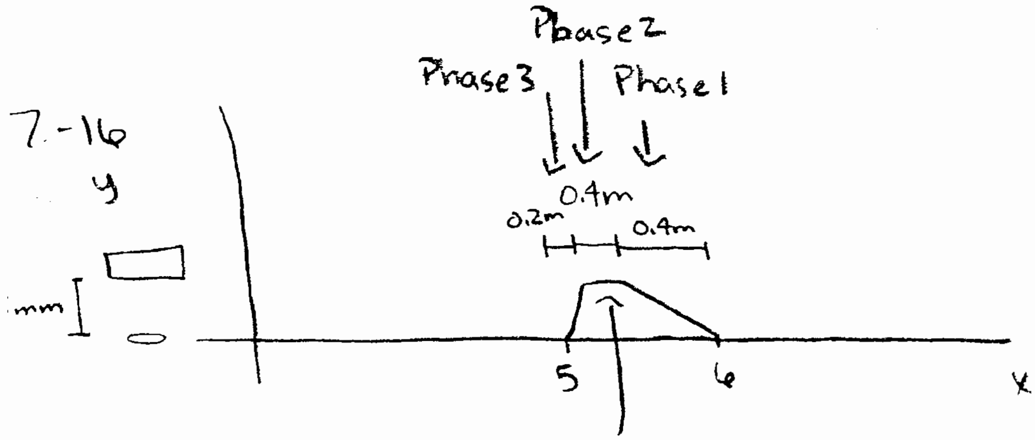
Ph 2a 2011 HW 4 Sol^n

7-16



(a)

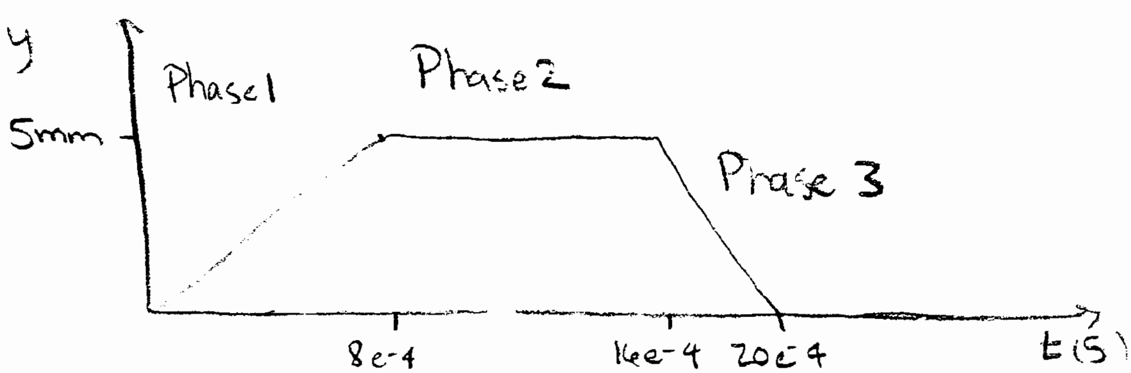
1/2 $v = \left(\frac{T}{\mu}\right)^{\frac{1}{2}}$ $\mu = \frac{M}{L} = \frac{5\,g}{12.5\,m} = \frac{2}{5 \cdot 1000} = \frac{1}{2500}$

$T = mg = 10kg \cdot 10\,m/s^2 = 100\,N$

$v = \sqrt{\frac{100}{1/2500}} = 500\,m/s$

$\therefore\ t = \frac{d}{v} = \frac{0.4\,m}{500\,m/s} = \boxed{8e\text{-}4\,s}$ //

(b) Phase 1 happens first $t = 8e\text{-}4\,s$ $\Delta y = 5\,mm$
Phase 2 " next " " $= 0\,mm$
1/3 Phase 3 " " $t = 4e\text{-}4\,s$ " $= -5\,mm$



2

(c) Phase 1 $v = \frac{d}{t} = \frac{5mm}{8e\text{-}4s} = 6.25 \frac{m}{s}$

/3

" 2 $v = \frac{0}{8e\text{-}4s} = 0 \frac{m}{s}$

" 3 $v = \frac{-5mm}{4e\text{-}4s} = \boxed{12.5 \frac{m}{s}}$ // V_{max}

Phase 3 was opening //.

(d) The beginning of Phase 1 has travelled 6 m's ∴

/2

$t = \frac{d}{v} = \frac{6m}{500m} s = \boxed{12 ms}$ //

7-18 (a) if $V_p = \left(\frac{2\pi S}{\rho \lambda}\right)^{\frac{1}{2}}$ we want in the form of ν/k

/3

$= \left(\frac{S}{\rho} k\right)^{\frac{1}{2}} = \left(\frac{S}{\rho}\right)^{\frac{1}{2}} k^{\frac{1}{2}}$

∴ $\nu = \left(\frac{S}{\rho} k^3\right)^{\frac{1}{2}}$ then $\nu_g = \frac{d\nu}{dk} = \frac{3}{2} k^{\frac{1}{2}} \left(\frac{S}{\rho}\right)^{\frac{1}{2}} = \frac{3}{2}\left(\frac{Sk}{\rho}\right)^{\frac{1}{2}}$

$= \frac{3}{2}\left(\frac{S 2\pi}{\rho \lambda}\right)^{\frac{1}{2}}$

$V_p = \frac{\nu}{k} = \left(\frac{S}{\rho} k\right)^{\frac{1}{2}}$ //

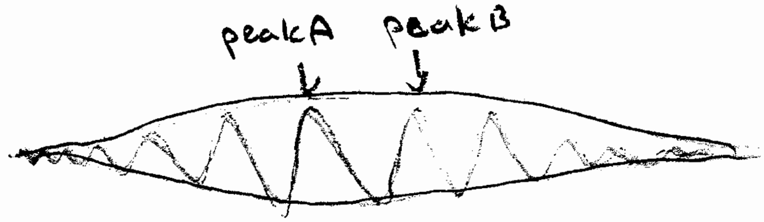
$= \frac{3}{2} \nu_g$ //

for wavelength close to λ

(b) This implies that the group of ripples will be travelling faster then

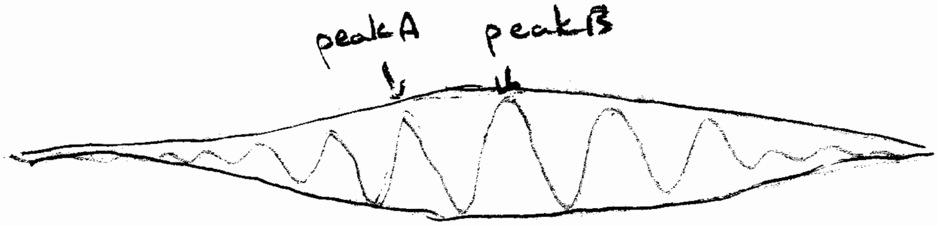
/4 any given ~~wave~~ ripple

ex



The velocity of the group (envelope) is higher then the actual motion of the component ~~waves~~ ripples

at t+dt



the ~~wave~~ ~~peaks~~ ripples move slower then the envelope.

$$V_p < V_g$$

(c)

$$V_{ripple} < V_{group}.$$

/3 recall

$$y = 2A\cos\underbrace{(\pi\Delta k - \pi t\Delta\nu)}_{\text{group}}\ \underbrace{\sin(2\pi kx - 2\pi \nu t)}_{\text{phase.}}$$

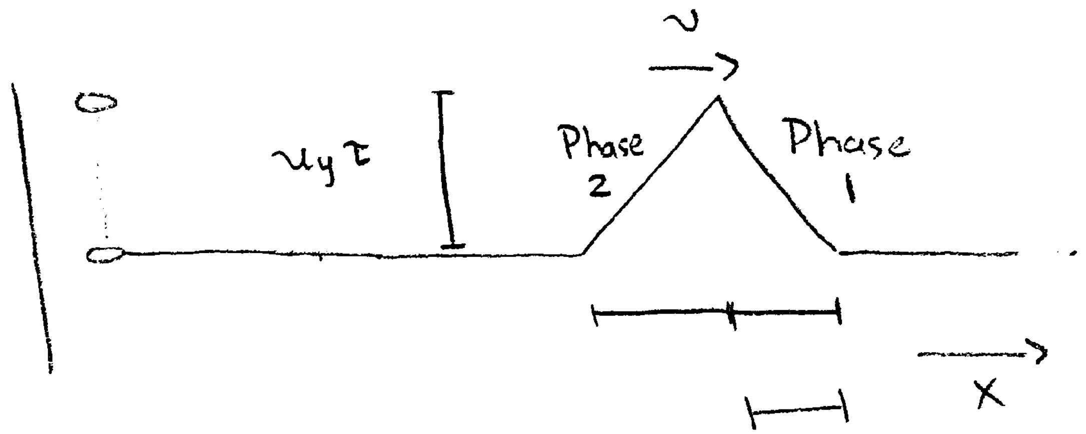
the group wavelength is $\frac{\Delta k}{2\pi} = \frac{1}{\lambda_g}$

$$\Delta k = \frac{2\pi}{\lambda_1} - \frac{2\pi}{\lambda_2} = 2\pi\left[\frac{1}{0.99} - \frac{1}{1.01}\right] = \frac{2\pi}{50\,cm} \quad //.$$

4.

7-23.

/10



Phase 1

$v_p = u_y \qquad KE = \int_0^{\Delta x} dx \left[\frac{1}{2}\mu (u_y)^2 \right]$

$= \frac{1}{2}\mu v \tau (u_y)^2$

Phase 2

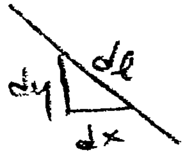
$v_p = -u_y \qquad KE = \int_0^{\Delta x} dx \left[\frac{1}{2}\mu (u_y)^2 \right]$

$= \frac{1}{2}\mu v \tau (-u_y)^2$

KE total = $\boxed{\mu v \tau u_y^2}$ //

$U = \int (dl - dx) T \qquad \int \left(dx \left(\sqrt{1 + \left(\frac{u_y}{v}\right)^2} \right) - dx \right) T$

$\approx \int dx \frac{u_y^2}{v^2} \frac{1}{2} T = \frac{1}{2} \frac{u_y^2}{v^2} T v \tau$



b/c the $(-u_y^2) = u_y^2$

Note $v^2 = \frac{T}{\mu}$

$dl^2 = dx^2 + dy^2$

$= dx^2 \left(1 + \left(\frac{dy}{dx}\right)^2 \right)$

$U_1 + U_2 = \boxed{u_y^2 T v \tau \, \mu / T}$ //

$dl = dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad$ but $\frac{dy}{dx} = \frac{u_y}{v}$, slope

5.

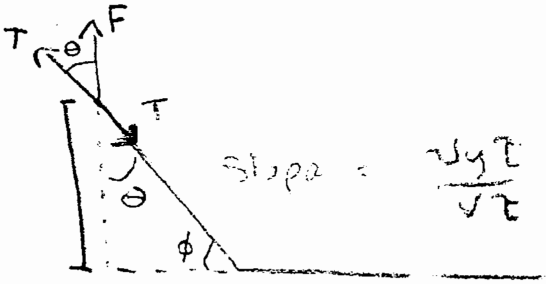
$$E_{tot} = KE + U$$

$$= \mu v \tau u_y^2 + u_y^2 v \tau \mu$$

$$= 2\mu v \tau u_y^2 \,//$$

(ii)

$dy = u_y \tau$



$$W = T \cdot dy = F\,dy = T\cos(\theta)\,dy = T\sin(\phi)\,u_y \tau$$

if ϕ is small (like we assumed in (a))

$$\sin(\phi) \approx \phi \approx \tan(\phi) = \frac{dy}{dx} = \frac{u_y}{v}$$

$W = \frac{T u_y^2 \tau}{v}$, the bending of the rope requires equal work

$$W_{tot} = \frac{2T u_y^2 \tau}{v} = \frac{2T u_y^2 \tau v}{v^2} = 2u_y^2 \tau v \mu \,//$$ as in (a).

but $v^2 = \frac{T}{\mu}$

8.1 $\qquad \mu_1 \qquad\qquad \mu_2$

$$\frac{A_r}{A_i} = \frac{g_1(t)}{f_1(t)} \qquad \frac{A_t}{A_i} = \frac{f_2(t)}{f_1(t)}$$, using French's notation.

note. $\dfrac{v_1}{v_2} = \dfrac{(T/\mu_1)^{1/2}}{(T/\mu_2)^{1/2}} = \left(\dfrac{\mu_2}{\mu_1}\right)^{1/2}$. $\qquad \dfrac{g_1(t)}{f_1(t)} = \dfrac{v_2 - v_1}{v_2 + v_1}$

(i) $\dfrac{\mu_2}{\mu_1} = 0 \qquad \mu_2 = 0 \qquad v_1 = 0 \qquad \dfrac{f_2(t)}{f_1(t)} = \dfrac{2v_2}{v_2 + v_1}$

1/2

$$\frac{A_r}{A_i} = \frac{v_2 - 0}{v_2 + 0} = 1$$

$$\frac{A_t}{A_i} = \frac{2v_2}{v_2} = 2 \quad //.$$

(ii) $\dfrac{\mu_2}{\mu_1} = 0.25 \quad v_1 = v_2\sqrt{\tfrac{1}{4}} = \dfrac{v_2}{2}$

1/2

$$\frac{A_r}{A_i} = \frac{v_2 - v_2/2}{v_2 + v_2/2} = \frac{v_2/2}{3v_2/2} = \frac{1}{3}$$

$$\frac{A_t}{A_i} = \frac{2v_2}{v_2 + v_2/2} = \frac{2}{3/2} = \frac{4}{3} \quad //.$$

(iii) $\frac{\mu_2}{\mu_1} = 1 \quad v_1 = v_2$

1/2 $\frac{A_r}{A_i} = \frac{v_2 - v_2}{v_2 + v_2} = 0$

$\frac{A_t}{A_i} = \frac{2v_2}{v_2 + v_2} = \cancel{2}\ 1$ //

(iv) $\frac{\mu_2}{\mu_1} = 4 \quad v_1 = 2v_2$

1/2 $\frac{A_r}{A_i} = \frac{v_2 - 2v_2}{v_2 + 2v_2} = -1/3$

$\frac{A_t}{A_i} = \frac{2v_2}{v_2 + 2v_2} = 2/3$ //

1/2 (v) $\frac{\mu_2}{\mu_1} = \infty \quad v_1 \gg v_2$

$\frac{A_r}{A_i} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{-v_1}{v_1} = -1$

$\frac{A_t}{A_i} = \frac{2v_2}{v_2 + v_1} = \frac{2v_2}{v_1} << 1 \approx 0$ //