Lecture 4: Forces of Nature

Fundamental Forces

affects day to day life

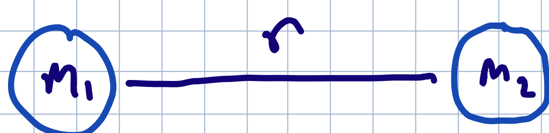
- **gravity**
  - free fall
  - celestial motion
- **E&M**
  - electricity
  - magnetism
    - springs
    - normal
    - friction

subtle effects

- **"weak force"**
- **"strong force"**

the origin of all mass, and nuclear physics + radioactivity.

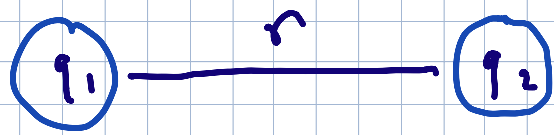
gravity



$$F_{grav} = \frac{G \overbrace{m_1 m_2}^{\text{attractive}}}{r^2}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

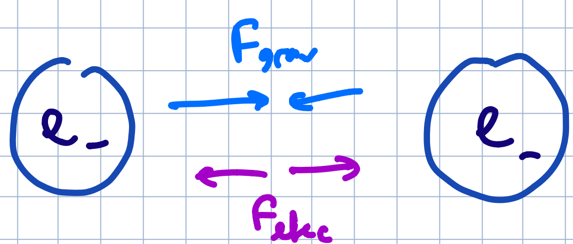
E&M



$$F_{elec} = \frac{K \overbrace{q_1 q_2}^{\text{attractive or repulsive}}}{r^2}$$

$K = 8.988 \times 10^9 \ Nm^2/C^2$

Consider the relative strengths of each force for an electron.



《demo: "Vandergraff"》

$$\left| \frac{F_{grav}}{F_{elec}} \right| = \frac{G m_e^2}{K q_e^2} \sim \frac{6.7 \times 10^{-11} \ Nm^2/kg \times (9.1 \times 10^{-31} \ kg)^2}{9 \times 10^9 \ Nm^2/C^2 \times (1.6 \times 10^{-19} \ C)^2}$$

$$\sim 10^{-43} \ !!$$

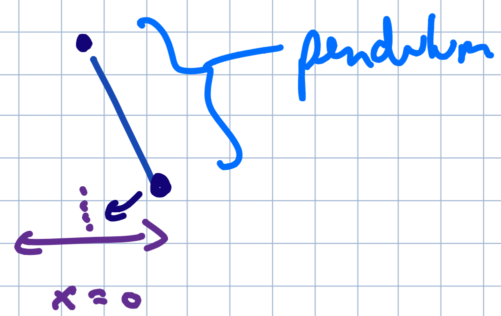
(a.k.a. fridge magnets are stronger than the Earth)

Mystery of Universe #1: "Why is gravity so weak?"

Effective Forces

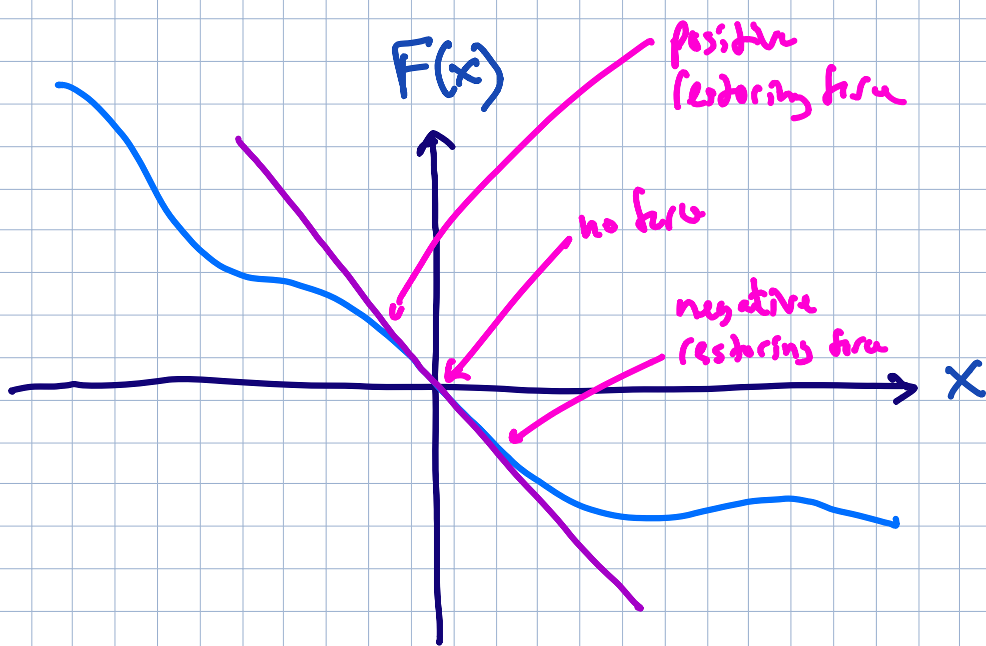
this means the force pushes towards eq. position

Consider any system w/ "stable equilibrium configuration" defined at $x = 0$ where $F(x=0) = 0$.



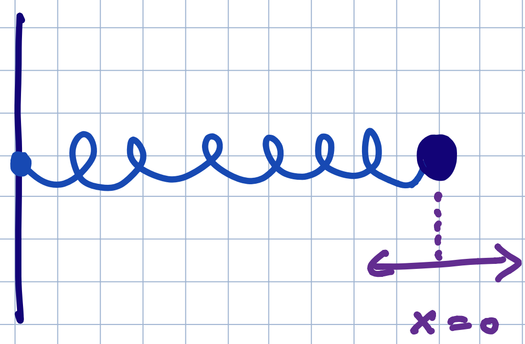
《《《《《 demo: "pendulum" 》》》》》

By continuity, $F(x)$ looks like



approximate linearly with: $F(x) \propto -x$

Spring Force



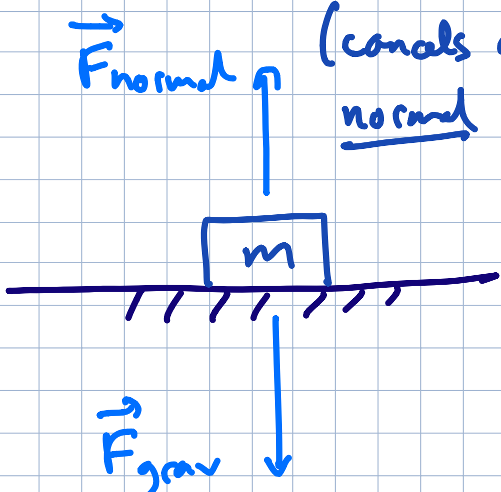
$$\boxed{F(x) = -kx}$$

Hooke's Law

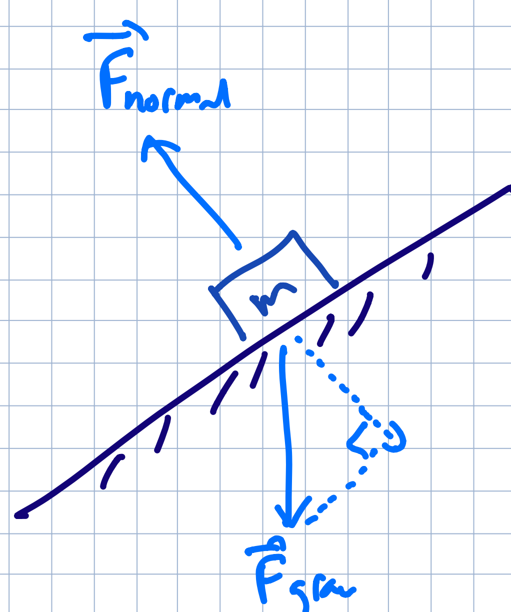
《《《《《demo: "springs"》》》》》

no mass	$\longrightarrow$	50 cm
25g	$\longrightarrow$	54 cm
50g	$\longrightarrow$	58 cm
100g	$\longrightarrow$	66 cm

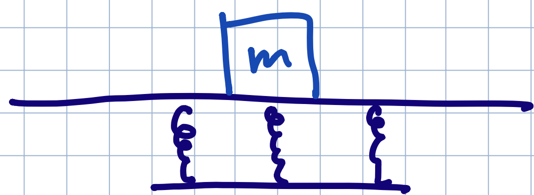
Normal Force

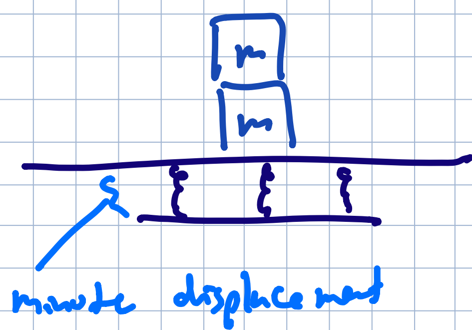


(cancels all force <u>normal</u> to surface)

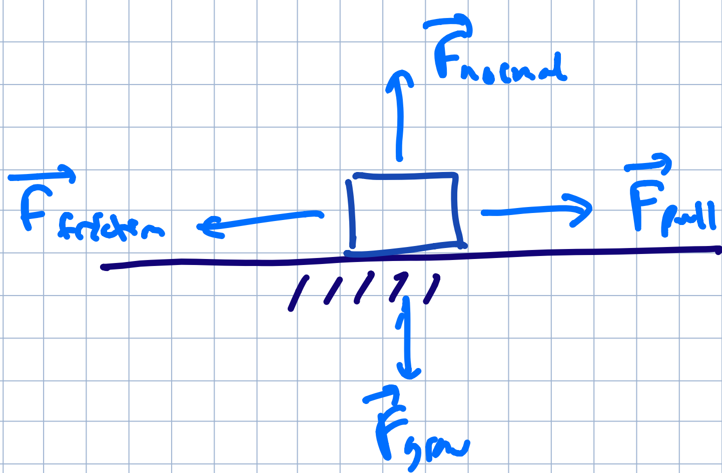


How does the table know to balance the force?





Friction Forces



$F_{friction} \le \mu F_{normal}$

where μ is either

μ_s = coeff of static friction

or

μ_k = coeff of kinetic friction

Typically, $\mu_s > \mu_k$, which is how anti lock brakes work.

《《demo: "measuring friction"》》

Drag Forces

Drag forces are velocity dependent and act to reduce it

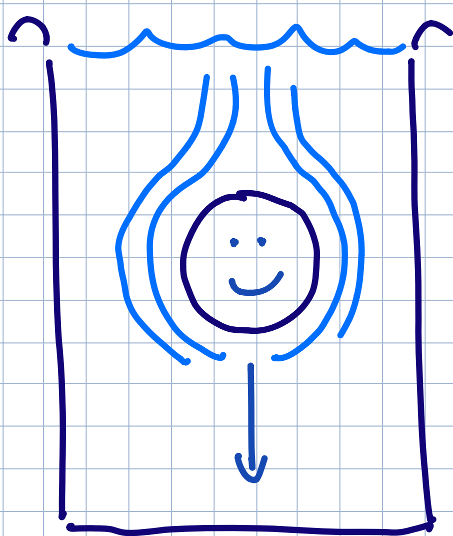
$F \propto -v$

(non-turbulent, slow)

$F \propto -v^2$

(turbulent, fast)

Consider non-turbulent flow.



$$F = -\Gamma v$$

radius of sphere

$$\Gamma = 6\pi R \eta$$

viscosity of medium

Consider the trajectory of bead in water.

positive direction is up

$$F = ma = -mg - \Gamma v$$

$$\rightarrow \quad m\ddot{x} = -mg - \Gamma \dot{x}$$

$$\ddot{x} = -g - \frac{\Gamma}{m}\dot{x}$$

$$= -\frac{\Gamma}{m}\left(\dot{x} + \frac{mg}{\Gamma}\right)$$

$$\rightarrow \quad \int \left[\frac{\ddot{x}}{\dot{x} + mg/\Gamma} = -\frac{\Gamma}{m} \right] dt$$

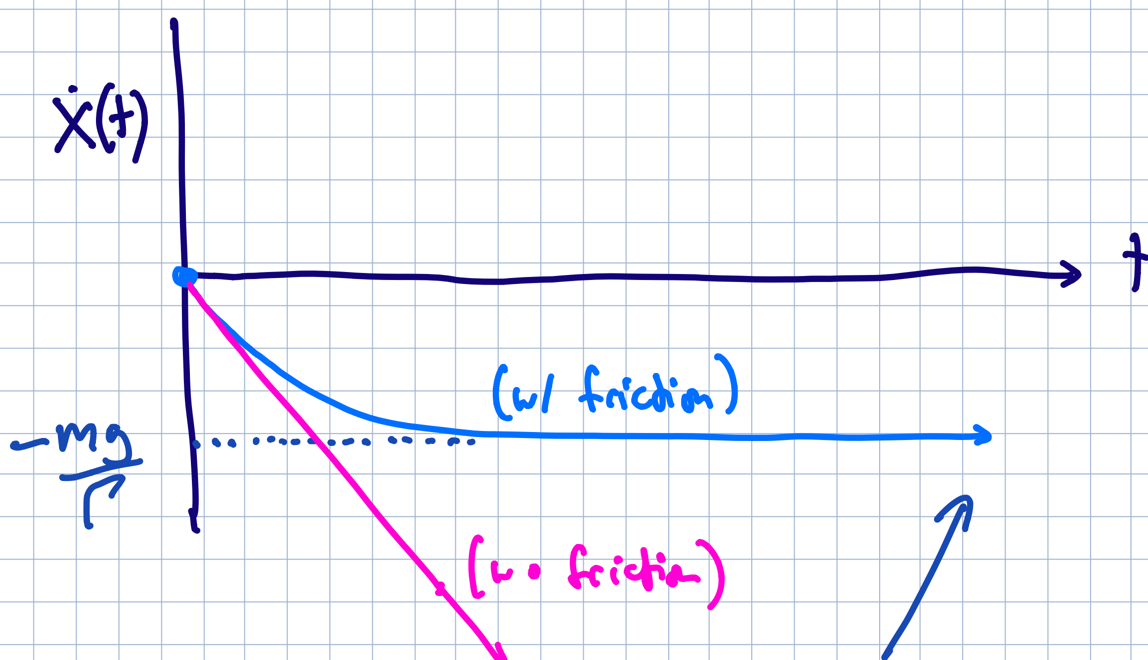
$$\log\left(\dot{x} + mg/\Gamma\right) = -\frac{\Gamma}{m}t + \text{const}$$

$$\dot{x} = -\frac{mg}{\Gamma} + \underbrace{e^{\text{const}}}_{=\,\text{const}}\, e^{-\frac{\Gamma}{m}t}$$

Assume bead starts at rest. So,

at $t=0$, $\dot{x} = 0 = -\frac{mg}{\Gamma} + \text{const}$

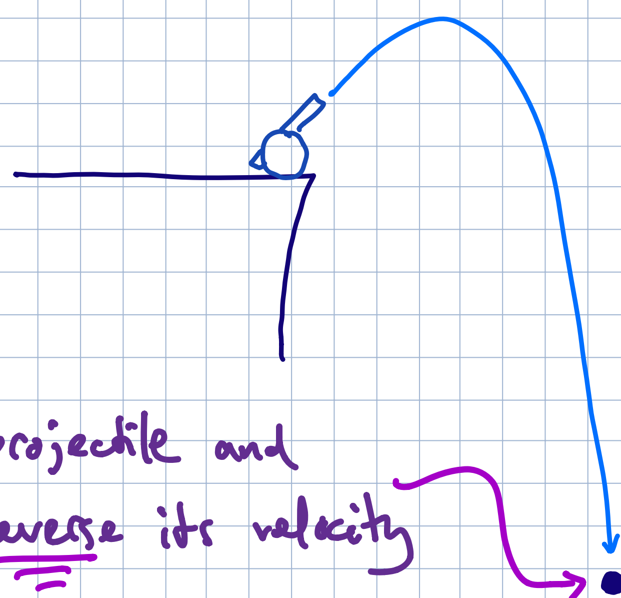
$$\Rightarrow \boxed{\dot{x} = -\frac{mg}{\Gamma}\left(1 - e^{-\Gamma/m\, t}\right)}$$



$\frac{mg}{\Gamma}$ = terminal velocity

⟪⟪ demo: "beads in syrup" ⟫⟫

Note a crucial difference w/ vs w/o friction.



stop the projectile and exactly ~~reverse~~ its velocity

w/o friction: it goes right back to the cannon

w/ friction: it <u>does not</u> return to the cannon

$\Longrightarrow$ the laws of physics including friction look different when time runs <u>forwards</u> versus <u>backwards</u>

Super Deep Fact #2:	friction prescribes an "arrow of time"

$$\left(\overbrace{m \frac{d^2x}{dt^2} = -mg}^{\text{even under } t \to -t} - \overbrace{\Gamma \frac{dx}{dt}}^{\text{odd under } t \to -t} \right)$$