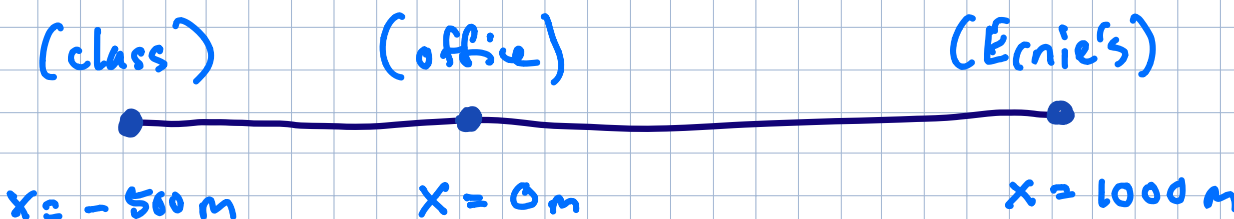
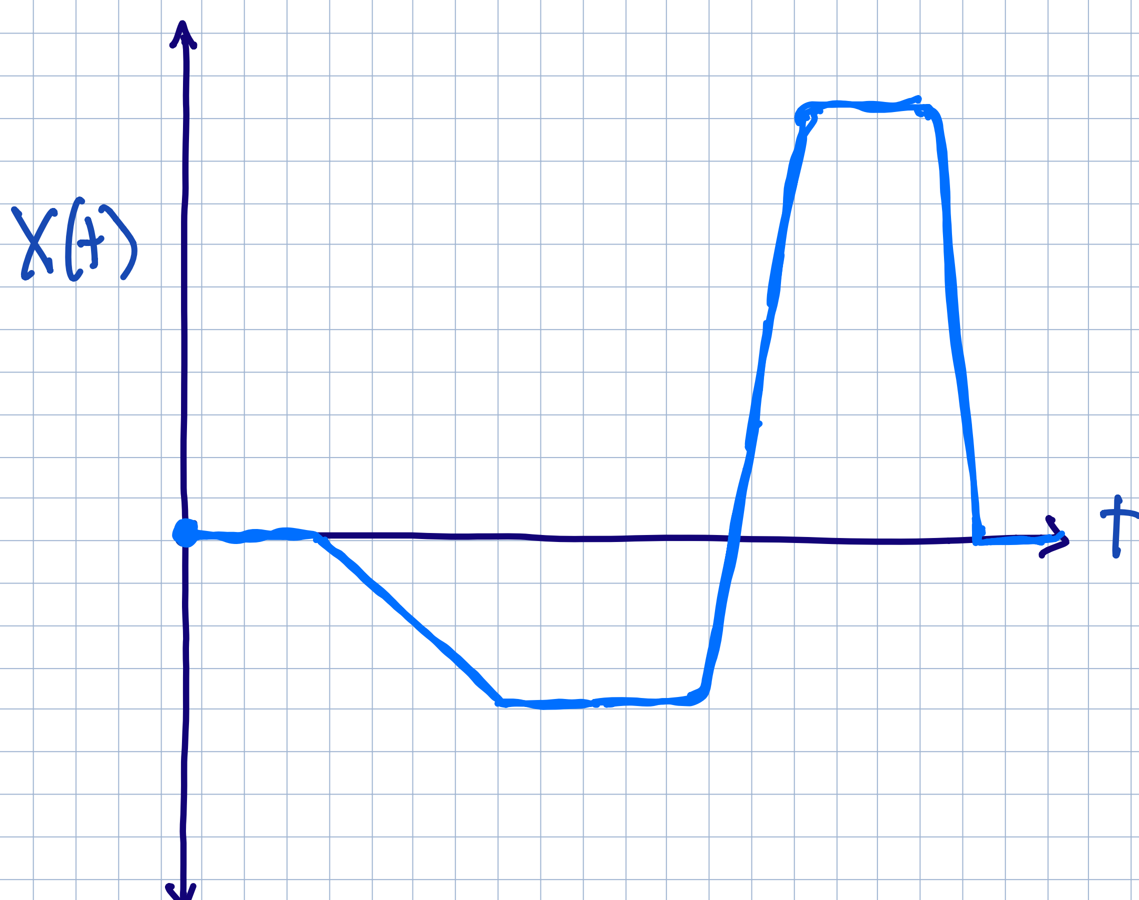
Lecture 2: Kinematics + Reference Frames

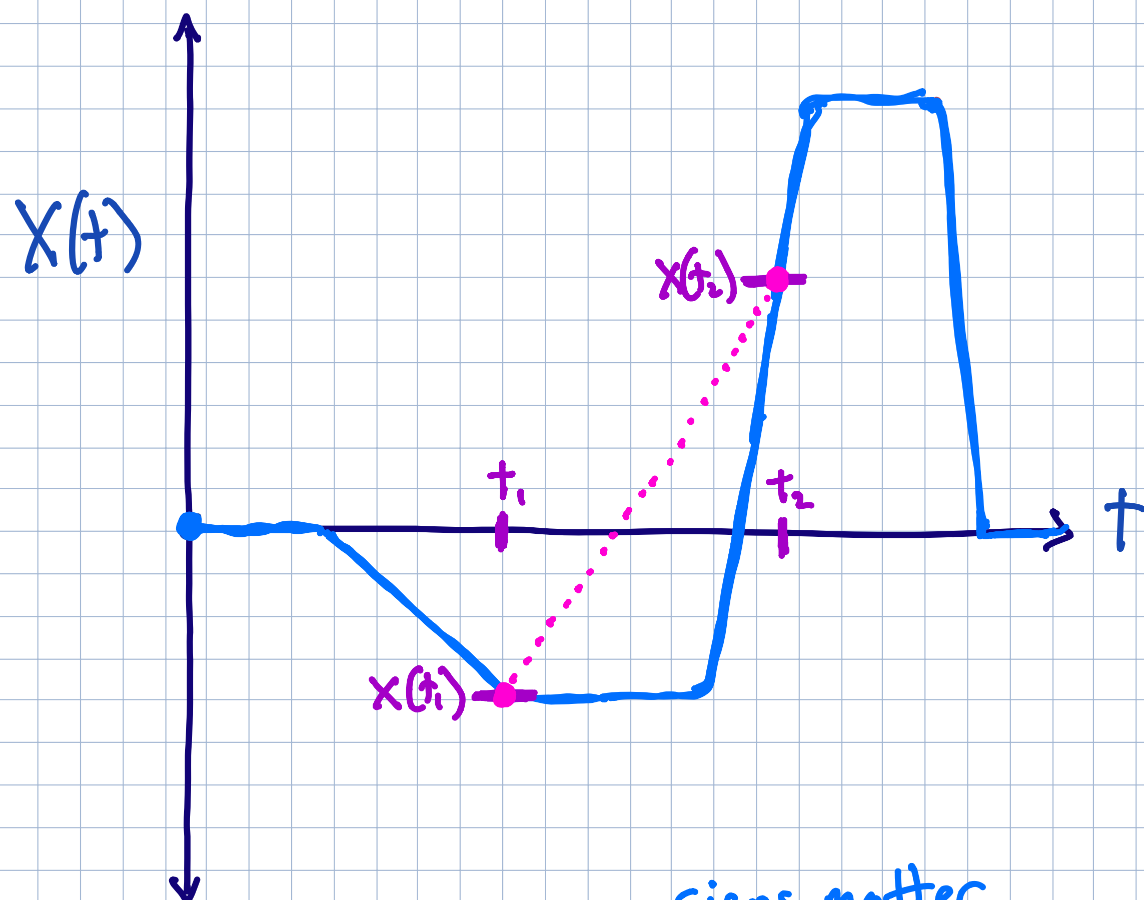
Kinematics



$x(t)$ = "position as a function of time"



$\bar{v}$ = "average velocity" (defined for a finite time interval $[t_1, t_2]$)

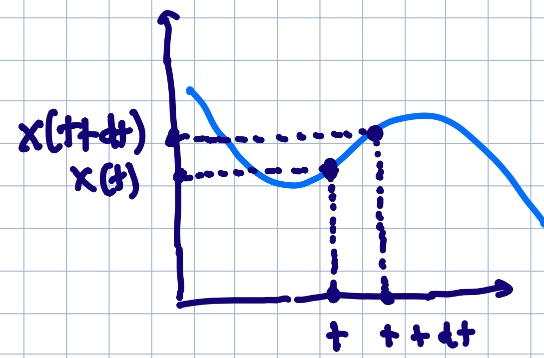


$$\bar{v} = \frac{\overbrace{x(t_2) - x(t_1)}^{\text{signs matter}}}{t_2 - t_1}$$

imperfect observable

Taking the limit $t_2 \to t_1$ yields the instantaneous velocity at time t_1.

More generally,



"instantaneous velocity"

$$v(t) = \lim_{dt \to 0} \frac{x(t+dt) - x(t)}{dt}$$

sometimes "t" dependence is suppressed

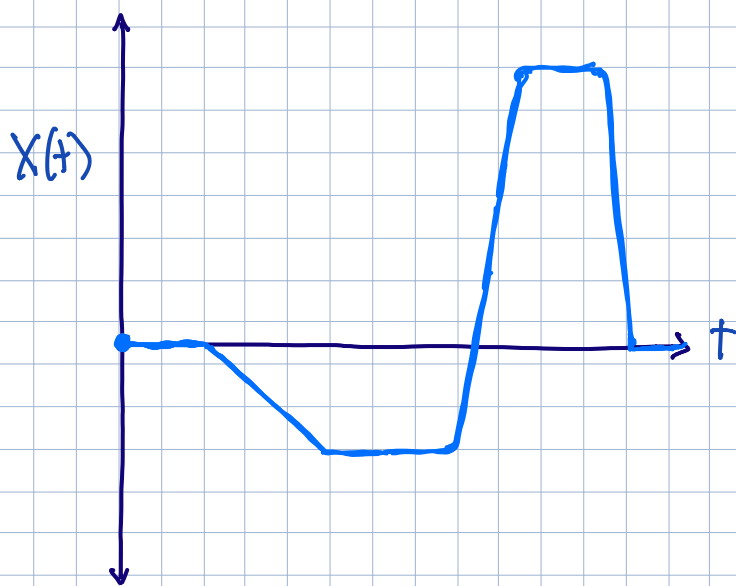
$$= \underbrace{\frac{dx(t)}{dt} = \frac{dx}{dt} = \dot{x}}_{\text{lots of different notation}}$$

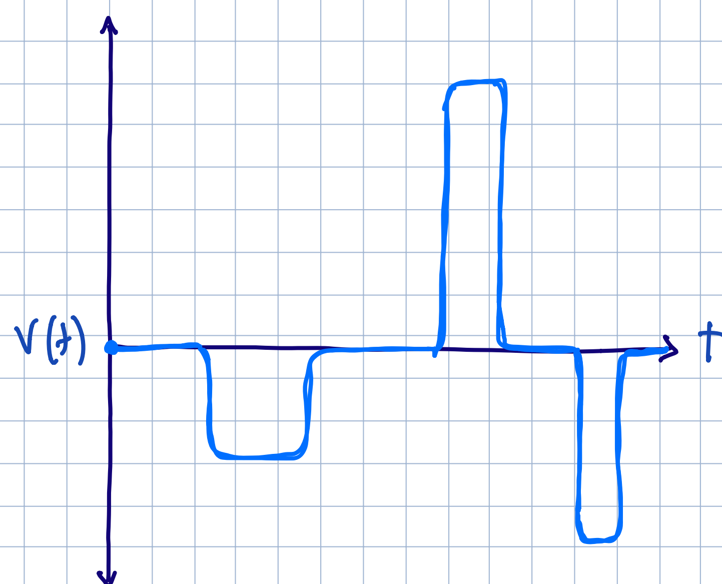
"instantaneous acceleration"

$$a(t) = \lim_{dt \to 0} \frac{v(t+dt) - v(t)}{dt} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$= \dot{v} = \ddot{x}$$

... and so on ...


x(t)
t


v(t)
t


a(t)
t

Recall that $\frac{d}{dt}(\text{constant}) = 0$.

Thus, each derivative has <u>less</u> information.

$$\Downarrow$$

<u>various functions of t</u>

x = position $\longleftarrow$ contains <u>all</u> information

$\frac{dx}{dt}$ = velocity $\longleftarrow$ initial position gone

$\frac{d^2x}{dt^2}$ = acceleration $\longleftarrow$ initial position and velocity gone

$\frac{d^3x}{dt^3}$ = jerk $\longleftarrow$ initial position, velocity, and acceleration gone.

e.g. consider a trajectory,

general parametrization of trajectory

$$x(t) = c_0 + c_1 t + \frac{c_2}{2!} t^2 + \frac{c_3}{3!} t^3 + \cdots$$

c_0 gone

$$\rightarrow v(t) = \frac{dx(t)}{dt} = c_1 + c_2 t + \frac{c_3}{2!} t^2 + \cdots$$

c_0, c_1 gone

$$\rightarrow a(t) = \frac{d^2x(t)}{dt^2} = c_2 + c_3 t + \cdots$$

$$\vdots$$

Thus $x(0) = c_0 =$ initial position

$v(0) = c_1 =$ initial velocity

$a(0) = c_2 =$ initial acceleration

$\vdots$

If you know $x(t)$, there is no work to do.

Every physics problem has as input partial info, e.g. you are given velocity or acceleration, etc.

$$a(t) \rightarrow v(t) \rightarrow x(t)$$

Free fall

$$\boxed{\text{fact of the day: things fall down}}$$

$a(t)$ = vertical acceleration on Earth

$$= -g = -9.8\ m/s^2 = \underline{\underline{\underline{\text{constant}}}}$$

$-$: downwards

for comparison,

$$g_{TESLA} \sim 1.1\,g$$

(ludicrous mode)

$$g_{\substack{\text{sun}\\\text{surface}}} \sim 28\,g$$

$$v(t) = \int a(t)\,dt = \int -g\,dt = -gt + \text{constant}$$

$$= -gt + V_0$$

V_0: initial velocity

$$x(t) = \int v(t)\,dt = \int -gt + v_0 \, dt$$

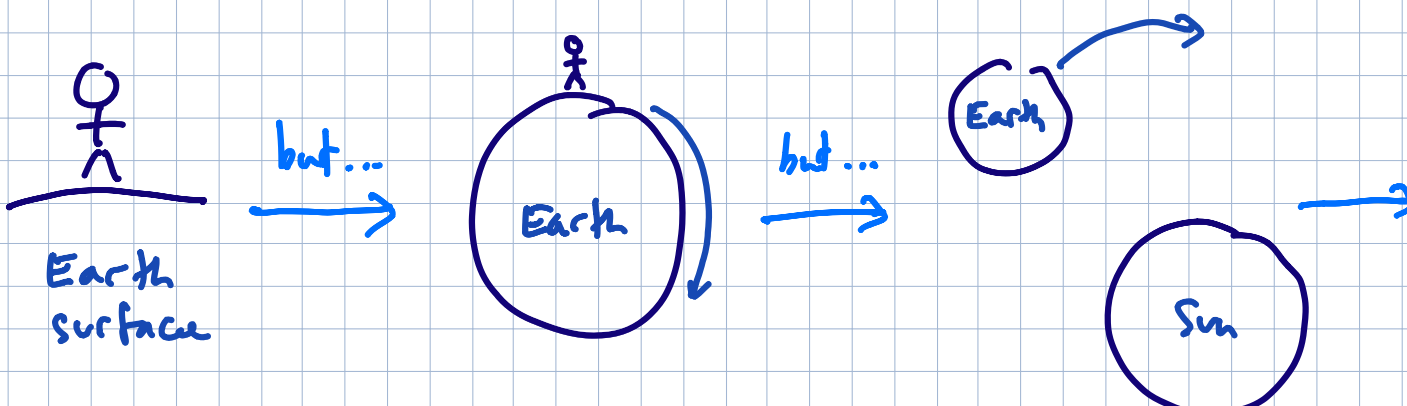
$$= -\tfrac{1}{2} g t^2 + v_0 t + \text{const}$$

$$= -\tfrac{1}{2} g t^2 + v_0 t + \underbrace{x_0}_{\text{initial position}}$$

《《《《《 demo: "penny + feather" 》》》》》

Reference Frames

The laws of physics are the same irrespective of whether you are at rest or at constant velocity.



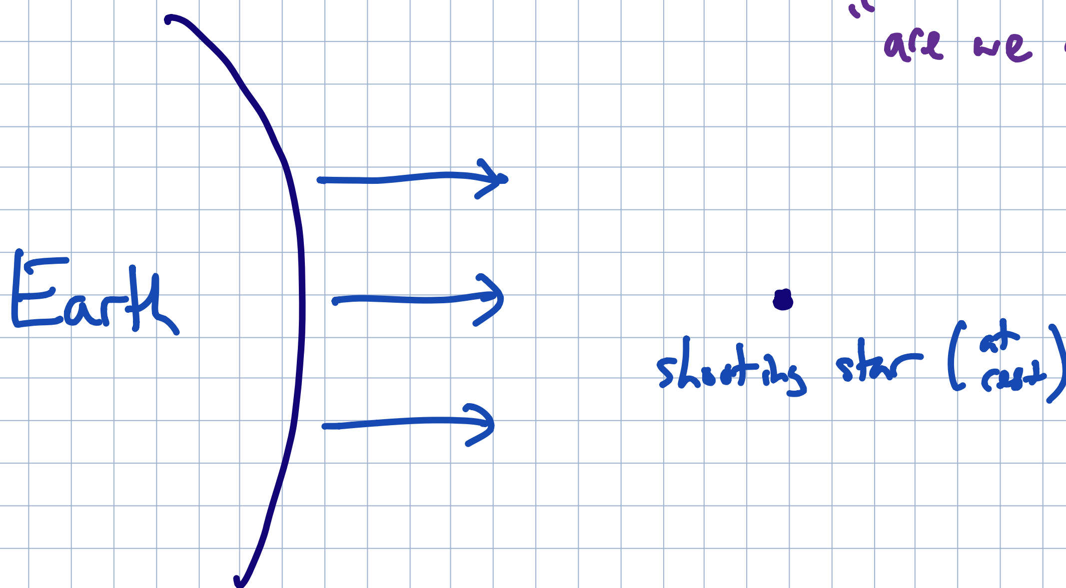
(ignore acceleration from gravity for now)

Super Deep Fact #1 : the question "are we motionless" is intrinsically meaningless

→ as meaningless as asking "am I upside down" in empty space

→ instead ask: "are we moving *relative* to ...?"

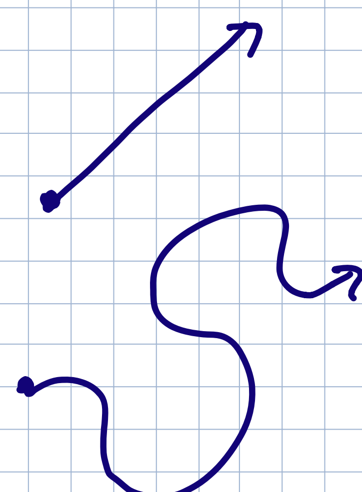
"are we accelerating?"



important nomenclature:

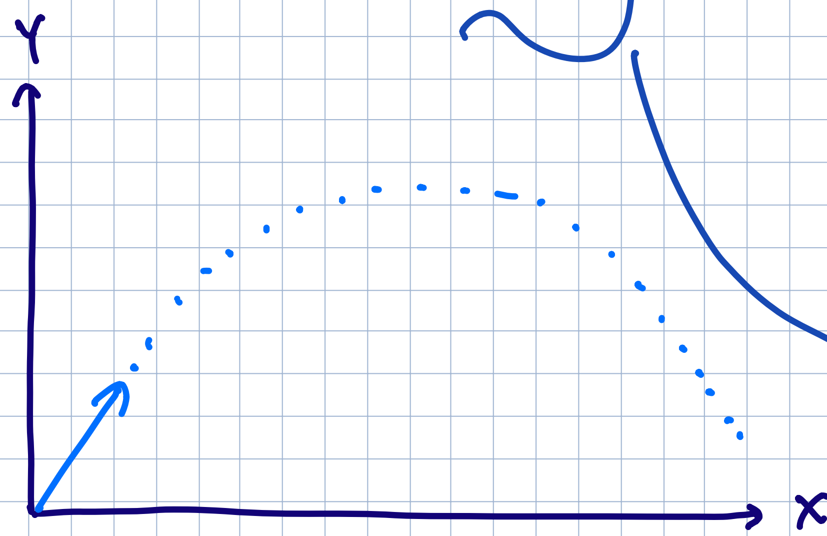
"inertial" $\left(\begin{array}{l} a = 0 \\ v = \text{const} \end{array} \right)$

"non-inertial" $\left(\begin{array}{l} a \neq 0 \\ v \neq \text{const} \end{array} \right)$

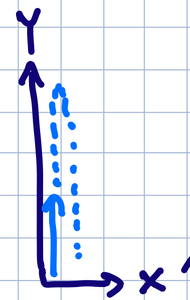


《《《 demo: "vertical canon" 》》》

Projectile Motion



equivalent to vertical motion in a moving frame



$$X(t) = V_{x0}\, t$$

initial velocity in x direction

$$Y(t) = -\frac{1}{2} g t^2 + V_{y0}\, t$$

initial velocity in y direction

Q1) What's the shape of the path?

Solve for t and plug back $\Rightarrow$ $t = X / V_{x0}$

$$\text{so} \quad \underbrace{y = -\frac{1}{2} g \left(\frac{x}{V_{x0}}\right)^2 + V_{y0}\left(\frac{x}{V_{x0}}\right)}_{\text{defines a parabola}} = y(x)$$

Q2) Where does it land?

Solve for $y = 0 = \frac{x}{V_{x0}}\left(-\frac{1}{2} g \frac{x}{V_{x0}} + V_{y0}\right)$

$\Rightarrow$ either $x = 0$ ← initial

$x = \frac{2 V_{x0} V_{y0}}{g}$ ← final

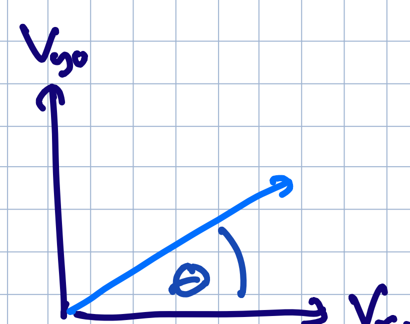
So $\boxed{X_{Land} = \frac{2 V_{x0} V_{y0}}{g}}$

Q3) What angle maximizes distance?

Define $V_{x0} = V_0 \cos\theta$

$V_{y0} = V_0 \sin\theta$

$V_0 = \sqrt{V_{x0}^2 + V_{y0}^2}$



$$X_{Land} = \frac{2}{g} V_0^2 \cos\theta \sin\theta$$

$$= \frac{V_0^2}{g} \sin 2\theta$$

Extremize $\frac{dX_{Land}}{d\theta} \sim \cos 2\theta = 0 \Rightarrow \boxed{\theta = 45°}$

《《《《《《 demo: "colliding marbles" 》》》》》》

《《《《《《 demo: "monkey shoot" 》》》》》》