2015.16 PH2b – WINTER TERM

QUIZ 3

Your solutions are due on FEBRUARY 16, 2016 at 11:00am, in the locked section boxes outside 201 E. Bridge. Late quizzes will not be accepted, except in very special circumstances.

The quiz is open book, open lecture, open section notes, open homework sets and solutions. Calculators may be used. Symbolic manipulators, other than your brain are not allowed. Please justify your answers and show all work.

TIME LIMIT: 90 MINUTES

IMPORTANT: Please write your name, your section number and your T.A’s name on the front of your solution sheet.

THERE WILL BE A QUIZ REVIEW ON 
SATURDAY FEBRUARY 13, 2016 AT 1:00PM 
IN 201 E. BRIDGE.
Problem 1) (3 pts)
Let $A$ and $B$ be a pair of commuting operators,

$$[A, B] = 0. \quad (1)$$

Prove the following assertions:

a) [1pt] If $|\psi\rangle$ is an eigenvector of $A$ then $B|\psi\rangle$ is an eigenvector of $A$ with the same eigenvalue.

b) [1pt] If $|\psi_1\rangle$ and $|\psi_2\rangle$ are two eigenvectors of $A$ with different eigenvalues then $\langle\psi_1|B|\psi_2\rangle = 0$.

c) [1pt] If $A$ has distinct eigenvalues, then its eigenvectors are also eigenvectors of $B$.

Problem 2) (3 pts)

If two operators commute then their corresponding observables are compatible: that is, both observables can be known to arbitrary precision. Let’s consider this for the “parity” operator,

$$P\psi(x) = \psi(-x), \quad (2)$$

which flips the wavefunction.

a) [0.5pt] What is $P^2\psi(x)$?

b) [0.5pt] What are the possible eigenvalues of $P$?

c) [1pt] If $H = \frac{p^2}{2m} + V(x)$ where $V(x) = V(-x)$ then what is $[H, P]$?

d) [1pt] Show that if $\psi(x)$ is a stationary state, then so is $\psi(-x)$. What is the energy of $\psi(-x)$?

Problem 3) (4 pts)

Parts a), b), c) will assume a Hydrogen wavefunction in the $(n, l, m) = (2, 1, 1)$ state, $\psi_{211}$.

a) [1pt] Let’s define $P(r)$ as the probability density per unit radius for this state, so $\int_0^{\infty} dr \ P(r) = 1$. Compute $P(r)$.

b) [1pt] At what radius $r$ is $P(r)$ at a maximum?

c) [1pt] What are the expectation values of $r$ and $r^2$ in this state?

d) [1pt] Ignoring the effects of electron spin, how many different wavefunctions are there in the $n = 3$ level of Hydrogen?

[Useful integral: $\int_0^{\infty} x^n e^{-x} dx = n!$ for positive integers $n$.]