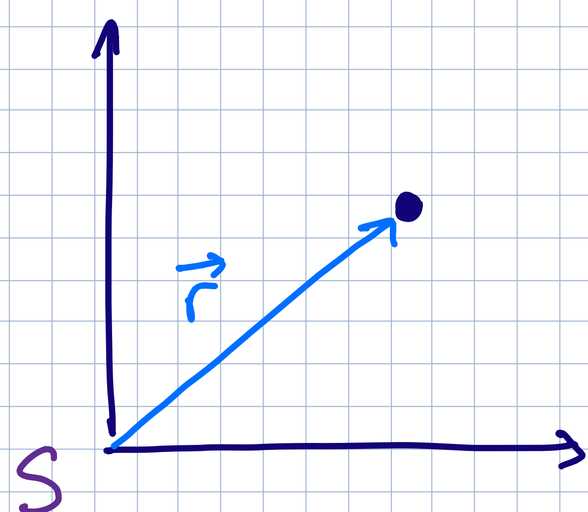
Lecture 6: Accelerating Reference Frames

We saw before that the laws of physics are the same in all inertial, i.e. constant velocity frames.

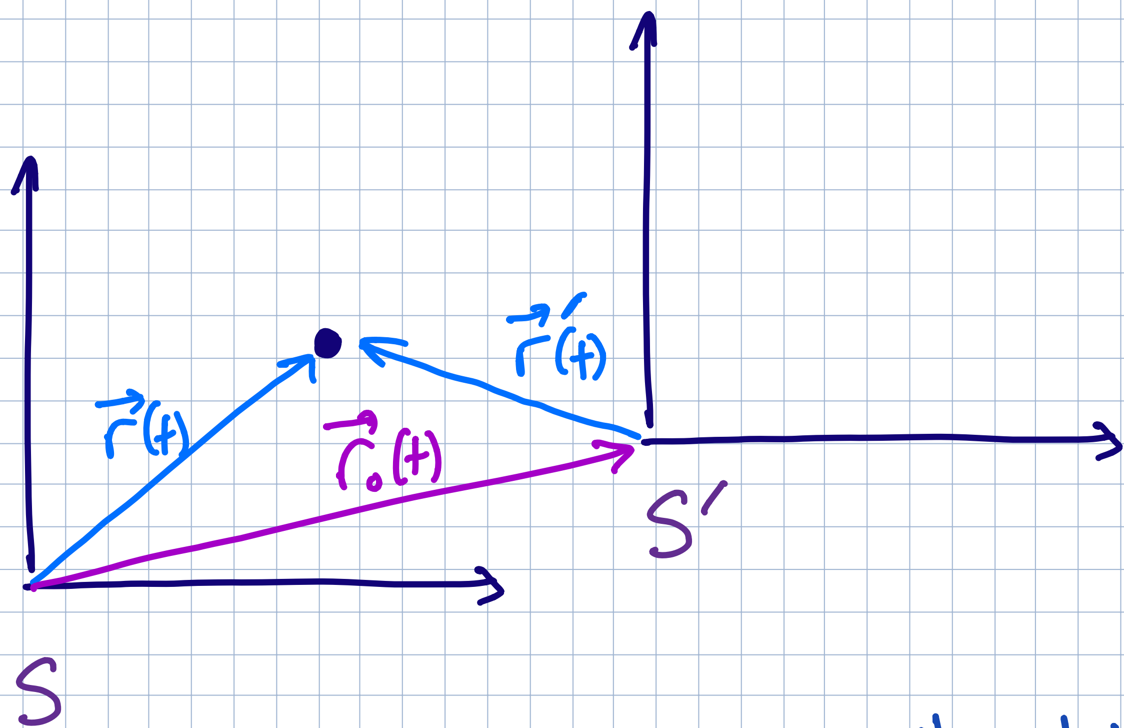
However it is sometimes convenient to go to a non-inertial = accelerating frame.



S = reference frame

$\vec{r}$ = coordinate of object in reference frame S

Define a new reference frame S'



position, velocity, and acceleration of S' reference frame

So, $\vec{r} = \vec{r}\,' + \vec{r}_0$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \frac{d\vec{r}_0}{dt} \longrightarrow \vec{v} = \vec{v}\,' + \vec{v}_0$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}\,'}{dt^2} + \frac{d^2\vec{r}_0}{dt^2} \longrightarrow \vec{a} = \vec{a}\,' + \vec{a}_0$$

Consider various cases :

1) $\vec{r}_0 = \text{const}$: trivial shift of the "origin"

2) $\vec{v}_0 = \text{const}$

$\downarrow$

$\vec{r}_0 = \vec{v}_0 t + \text{const}$: inertial frame

3) $\vec{v}_0 \neq \text{const}$: non-inertial, ie accelerating frame

$\downarrow$

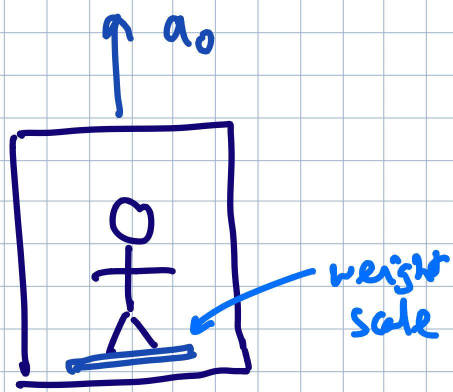
$\vec{a}_0 \neq 0$

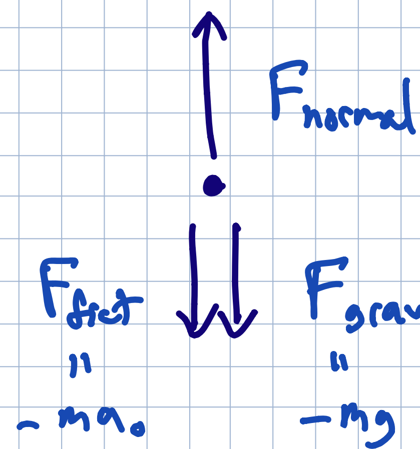
Since $\vec{a} = \vec{a}' + \vec{a}_0$

$\Downarrow$

$\vec{F} = \vec{F}' + m\vec{a}_0$ ← fictitious force

(ex 1) Fun with Elevators



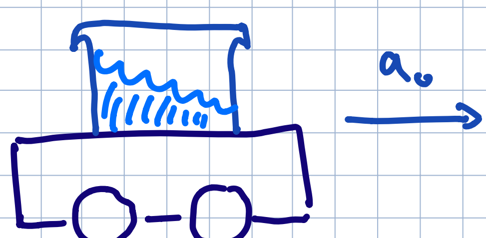


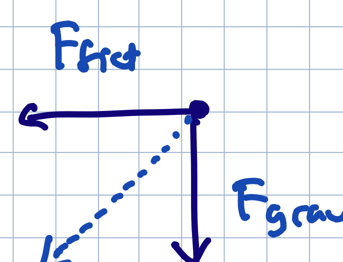
$$\Rightarrow F_{tot} = 0 = F_{normal} - mg - ma_0$$

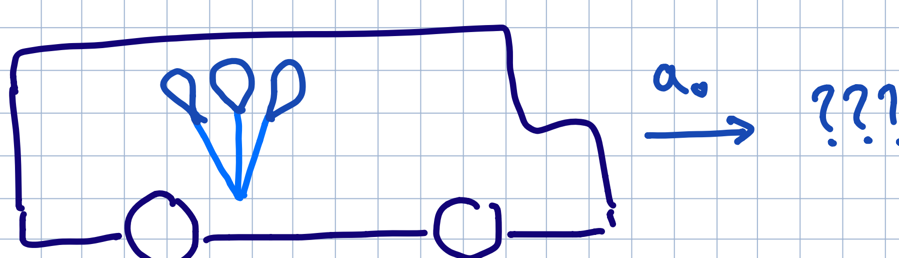
$$F_{normal} = m(g + a_0)$$

↑ effectively heavier

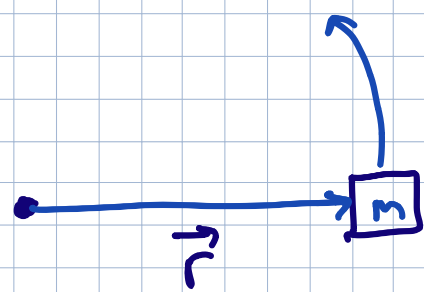
(ex 2) Fun with Cars







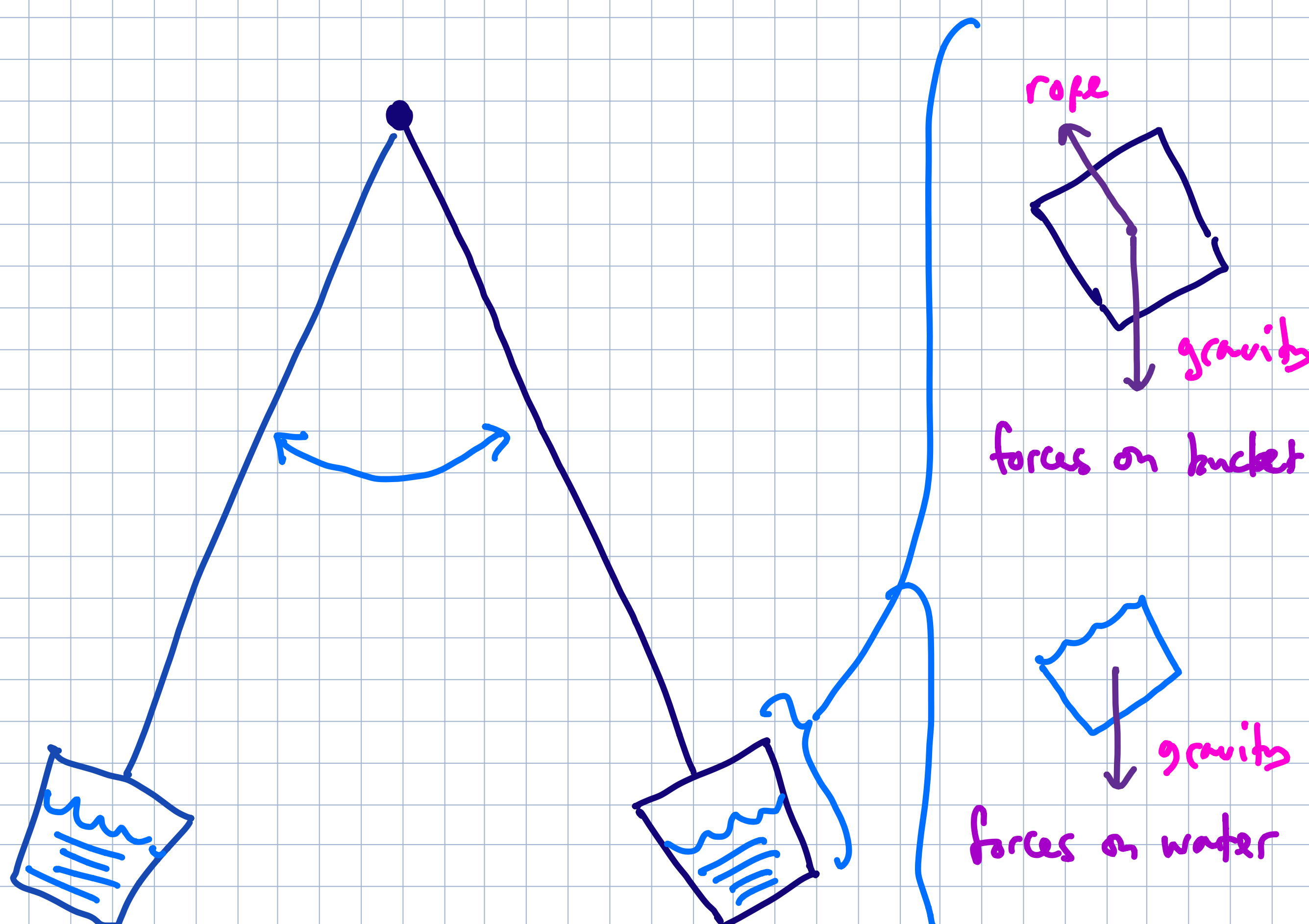
(ex 3) Centrifugal Force



$\vec{a}_0 = -\omega^2 \vec{r}$

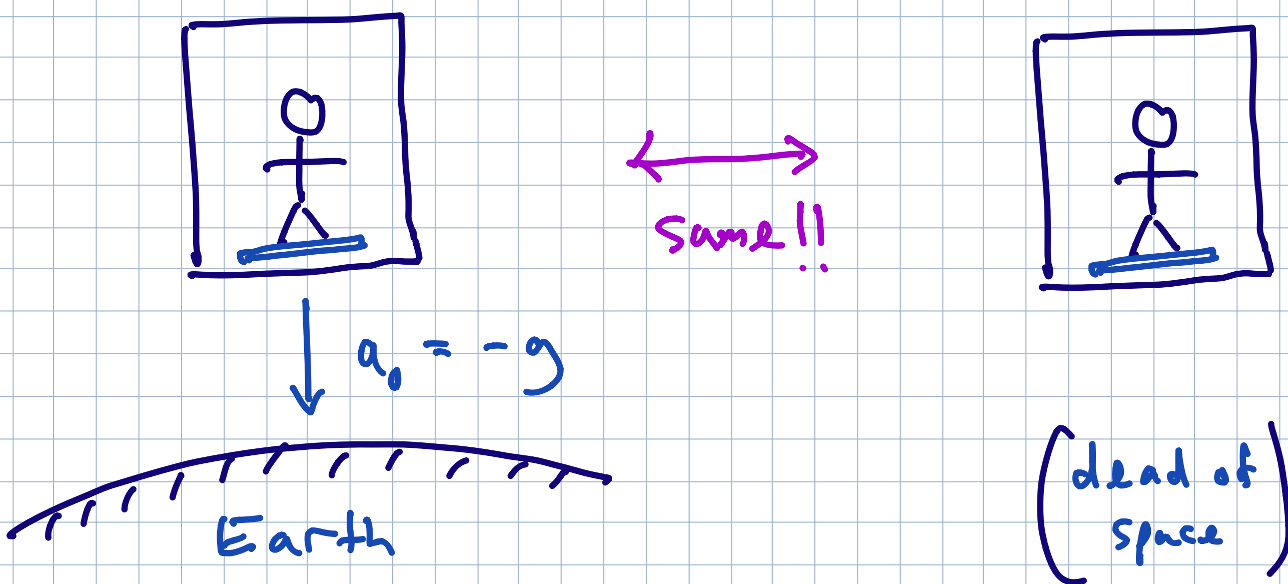
so $\vec{F}_{centrifugal} = m\omega^2 \vec{r}$

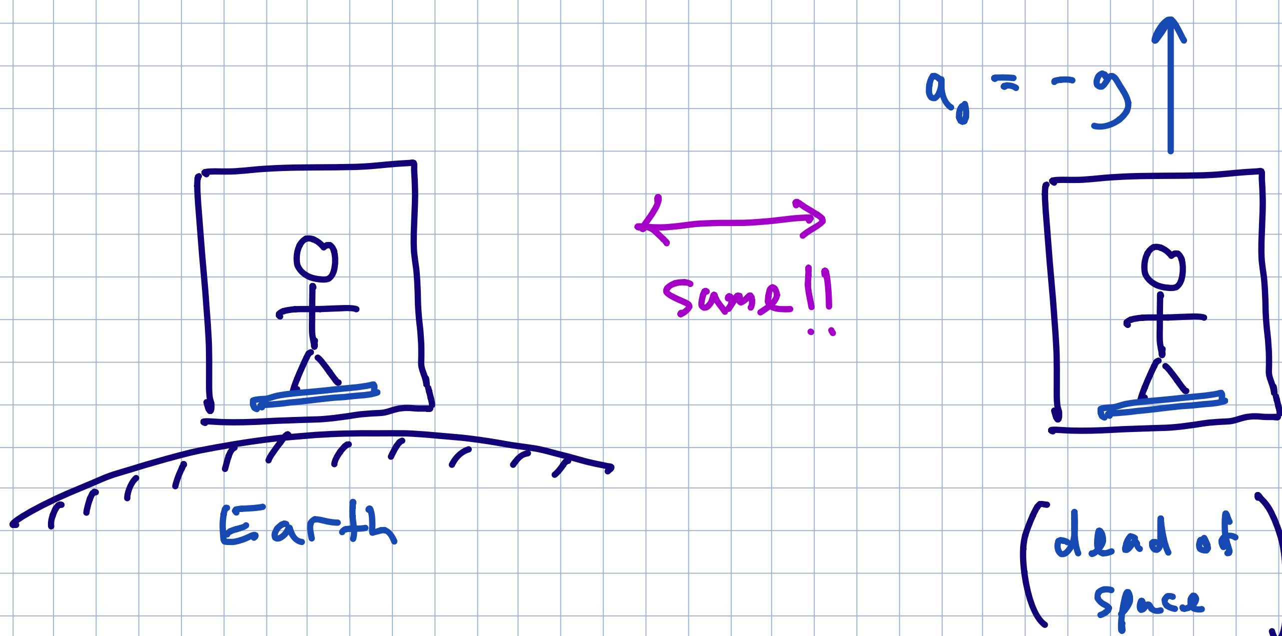
《 demo: "swinging bucket" 》



《 demo: "bucket on ramp" 》

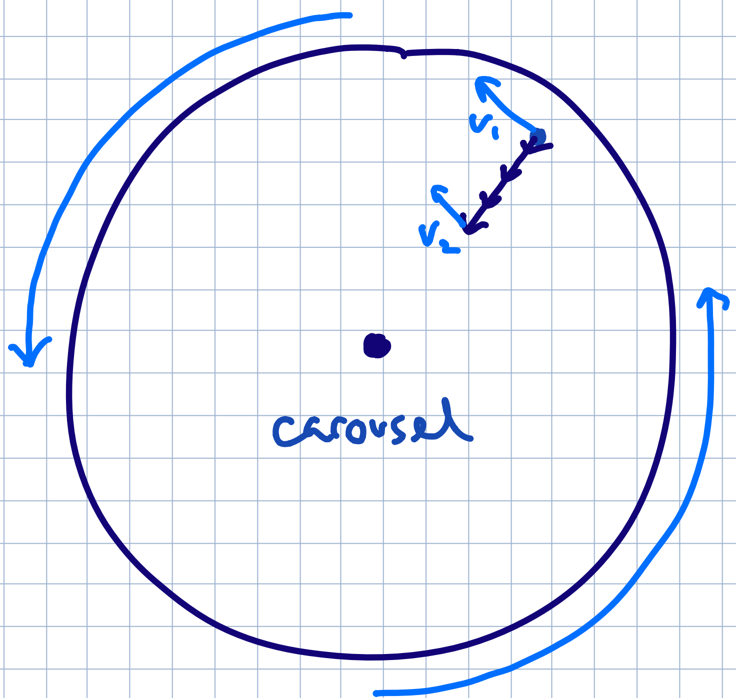
(ex 4) Fun with Einstein





Super Deep Fact #3: You feel the pull of gravity because you are ***accelerating***

(ex 5) Coriolis Force



Walk from outside to inside.

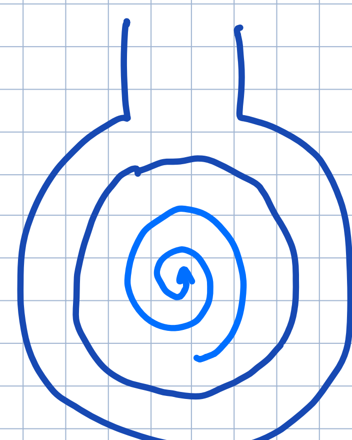
$$v_1 = \omega r_1 > \omega r_2 = v_2$$

Since $v_1 > v_2$, your frame accelerates to your left as you walk. Hence there is a phantom force to your right.

$$\Rightarrow \quad \vec{F}_{Coriolis} = 2m\,\vec{v} \times \vec{\omega}$$

($\vec{v}$: your velocity; $\vec{\omega}$: angular freq vector of the carousel)

Physics of Toilets 101:



angular freq of Earth

$$a \sim v \cdot \omega$$

Coriolis acceleration of toilet water

velocity of toilet water

$$\sim \left(\frac{m}{s}\right) \cdot \frac{2\pi}{yr} \sim 10^{-8}\ m/s^2$$

$$a \sim 10^{-9}\ g$$

too small to matter!