Lecture 3: Newton's Laws

The Laws

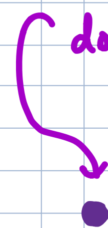
i) Principle of Inertia:

"in the absence of force, an object stays at rest or in uniform motion"

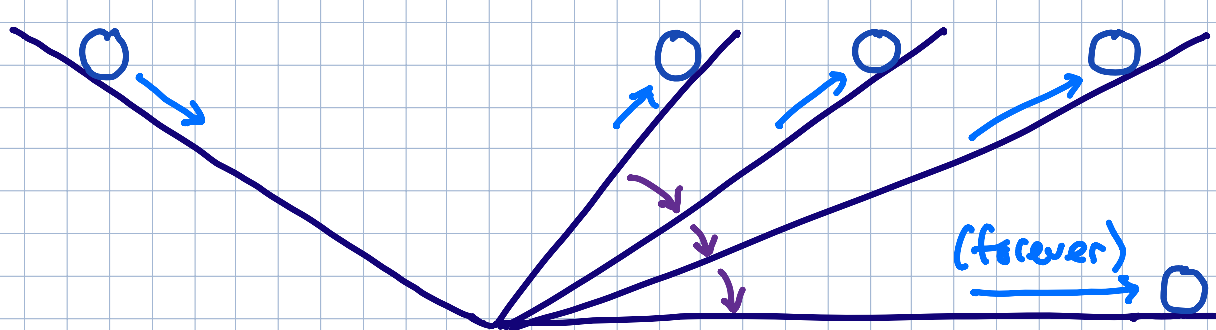
$\Updownarrow$ (equivalent)

"laws of physics are the same for inertial reference frames"

ball at rest doesn't move



Galileo's Thought Experiment:



ii) Force Law:

"an object accelerates proportionally to the force exerted on it"

proportionality constant defines mass!!!

$$\vec{F} = m\vec{a}$$

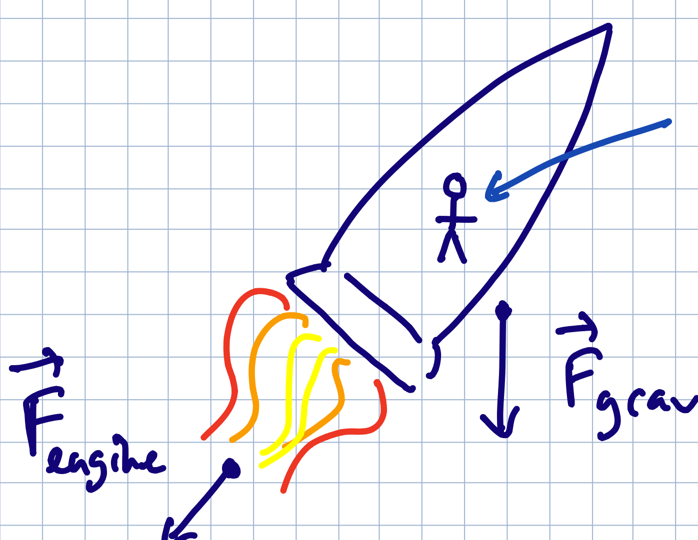
→ heavy masses accelerate slower

→ lighter masses accelerate heavier

acceleration of object

《《《demo: "hammer your hand"》》》

sum of all external forces on object



forces exerted "internally" don't effect the trajectory of the rocket

$\vec{a} = \frac{d\vec{v}}{dt}$ and $\vec{v} = \frac{d\vec{x}}{dt}$

$$\vec{F} = (F_x, F_y, F_z)$$
$$\vec{a} = (a_x, a_y, a_z)$$
$$\vec{v} = (v_x, v_y, v_z)$$
$$\vec{x} = (x, y, z)$$

Crucially, x, y, z are independant!

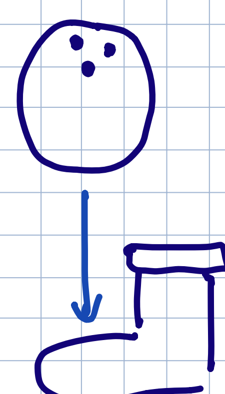
(This is the origin of parabolic motion.)

iii) Action / Reaction: "the force exerted on object 1 by object 2 is equal and opposite to the force exerted on object 2 by object 1"

$$\vec{F}_{12} = -\vec{F}_{21}$$

confusion iii a)

Seems unintuitive but that's because we "feel" accelerations.



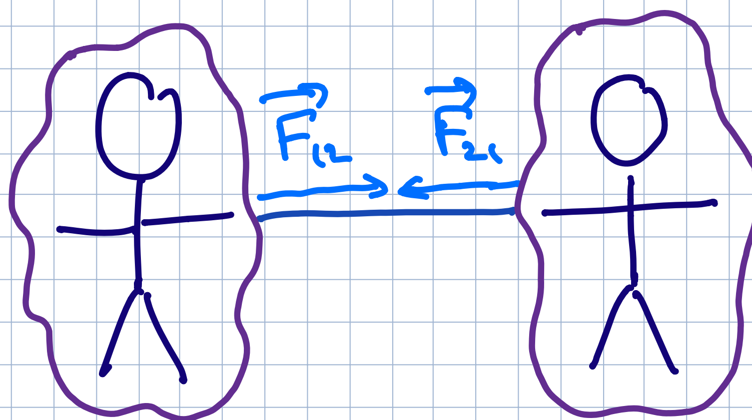
demo: "skateboard + ball"

confusion iiib)

Seems false because not all forces cancel...

but we are not saying that!

object #1



object #2

《《 demo: "recoiling cars" 》》

Equilibrium of Forces

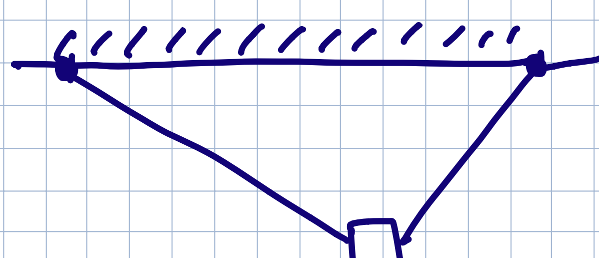
"the study of at-rest objects"

$$\vec{a} = 0 \implies \underbrace{\vec{F}}_{\text{total force on object}} = m\vec{a} = 0$$

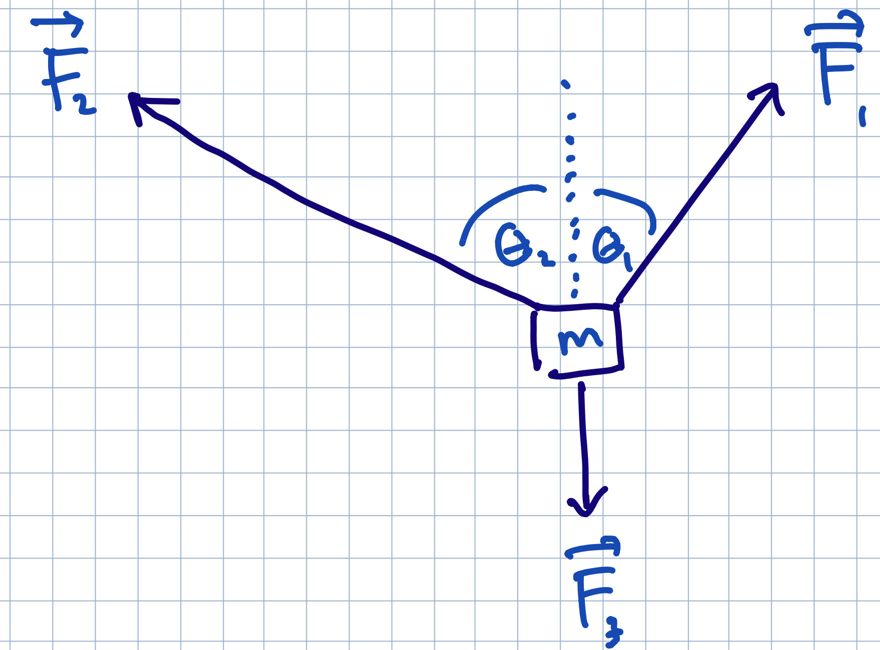
so $\vec{F} = \sum_i \vec{F}_i$

(ex. 1)

hanging box



Construct the "force diagram":



$$\vec{F}_1 = \left(T_1 \sin\theta_1 \; , \; T_1 \cos\theta_1 \right)$$

tension of each rope

$$\vec{F}_2 = \left(-T_2 \sin\theta_2 \; , \; T_2 \cos\theta_2 \right)$$

$$\vec{F}_3 = \left(0 \; , \; -mg \right)$$

gravitational force

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (0, 0)$$

$$\parallel$$

$$\left(T_1 \sin\theta_1 - T_2 \sin\theta_2 \; , \; T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg \right)$$

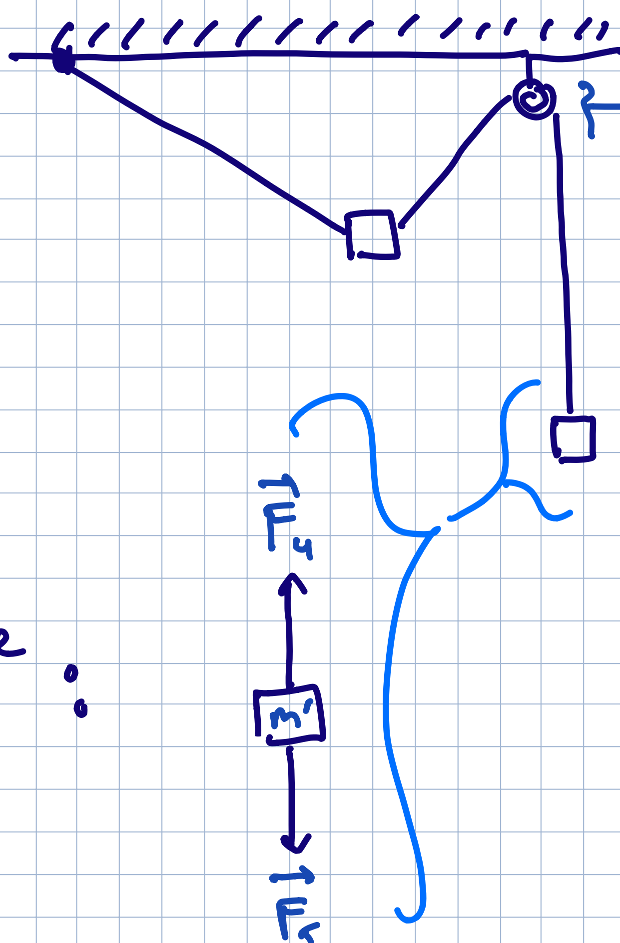
$$\Rightarrow T_1 = T_2 \frac{\sin\theta_2}{\sin\theta_1}$$

$$\Rightarrow T_2 \frac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 + T_2\cos\theta_2 = mg$$

So
$$\boxed{\begin{aligned} T_2 &= mg\,\frac{\sin\theta_1}{\sin\theta_1+\theta_2} \\ T_1 &= mg\,\frac{\sin\theta_2}{\sin\theta_1+\theta_2} \end{aligned}}$$

(ex. 2)

hanging box and pulley



perfect ideal pulley

$\Rightarrow$ add a force diagram:

$\Rightarrow$ add an equation:

$$\vec{F}_4 = (0, T_1)$$

$$\vec{F}_5 = (0, -m'g)$$

$$\vec{F}_4 + \vec{F}_5 = 0 \Rightarrow \boxed{T_1 = m'g}$$

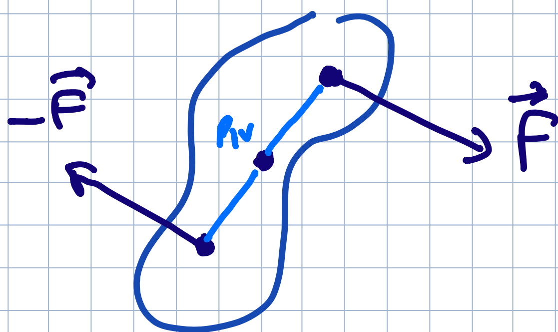
Check if it's reasonable: As $m' \to \infty$, $T_1 \to \infty$

so $\sin \theta_1 + \theta_2 \to 0$

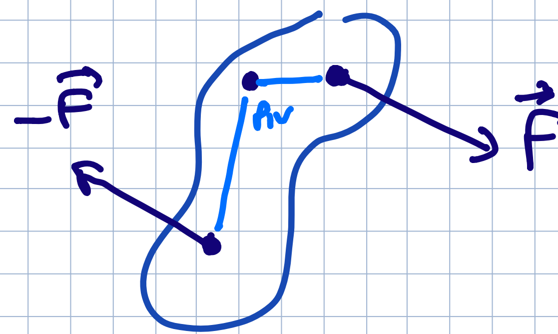
and $\theta_1 + \theta_2 = \pi$

demo: "methanol rocket"

Finally, note that for statics ($\vec{F} = \sum_i \vec{F}_i = 0$) we can measure the torques from <u>any</u> pivot point.



OR



Proof: $\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i$

$$\Big\downarrow \text{(shift pivot by } \vec{d}\text{)}$$

$$\sum_i (\vec{r}_i + \vec{d}) \times \vec{F}_i$$

$$=$$

$$\vec{\tau} + \sum_i \vec{d} \times \vec{F}_i$$

$$=$$

$$\vec{\tau} + \vec{d} \times \underbrace{\sum_i \vec{F}_i}_{=0}$$