HW # 6 Solutions

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Phillips 1-6 $\Delta p > \frac{h}{\Delta x}$ $\Delta x \sim 0.53\,nm$ (Bohr radius)

/10 $\Delta p > 1.25e{-24}\ \frac{m\,kg}{s}$

if $\Delta p \sim p$

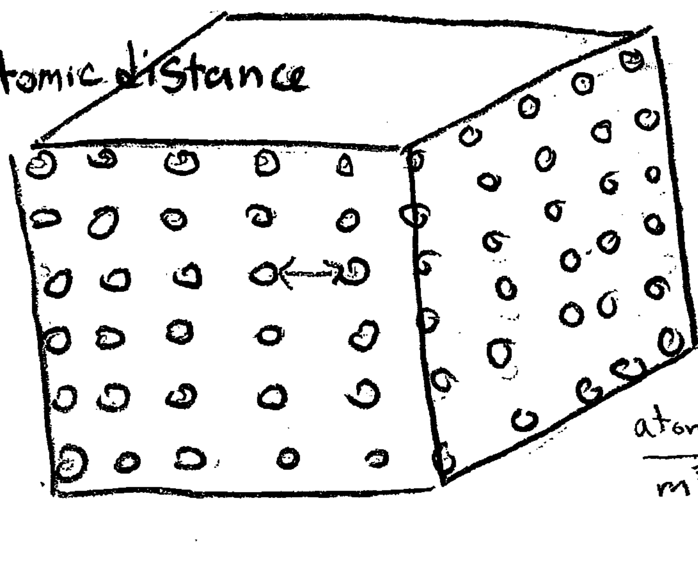
$KE = \frac{p^2}{2m_e} = 5.4\,eV$ // very close to the binding energy.

1-10 The de Broglie wavelength is

/10 $\lambda = \frac{h}{\sqrt{2mE}}$ with $E = 7eV$

$= 0.464\,nm$

Interatomic distance



$1m^3 = 8.9\times10^3\,kg$
$1mol = 60g$

$density = \frac{8.9e3\,kg\cdot mol}{0.06\,kg\cdot m^3}$

$\frac{atoms}{m^3} = \frac{8.9e3\,mol\cdot 6.02e23\,ato}{mol\cdot 0.06\,m^3}$

Inter atomic spacing

$$l = \sqrt[3]{\frac{m^3}{atom}} = \sqrt{\frac{0.06\,m^3}{8.9e3 \cdot 6.02e23 \text{ atoms}}}$$

$$= 0.22 \text{ nanometers}$$

The wavelike properties are very important as $\lambda \sim$ interatomic spacing.

Assignment #6

2-3.

/10 (i) $\psi = A\cos(kx - \omega t)$

$$S.E = i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \cancel{V(x)\psi}$$

0 b/c free particle

$$\frac{\partial \psi}{\partial t} = A\omega \sin(kx - \omega t)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -Ak^2 \cos(kx - \omega t)$$

clearly $i\hbar A\omega \sin(kx - \omega t) \neq +\frac{\hbar^2}{2m} A k^2 \cos(kx - \omega t)$

b/c LHS is imaginary and RHS is real
for arbitrary x and t

similerly $\psi = A\sin(kx - \omega t)$

$$\frac{\partial \psi}{\partial t} = -A\omega \cos(kx - \omega t)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -Ak^2 \sin(kx - \omega t)$$

and $i\hbar A\omega \cos(kx - \omega t) \neq +\frac{Ak^2\hbar^2}{2m}\sin(kx - \omega t)$

again LHS is Imaginary
and RHS is real for orbitrary x, t

these do <u>not</u> satisfy S.E.

/10

2-4. (a)

/6 $$\Psi(x,t) = Ae^{i(kx-wt)} - Ae^{-i(kx+wt)}$$

$$\frac{\partial \Psi(x,t)}{\partial t} = Ae^{i(kx-wt)}(-iw) - Ae^{-i(kx+wt)}(-iw)$$

$$= -iw(\Psi(x,t))$$

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{\partial}{\partial x}\left[Ae^{i(kx-wt)}ik - Ae^{-i(kx+wt)}(-ik)\right]$$

$$= Ae^{i(kx-wt)}(ik)^2 - Ae^{-i(kx+wt)}(-ik)^2$$

$$= (ik)^2\,\Psi(x,t)$$

So S.E becomes $i\hbar(-iw)\Psi(x,t) = \frac{-\hbar^2(ik)^2}{2m}\Psi(x,t)$

$$\hbar w\,\Psi(x,t) = \frac{\hbar^2 k^2}{2m}\Psi(x,t)$$

S.E. is satisfied if $\boxed{\hbar w = \frac{\hbar^2 k^2}{2m}}$ //

(b)

/4 $$\Psi(x,t) = Ae^{i(kx-wt)} - Ae^{-i(kx+wt)}$$

$$= Ae^{-iwt}\left[e^{ikx} - e^{-ikx}\right]$$

$$= A2i\,e^{-iwt}\sin(kx) \quad b/c \quad \frac{e^{ix}-e^{-ix}}{2i} = \sin(x)$$

It is a complex standing wave with wavenumber k + & frequency w