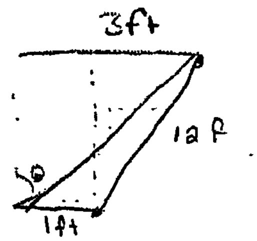
$r_n - r_0 = 12.5\,ft - 12\,ft = .5\,ft \approx .1524\,m$

$\nu \approx \frac{c}{\lambda} = \frac{340}{.1524\,m} \cong 2230$

ii) so $\nu \geq 2200$ Hz

or $2d\sin\theta = n\lambda$

$\lambda = 2(1ft)\sin\theta$



$\tan\theta = \frac{O}{A}$ $O = 3ft$

$A = (12^2 - 2^2)^{1/2} \approx 11.8$

$\lambda = 2ft\sin(\arctan(\frac{3}{11.8})) \approx .5ft$

so $\nu \approx 2200Hz$ integer multiples of

[redacted]

[redacted]

#8.15 (a) $\sin(\theta_n) = \frac{n\lambda}{d}$ $\quad \sin(\theta_{max}) = 1$

/5

$\therefore \frac{n_{max}\lambda}{d} = 1 \quad n_{max} = \frac{d}{\lambda} = \frac{0.05\,mm}{5000\,Å} = \boxed{100}$

(b) $\frac{A}{A_0} = \frac{\sin(N\delta/2)}{\sin(\delta/2)} \quad \delta = \frac{2\pi d \sin(\theta)}{\lambda}$

$\therefore \frac{A}{A_0} = \frac{\sin(100N\pi\sin(\theta))}{\sin(100\pi\sin(\theta))}$

(c) recall the double slit result was

$A/A_0 = 2\cos\left(\frac{\pi d \sin(\theta)}{\lambda}\right)$

if we take $N=2$ in (b) then

$A/A_0 = \frac{\sin(2(100\pi\sin(\theta)))}{\sin(100\pi\sin(\theta))}$

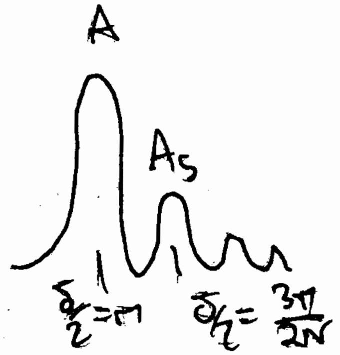
Since $\sin(2A)$ $\quad 2\sin(A)\cos(A)$

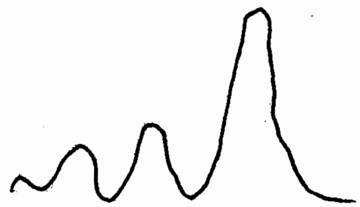
$A/A_0 = 2\cos(100\pi\sin(\theta))$

since $d/\lambda = 100$ this agrees w doubleslit

/5

(d)





$A_i = A_0 \dfrac{\sin(N\delta_i/2)}{\sin(\delta_i/2)}$ A_i is max when $\delta_i/2 = \pi$ note this is okay $N\frac{\delta_i}{2} = N\pi$ we have 0/0 but you can show that this number is finite

1^{st} max to right will occur when $\frac{N\delta_i}{2} = N\pi + \frac{3\pi}{2}$ (pg#3 on 287)

$$\frac{\delta_i}{2} = \pi + \frac{3\pi}{2N}$$

$$A_s = A_0 \frac{\sin(N\pi + \frac{3\pi}{2})}{\sin(\pi + \frac{3\pi}{2N})} = A_0 \frac{\sin(N\pi)\cos(3\pi/2) + \cos(N\pi)\sin(\frac{3\pi}{2})}{\sin(\pi)\cos(\frac{3\pi}{2N}) + \sin(\frac{3\pi}{2N})\cos(\pi)}$$

$$= A_0 \frac{1}{\sin(\frac{3\pi}{2N})}$$

$\therefore \frac{A_s}{A_0} = \left[\sin\left(\frac{3\pi}{2N}\right)\right]^{-1} \approx 20$

since $A = A_0 N$ $\therefore$ $\boxed{\frac{A_s}{A} = \frac{1}{100}\frac{A_s}{A_0} = \frac{1}{5}}$ //

QP3

8

[redacted]

1 Evaluate ω_p. [2 Points]

$$\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0} = \frac{(2 \times 10^{-8} m^{-3})(1.6 \times 10^{-19} C)^2}{(9.11 \times 10^{-31} kg)(8.85 \times 10^{-12} \frac{F}{m})} = 6.3505 \times 10^{-5} \frac{C^2}{m^2 kg F} \quad (1)$$

$$Units : 1 \frac{C^2}{m^2 kg F} = 1 s^{-2} \quad (2)$$

$$\omega_p = .0007969 Hz \quad (3)$$

2 Find the phase velocity v_{ph}. [2 Points]

So we begin by looking at the dispersion relations we've been given $\omega^2 = \omega_p^2 + k^2 c^2$

$$v_{ph} = \frac{\omega}{k} = \frac{\sqrt{\omega_p^2 + k^2 c^2}}{k} = \frac{1}{k} \sqrt{\omega_p^2 + k^2 c^2} = \sqrt{\left(\frac{\omega_p}{k}\right)^2 + c^2} \quad (4)$$

3 Find the group velocity v_g. [2 Points]

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk} \sqrt{\omega_p^2 + k^2 c^2} = \frac{1}{2} (\omega_p^2 + k^2 c^2)^{-1/2} (2kc^2) = \frac{kc^2}{\sqrt{\omega_p^2 + k^2 c^2}} \quad (5)$$

4 Find the time difference in arrival times of two pulses moving through interstellar space with ω_1 = 1GHz and ω_1 = 2GHz. [2 Points]

First we begin by putting the v_g in terms of ω. We use v_g because this is the speed at which a pulse will travel.

$$v_g = \frac{kc^2}{\sqrt{\omega_p^2 + k^2 c^2}} \quad (6)$$

We now solve for k using the dispersion relation and plug into (6). $k = \frac{\sqrt{\omega^2 - \omega_p^2}}{c}$

$$v_g = \frac{\sqrt{\omega^2 - \omega_p^2} c}{\omega} = \frac{1}{\omega} \sqrt{\omega^2 - \omega_p^2} = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} c \quad (7)$$

We now use our last equation for v_g to find the time difference in arrival times of the two pulses across $D = 3 \times 10^{20}$. Now we just use $D = v_g t$.

t_1: is the time is takes the pulse at 1 GHz to travel D.

t_2: is the time is takes the pulse at 2 GHz to travel D.

$$\Delta t = t_1 - t_2 = \frac{D}{v_{g1}} = \frac{D}{v_{g2}} = D\left(\frac{1}{v_{g1}} = \frac{1}{v_{g2}}\right) = \frac{D}{c}\left[\left(1 - \left(\frac{\omega_p}{\omega_1}\right)^2\right)^{-1/2} - \left(1 - \left(\frac{\omega_p}{\omega_2}\right)^2\right)^{-1/2}\right] \quad (8)$$

Now the trick here is that if we were to calculate this outright, it would extremely small. Best thing to do to get an answer that makes sense is to taylor expand our group velocity such that $(1-x)^n \approx 1 - nx$ for $x << 1$. Since $\omega_p << \omega_{1or2}$ then this approximation will work very well.

$$D/c\left[1+\frac{1}{2}\left(\frac{\omega_p}{\omega_1}\right)^2 - 1 - \frac{1}{2}\left(\frac{\omega_p}{\omega_2}\right)^2\right] = \frac{D}{2c}\left[\left(\frac{\omega_p}{\omega_1}\right)^2 - \left(\frac{\omega_p}{\omega_2}\right)^2\right] = \frac{D\omega_p}{2c}\left[\left(\frac{1}{\omega_1}\right)^2 - \left(\frac{1}{\omega_2}\right)^2\right] \quad (9)$$

$$= \frac{(3\times 10^{20}m)(6.35\times 10^{-5}Hz^2)}{2(3\times 10^8)}\left[\frac{1}{(10^9Hz)^2} - \frac{1}{(2\times 10^9Hz)^2}\right] = 2.3814\times 10^{-11}Seconds \quad (10)$$

5 Telescope has 305 m diameter. Find the angular resolution of the telescope for the 1 GHz Pulse [2 Points]

$$\omega = 2\pi f = 2\pi\left(\frac{v_1}{\lambda}\right) \quad (11)$$

$$\lambda = \frac{2\pi v_1}{\omega} = \frac{2\pi\sqrt{1-\left(\frac{\omega_p}{\omega_1}\right)^2}c}{\omega_1} \quad (12)$$

$$\theta = \frac{\lambda}{d} = \frac{2\pi\sqrt{1-\left(\frac{\omega_p}{10^9}\right)^2}c}{305\omega_1} = .0062Rads \quad (13)$$