1. (a) Using eq.(7-11) in the textbook by French, we have

\[ v = \sqrt{\frac{\gamma P}{\rho}}. \]  

(1)

For air \( \gamma \approx 1.4, \), \( \rho = 1.2 \times 10^{3} \text{ kg/m}^3 \) and \( P = 1.5 \times 10^{5} \text{ N/m}^2 \), this gives

\[ v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.5 \times 10^{5} \text{ N/m}^2}{1.2 \text{ kg/m}^3}} \approx 418.33 \text{ m/s}. \]  

(2)

(b) The frequency of the normal mode of the pipe opened at both ends is given by

\[ f_n = \frac{nv^2}{L} \approx 209.17 \frac{n}{L}. \]  

(3)

The fundamental and 1st four normal modes are then

\[ f_1 = \frac{v}{2L} = \frac{209.17}{L} \text{ (fundamental mode)}, \]  

(4)

\[ f_2 = \frac{v}{L} = \frac{418.33}{L}, \quad f_3 = \frac{3v}{2L} = \frac{627.51}{L}, \quad f_4 = \frac{2v}{L} = \frac{836.66}{L}, \quad f_5 = \frac{5v}{2L} = \frac{1045.85}{L}, \]  

(5)

all in Hz if the unit of \( L \) is in meter.

(c) The frequency of the normal mode of the pipe closed at one end only is given by

\[ f_n = \frac{(2n-1)v}{4L} \approx 104.58 \frac{2n-1}{L}. \]  

(6)

The fundamental and 1st four normal modes are then

\[ f_1 = \frac{v}{4L} = \frac{104.58}{L} \text{ (fundamental mode)}, \]  

(7)

\[ f_2 = \frac{3v}{4L} = \frac{313.75}{L}, \quad f_3 = \frac{5v}{4L} = \frac{522.91}{L}, \quad f_4 = \frac{7v}{4L} = \frac{732.08}{L}, \quad f_5 = \frac{9v}{4L} = \frac{941.22}{L}, \]  

(8)

again all in Hz if the unit of \( L \) is in meter.

(d) From eq.(7-12) in the textbook by French, we can know that

\[ v = \sqrt{\frac{\gamma P}{\rho}} \propto T^{1/2}. \]  

(9)

Therefore

\[ v(T = 20^\circ) = \left(\frac{-20 + 273}{20 + 273}\right)^{1/2} v(T = 20^\circ) = \left(\frac{253}{293}\right)^{1/2} \times 418.33 \text{ m/s} \approx 388.73 \text{ m/s}. \]  

(10)

The beat frequency at the fundamental mode is then

\[ \Delta f_1 = f_1(T = 20^\circ) - f_1(T = -20^\circ) = \frac{1}{2L} \left[ v(T = 20^\circ) - v(T = -20^\circ) \right] \]

\[ = \frac{1}{2L} \left[ 418.33 \text{ m/s} - 388.73 \text{ m/s} \right] = \frac{14.8}{L}. \]  

(11)

in Hz if the unit of \( L \) is in meter.
2.

(a) The wave equation for sound displacement in 3D is given by

\[ \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \xi}{\partial t^2}, \]

(12)

where \( \xi = \xi(x, y, z, t) \) and \( v \) is the speed of sound.

(b) Plugging the solution into the wave equation, we find

\[ k^2 \equiv k_x^2 + k_y^2 + k_z^2 = \frac{w^2}{v^2}. \]

(13)

Also, the boundary conditions give us \( k_i = \pi n_i / L_i \), it follows that

\[ w(n_x, n_y, n_z) = v k = v \sqrt{k_x^2 + k_y^2 + k_z^2} = \pi v \sqrt{n_x^2 + n_y^2 + n_z^2}. \]

(14)

The first 4 normal mode frequencies are then

\[ w(1, 1, 1) = \frac{7}{6} \pi v \simeq 1.17 \pi v, \quad w(1, 1, 2) = \frac{\sqrt{61}}{6} \pi v \simeq 1.30 \pi v, \]

\[ w(1, 2, 1) = \frac{\sqrt{19}}{3} \pi v \simeq 1.45 \pi v, \quad w(1, 1, 3) = \frac{3}{2} \pi v = 1.5 \pi v. \]

(15)

all in Hz if the unit of \( v \) is in m/s.

3.

(a) Using \( 2 \cos x = e^x + e^{-x} \), it is easy to write out

\[ y(x, t) = A \cos (k_1 x - w_1 t) + A \cos (k_2 x - w_2 t) = \frac{A}{2} \left[ e^{i(k_1 x - w_1 t)} + e^{-i(k_1 x - w_1 t)} + e^{i(k_2 x - w_2 t)} + e^{-i(k_2 x - w_2 t)} \right] \]

\[ = \frac{A}{2} \left[ e^{i\left(k_1 x - \frac{w_1 + w_2}{2} t\right)} + e^{-i\left(k_1 x - \frac{w_1 + w_2}{2} t\right)} \right] \left[ e^{i\left(k_2 x - \frac{w_1 + w_2}{2} t\right)} + e^{-i\left(k_2 x - \frac{w_1 + w_2}{2} t\right)} \right] \]

\[ = 2A \cos \left[ \frac{k_1 + k_2}{2} x - \frac{w_1 + w_2}{2} t \right] \cos \left[ \frac{k_1 - k_2}{2} x - \frac{w_1 - w_2}{2} t \right] \]

\[ = 2A \cos \left( k_{av} x - w_{av} t \right) \cos \left( k_{\Delta} x - w_{\Delta} t \right). \]

(16)

(b) The group velocity is given by

\[ v_g = \frac{dw(k)}{dk} = \frac{d}{dk} \left[ \frac{1}{v} \left( k + \frac{k^2}{10k_{av}} \right) \right] = \left( 1 + \frac{k}{5k_{av}} \right). \]

(17)
4. 

(a)(b) To find the reflected amplitude and transmitted amplitude of the strings, we use the boundary conditions (i) \( \xi(x, t) \) is continuous (ii) \( d\xi(x, t)/dx \) is continuous. Let \( \xi_I(x, t) = A \sin(k_I x - wt) \) the incident wave, \( \xi_R(x, t) = A_R \sin(-k_R x - wt) \) the reflected wave, and \( \xi_T(x, t) = A_T \sin(k_T x - wt) \) the transmitted wave, where \( k_R = k_1 \) since \( k = 2\pi/\lambda = 2\pi f/v = w/v = w\sqrt{\mu/T} \), so \( k \) only depends on \( \mu \). Now, from the boundary conditions, we have

\[
\xi_I(x, t) \bigg|_{x=0} + \xi_R(x, t) \bigg|_{x=0} = \xi_T(x, t) \bigg|_{x=0},
\]

\[
d\xi_I(x, t) \bigg|_{x=0} + d\xi_R(x, t) \bigg|_{x=0} = d\xi_T(x, t) \bigg|_{x=0}.
\]

Plugging the wave solutions, one can find

\[
A + A_R = A_T, \quad A k_1 - A_R k_1 = A_T k_T
\]

The solution is then

\[
A_R = \frac{k_1 - k_T}{k_1 + k_T} A, \quad A_T = \frac{2 k_1}{k_1 + k_T} A,
\]

Because \( k \propto \sqrt{\mu} \), it follows that

\[
A_R = -\frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} A, \quad A_T = \frac{2 \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} A.
\]

Since \( \mu_2 > \mu_1 \), then there is a phase shift for reflected wave. The power of the wave is given by

\[
P_R = \frac{1}{2} \mu v w^2 A^2,
\]

We obtain

\[
P_R = \frac{1}{2} \mu_1 v_1 w^2 A_R^2 = \frac{1}{2} \mu_1 \sqrt{\frac{T}{\mu_1}} w^2 \left( \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} A \right)^2 = \frac{1}{2} \sqrt{\mu_1 T} w^2 A^2 \left( \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} \right)^2,
\]

\[
P_T = \frac{1}{2} \mu_2 v_2 w^2 A_T^2 = \frac{1}{2} \mu_2 \sqrt{\frac{T}{\mu_2}} w^2 \left( \frac{2 \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} A \right)^2 = \frac{1}{2} \sqrt{\mu_2 T} w^2 A^2 \left( \frac{2 \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} \right)^2,
\]

One can check that

\[
P_R + P_T = \frac{1}{2} \sqrt{\mu_1 T} w^2 A^2 = P_I
\]

so the energy is conserved.