2015.16 PH2b – WINTER TERM

QUIZ 2

Your solutions are due on FEBRUARY 02, 2016 at 11:00am, in the locked section boxes outside 201 E. Bridge. Late quizzes will not be accepted, except in very special circumstances.

The quiz is open book, open lecture, open section notes, open homework sets and solutions. Calculators may be used. Symbolic manipulators, other than your brain are not allowed. Please justify your answers and show all work.

TIME LIMIT: 90 MINUTES

IMPORTANT: Please write your name, your section number and your T.A’s name on the front of your solution sheet.

THERE WILL BE A QUIZ REVIEW ON SATURDAY JANUARY 30, 2016 AT 1:00PM IN 201 E. BRIDGE.
Problem 1) (5 pts)

Consider the following potential well:

\[ V(x) = \begin{cases} 
-V_0, & 0 \leq x \leq \frac{a}{2} \text{ (Region I)} \\
0, & \frac{a}{2} \leq x \leq a \text{ (Region II)} \\
\infty, & \text{otherwise}
\end{cases} \tag{1} \]

We'll compute the stationary states for the time-independent Schrödinger equation with \( E > 0 \).

a) [1pt] What are the boundary conditions on the wavefunction in Regions I and II?

b) [1pt] Write down a general wavefunction for regions I and II. (Hint: it may be useful to use that \( \sin(k(x - b)) \) for some constant \( b \) is a solution to the free particle Schrödinger equation.

c) [1pt] Use your answers from a) and b) to find an equation for the energy \( E \). Do not attempt to solve the equation! Is the spectrum of energies continuous or discrete?

d) [1pt] From physics arguments, what’s the spectrum of energies for large energies: \( E \gg V_0 \)?

e) [1pt] Use equations to justify your answer in d) by studying the \( E \gg V_0 \) limit of the boundary conditions you computed earlier.

Problem 2) (5 pts)

Let’s study how a wavefunction evolves in time in the quantum harmonic oscillator.

a) [1pt] Write down the most general solution \( \Psi(x, t) \) to the time-dependent Schrödinger in terms of the \( \psi_n \) solutions \((n = 0, 1, 2, \ldots)\) to the time-independent Schrödinger equation, so

\[ H\psi_n = \hbar\omega(n + 1/2)\psi_n \tag{2} \]

b) [2pt] Compute \( \langle X \rangle \) given \( \Psi(x, t) \). (Hint: write \( X \) in terms of raising/lowering operators!) You should get a bunch of terms that oscillate in time; at what frequency are these oscillations? Given that \( \langle X \rangle \) is the average position in a harmonic oscillator, does this make sense?

c) [1pt] I measure the energy of the oscillator to be \( E_n \), collapsing the wavefunction to a single stationary state, \( \psi_n \). Afterwards, what frequency does \( \langle X \rangle \) oscillate after the measurement?

d) [1pt] I can imagine initializing the wavefunction at \( t = 0 \) in various ways. For each way listed below, state whether \( \langle X \rangle \) will oscillate in time or not:

i) \( \Psi(x, 0) = \sum_{n \text{even}} c_n \psi_n \) \quad (3)

ii) \( \Psi(x, 0) = \sum_{n \text{odd}} c_n \psi_n \) \quad (4)

iii) \( \Psi(x, 0) = \sum_{n \text{prime}} c_n \psi_n \) \quad (5)