

Quiz Review

Identical particles vs Distinguishable Particles:

2 Bosons
3 identical particles $\left\{ \begin{array}{l} \text{Bosons - Symmetric wavefunction} \\ \text{Fermions - Antisymmetric wavefunction} \end{array} \right.$

Distinguishable particles: Direct product wavefunction

Consequences: 2 fermions cannot ^{have} ~~replace~~ same wavefunction

Example problem:

Three particles in 1D harmonic oscillator problem.
Determine ground state energy & degeneracy.

$$E = (n + \frac{1}{2}) \hbar \omega$$

(ground)

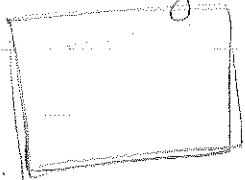
Bosons: $3 \hbar \omega$ ← Degeneracy: 1 (spinless)

Fermions: $6 \hbar \omega$ ← Degeneracy: 1 (spinless)

Distinguishable: $3 \hbar \omega$ Degeneracy (depends on spin)
(spin 0: 1, spin $\frac{1}{2}$: 8)

spin $\frac{1}{2}$ fermions: $8 \hbar \omega$ Degeneracy = 2

Closed system



definite E

↳ gives accessible no. of microstates Ω satisfying requirement of total energy E .
(find all states)

Energy of spins

example:

1 spin: $\mathcal{H} = -mB$ + $\frac{B}{2}$ \downarrow (1)

- $\frac{B}{2}$ \downarrow (1)

$E = -\frac{B}{2}$ or $E = +\frac{B}{2}$

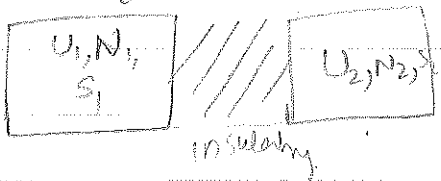
2 spins: depending on distinguishability / bosonic / fermionic different energies are allowed.

N spin problem discussed in class

$$g = \frac{N!}{N_+! N_-!}$$

Thermal Equilibrium Entropy, Temperature

consider 2 systems



Total energy: $U = U_1 + U_2$

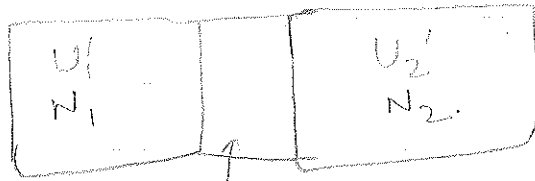
Entropy: $S_1 + S_2$

Why? $g(N_1)$ accessible states ^{in 1}, $g(N_2)$ accessible states in 2

Total accessible states in combined system: $g(N_1)g(N_2)$

$$\log(g(N_1)g(N_2)) = \log(g(N_1)) + \log(g(N_2)) = S_1 + S_2$$

Now bringing to thermal equilibrium



$$U = U_1 + U_2$$

$$= U_1' + U_2'$$

conductor
thermal contact
(allows energy exchange)

Now what is the accessible states of whole system?

Now we know what it is when U_1' & U_2' are fixed

$$\rightarrow g(N_1, U_1') g(N_2, U_2')$$

when they are not, all the states must be allowed!

$$g(N, U) = \sum_{U_1'} g(N_1, U_1') g(N_2, U - U_1')$$

For large N , approx

$$g(N, U) \approx \left(g(N_1, U_1') g(N_2, U - U_1') \right)_{\max}$$

what gives you $g(N, U)$?

$$\left(\frac{d}{dU} g(N, U) \right)$$