2015.16 PH2a

QUIZ 2

Your solutions are due on TUESDAY OCTOBER 27, 2015, at the beginning of lecture (11:00am). Place the solutions in the locked section boxes outside 201 E. Bridge. Late quizzes will not be accepted, except in very special circumstances.

The quiz is open book, open lecture notes, open section notes, open homework sets and solutions. Calculators may be used. Computers and smart devices, other than your brain are not allowed. Please justify your answers and show all work.

Time Limit: 90 minutes

IMPORTANT: PLEASE WRITE YOUR NAME, SECTION NUMBER AND YOUR T.A.’S NAME ON THE FRONT OF YOUR SOLUTION SHEET.

THERE WILL BE A QUIZ REVIEW ON
SATURDAY OCTOBER 24, 2015
2:00PM TO 3:00PM
IN
201 E. BRIDGE
Quiz 2 [all parts equal credit]

Two masses are connected by springs as shown in the picture and constrained to only make vertical (y) displacements. Note that the vertical displacements are SMALL, i.e., \( y_i \ll L \), which means that the length of the spring and the resulting force does not change. We can therefore write the force on each mass as a tension \( T_0 \). What matters is the component of the tension in the y-direction.

![Diagram of masses connected by springs with y-displacements](image)

a. Show that the y-component of the force on mass 1 is \( F_i = T_0 \frac{y_i}{L} + T_0 \left( \frac{y_i - y_2}{L} \right) \).

b. Write down the equations of motion in terms of \( y_1, y_2 \), and \( \omega_0^2 = \frac{T_0}{L} \).

c. From symmetry sketch the two normal modes and argue which one has the larger frequency. For the sketch, for example, you can use our usual notation showing the maximum amplitude vs. oscillator number for each mode.

d. Substitute a complex exponential solution into b. and find the algebraic equation set that results.

e. Find the normal mode frequencies in terms of \( \omega_0 \).

f. Find the normal mode amplitudes (to within an arbitrary constant) and write down the two normal modes (you can use the compact vector format if you like).

g. Write down the most general solution.

h. Now write down a solution which pertains if mass 1 is started at zero velocity and position \( y=0 \), while mass 2 is started with position 0 and velocity 0, i.e.,
\[
 y_1(t=0) = A, \quad \dot{y}_1(t=0) = 0, \quad y_2(t=0) = 0, \quad \dot{y}_2(t=0) = 0
\]