Lecture 1: Units and Dimensional Analysis

SI Units ← "Système Internationale"

length (m = "meter")
time (s = "second")
mass (kg = "kilogram")

} fundamental units (assumes: universe labelled by rigid rulers + clocks)

force (N = "Newton" = $kg\ m/s^2$)
energy (J = "Joule" = $kg\ m^2/s^2$)

} derived units

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i) avoid nonsensical answers w/ wrong units

(e.g.) "How long was class ?!?"

... 5 } all nonsense
... 5 hamburgers
... 5 parsecs
... 5 hours } consistent! albeit unfortunate...

ii) multiply by "1" to convert units

$$1 \text{ lifetime} \times \left(\frac{100 \text{ yrs}}{1 \text{ lifetime}}\right) \times \left(\frac{365 \text{ days}}{1 \text{ yr}}\right)$$

$$\times \left(\frac{24 \text{ hrs}}{1 \text{ dy}}\right) \times \left(\frac{60 \text{ min}}{1 \text{ hr}}\right)$$

$$= 52,560,000 \text{ min}$$

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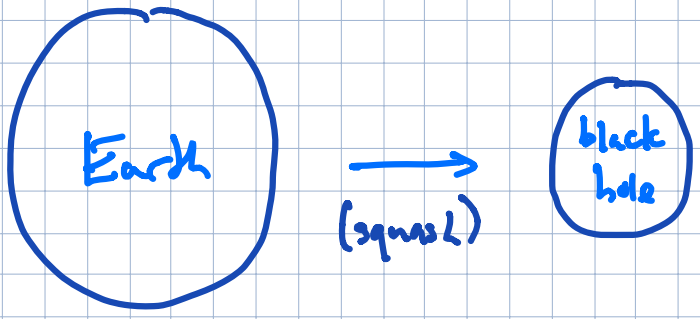
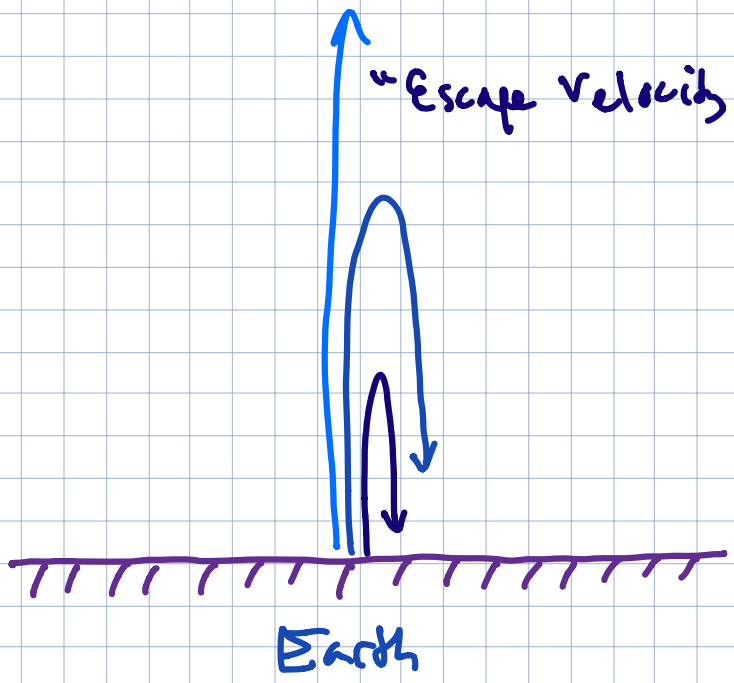
iii) use units to guess the answer

(e.g.) black hole Earth:

"At what radius does the Earth's mass collapse to a black hole???"



Light cannot escape a black hole.



$$v_{esc}^2 = (\text{dimensionless } \#) \times G \times M \times \frac{1}{R}$$

mandated by units

v_{esc}^2: $\frac{m^2}{s^2}$; G: $7 \times 10^{-11} \times \frac{m^3}{kg\, s^2}$; M: kg ; $\frac{1}{R}$: $\frac{1}{m}$

$$R = (\text{dimensionless } \#) \times \frac{GM}{v_{esc}^2}$$

$$= \# \times \frac{\left(7 \times 10^{-11} \frac{m^3}{kg\, s^2}\right)\left(6 \times 10^{24}\, kg\right)}{\left(3 \times 10^{8}\, m/s\right)^2} \sim \boxed{(\#) \times 0.5\ cm}$$

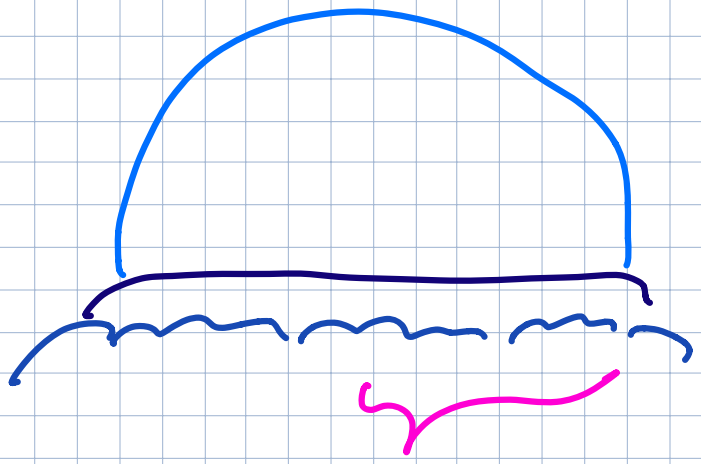
$= 2$ via actual calculation

(e.g.) Trinity Test

E = energy of blast

T = time elapsed

ρ = density of air



R = radius of blast

$$R = E^i T^j \rho^k$$

$$[m] = [kg\ m^2/s^2]^i [s]^j [kg/m^3]^k$$

$$= [kg]^{i+k} [m]^{2i-3k} [s]^{j-2i}$$

$$i + k = 0$$
$$2i - 3k = 1 \quad \rightarrow \quad i = 1/5,\ j = 2/5,\ k = -1/5$$
$$j - 2i = 0$$

$$R = \frac{E^{1/5} T^{2/5}}{\rho^{1/5}} \Rightarrow E = \rho \frac{R^5}{T^2}$$

$$= (1\,kg/m^3)\,(120\,m)^5 \Big/ (0.025\,s)^2$$

$$= 3.98 \cdot 10^{13}\,J$$

(correct within order of magnitude)

~ energy in 1 gram of matter

~ kinetic energy of the ISS