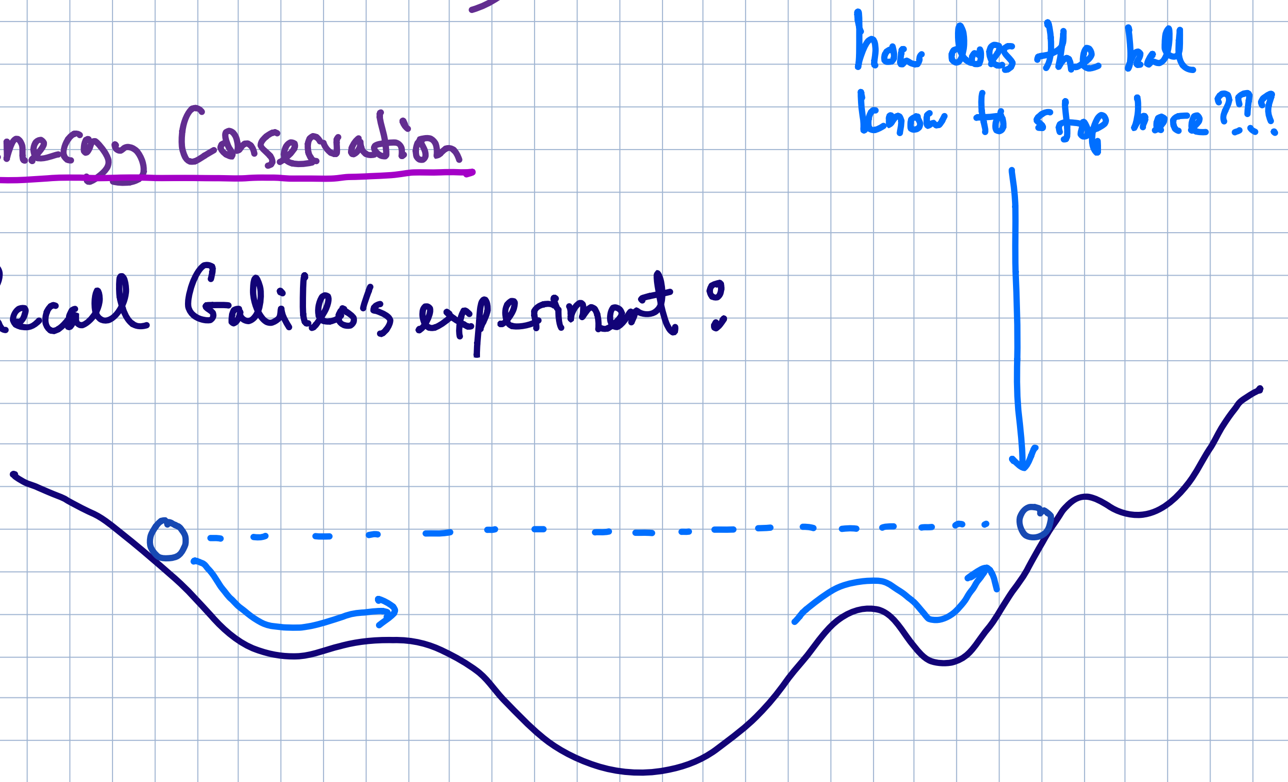
# Lecture 7: Energy

## Energy Conservation

Recall Galileo's experiment:


how does the ball know to stop here???

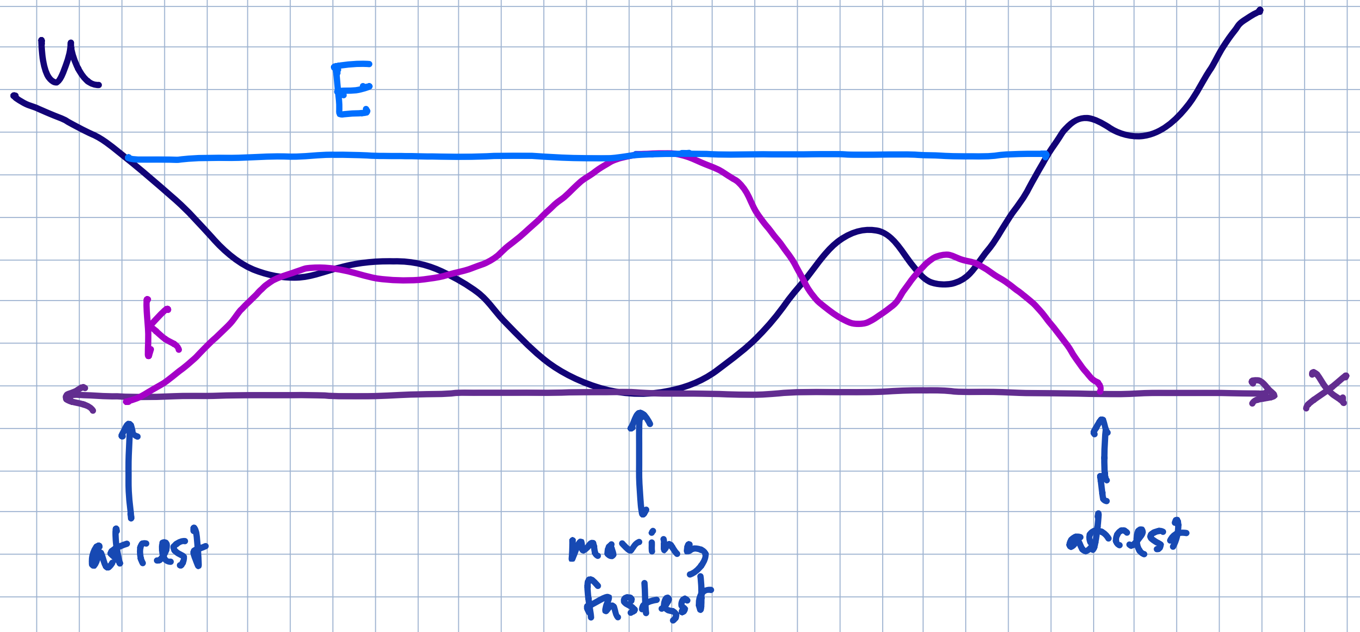
«demo: "peg & pendulum"»

total energy is conserved (i.e. constant in time)

$$\underbrace{E}_{\text{unit is Joules} = kg\, m^2/s^2} = \overbrace{K}^{= \frac{1}{2} m \vec{v}^2} + \overbrace{U}^{\text{depends on system}}$$

total energy = kinetic energy + potential energy

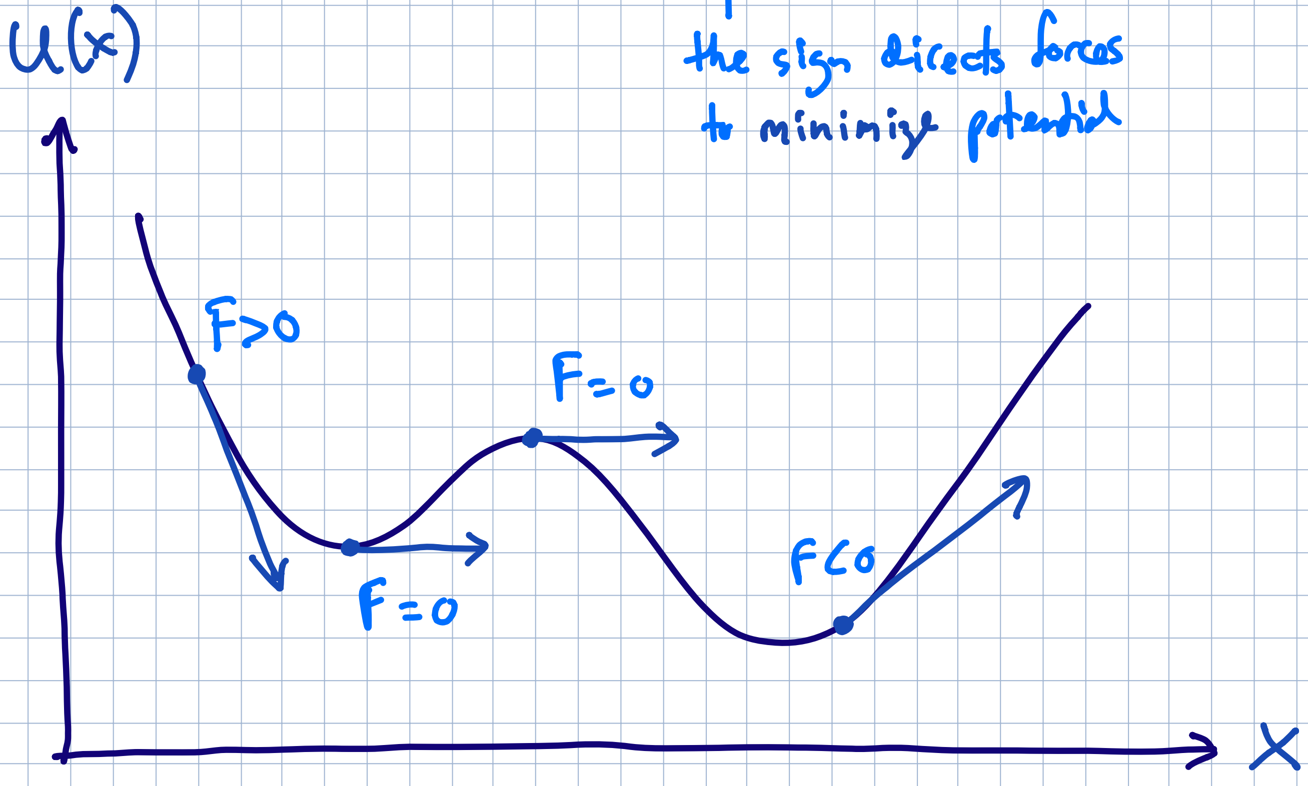
(energy of motion) (energy by virtue of position)



The notion of force comes from energy:

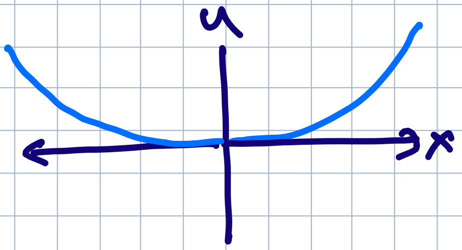
$$F = -\frac{dU}{dx}$$

the sign directs forces to minimize potential



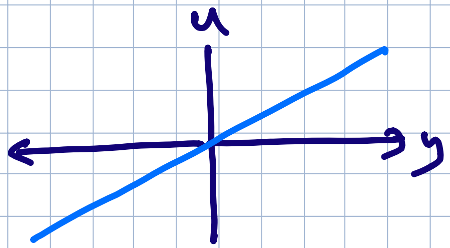
# Some example potentials:

$$F_{Hooke} = -kx \longrightarrow U_{Hooke} = \frac{1}{2}kx^2$$



---

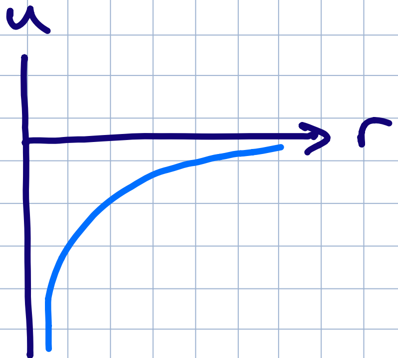
$$F_{\substack{grav \\ Earth}} = -mg \longrightarrow U_{\substack{grav \\ Earth}} = mgy$$



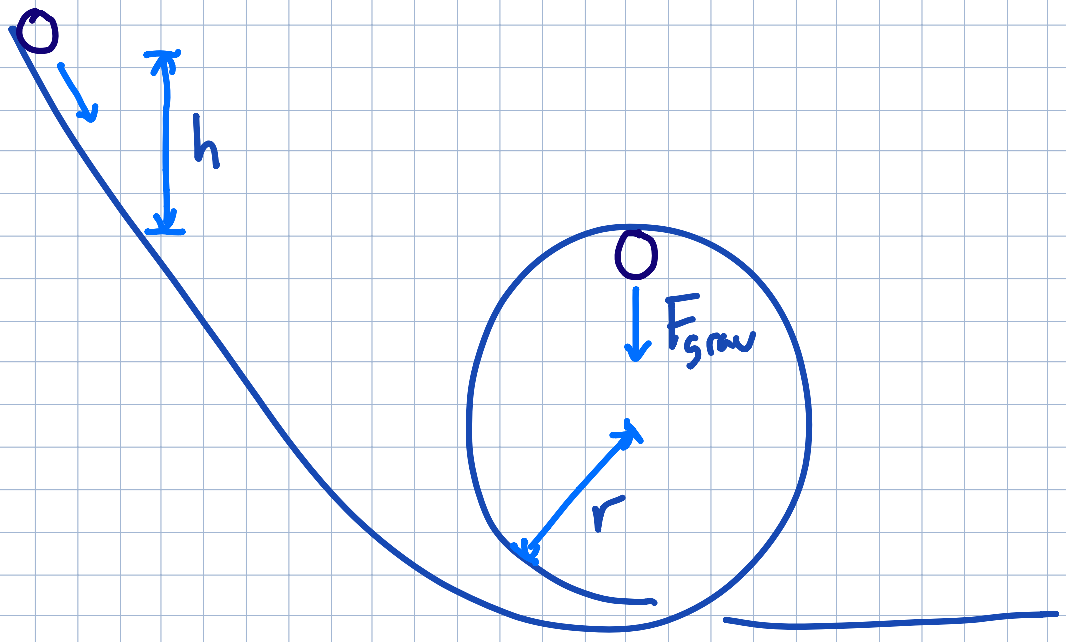
«« demo: “vertical cannon” »»

---

$$F_{\substack{grav \\ central}} = -\frac{GmM}{r^2} \longrightarrow U_{\substack{grav \\ central}} = -\frac{GmM}{r}$$



# (eg) loop-the-loop



energy conservation: $\frac{1}{2}mv^2 = mgh$

centripetal force: $\frac{mv^2}{r} = mg$

$$\Longrightarrow \quad \frac{1}{2}\cancel{m}\cancel{g}r = \cancel{m}\cancel{g}h \quad \Longrightarrow \quad \boxed{h = r/2}$$

«« demo: "loop-the-loop" »»

# Work

Let us define a useful concept,

$$W = \text{“amount of energy transferred to an object via a force”}$$

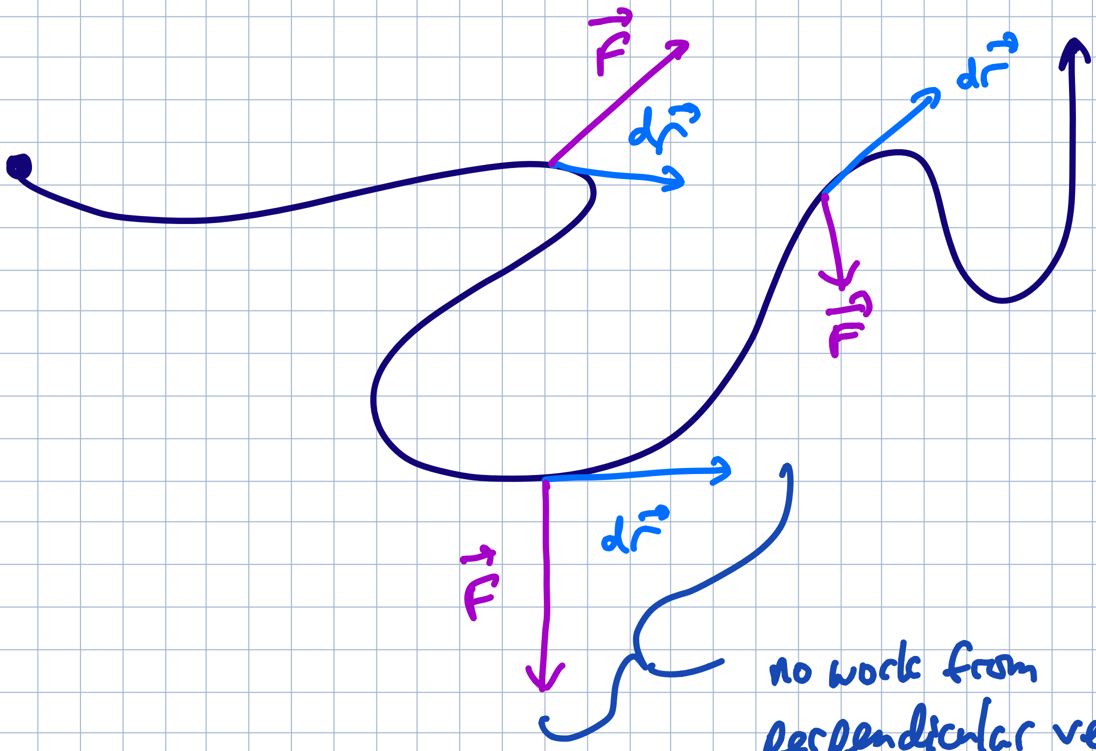
$$= \int_{\overset{\uparrow}{\underset{\text{initial}}{A}}}^{\overset{\text{final}\,\rightarrow\,B}{}} F\,dx = -\int_A^B \frac{dU}{dx}\,dx$$

$$= -\left[U(B) - U(A)\right] = -\Delta U$$

$$\text{so} \quad \underset{\substack{\uparrow \\ \text{energy gained} \\ \text{e.g. as kinetic}}}{W} = \underset{\substack{\uparrow \\ \text{energy lost} \\ \text{in potential}}}{-\Delta U}$$

This gets more complicated beyond one dimension.

$$W = \int_{A}^{B} \vec{F} \cdot d\vec{r}$$



no work from perpendicular vectors

## Power

$$P = \frac{dW}{dt} = \text{"work per time"}$$

in watts = Joules/sec

⟪⟪ demo: "small explosive" ⟫⟫

# Energy and power can range over immense scales!

- energy of a million $\underbrace{\text{tons}}_{= 10^3 \text{ kg}}$ of TNT $= \underbrace{1 \text{ MT}}_{= 4 \times 10^{15} \text{ J}}$
- energy needed for one household for 100,000 yrs $\sim 1 \text{ MT}$
- all energy from nuclear weapons tests + conventional war explosives $\sim 540 \text{ MT}$
- all solar energy received by Earth in one minute $\sim 62{,}500 \text{ MT}$
- black hole merger $\sim 10^{32} \text{ MT}$

---

$$P_{\text{house}} \sim \frac{1 \text{ MT}}{100{,}000 \text{ yrs}} \sim 1300 \text{ W}$$

$$\Rightarrow \quad P_{\text{mankind}} \sim \frac{540 \text{ MT}}{\text{century}} \sim 7 \times 10^{8} \text{ W}$$

$$P_{\text{solar}} \sim \frac{62{,}500 \text{ MT}}{\text{minute}} \sim 4 \times 10^{18} \text{ W}$$

# demo: "bowling ball"