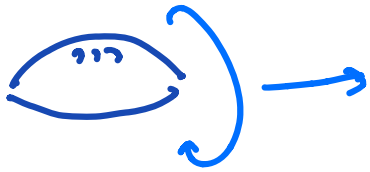
Lecture 18: Gyroscopes

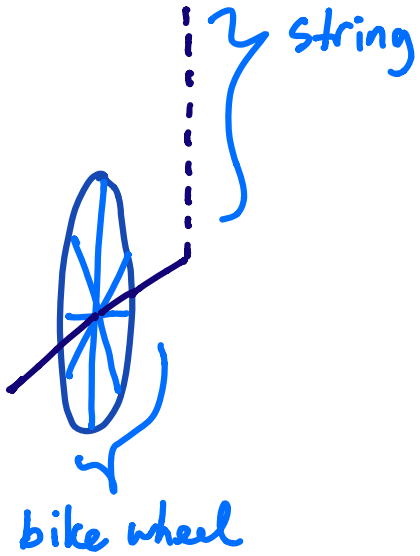
Many wondrous human inventions utilize angular momentum.

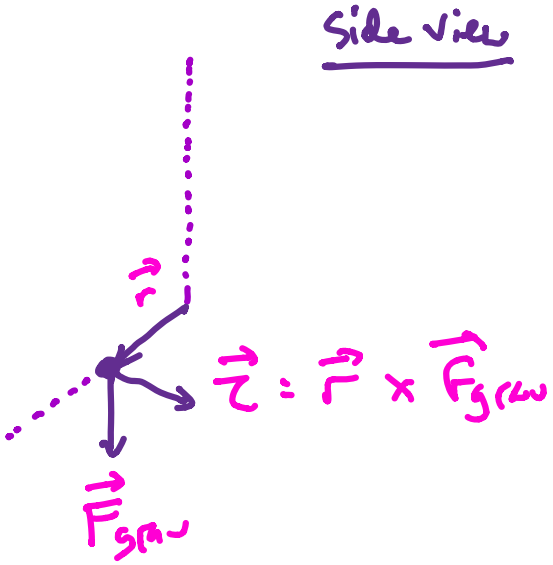
- frisbees, footballs 
- rifling 
- steadicam
- ship stabilizers

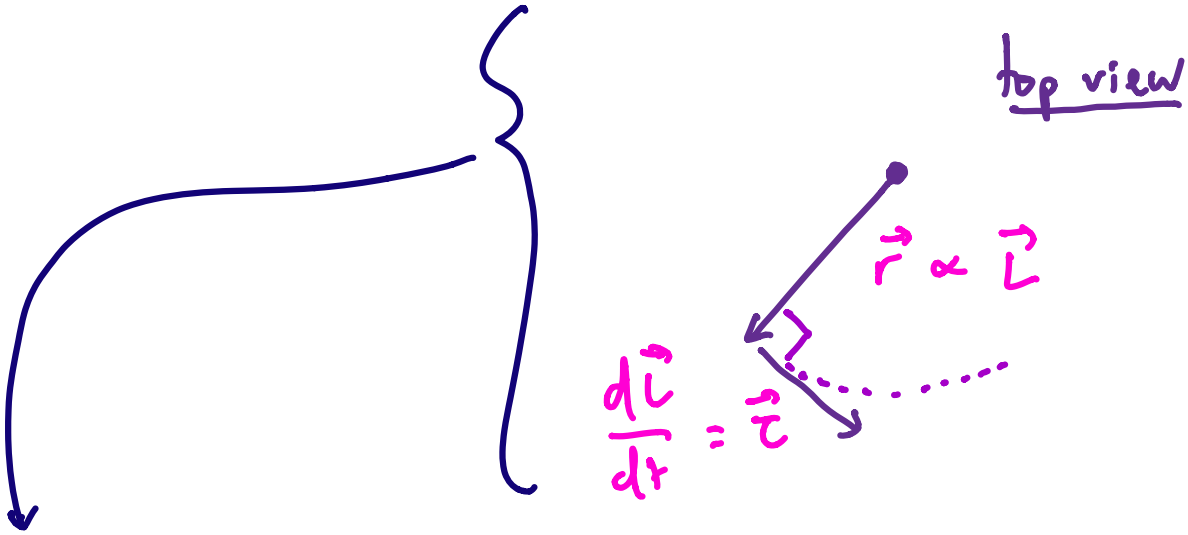
These all function to stabilize a direction.

Precession

Let's start w/ a peculiar demo.

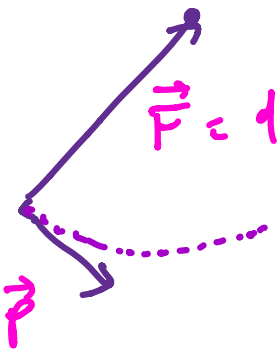




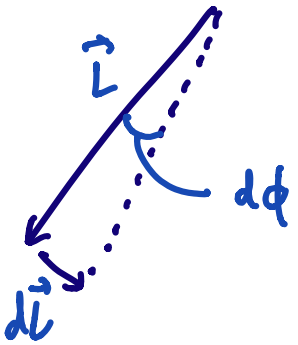


$\vec{L} \perp \frac{d\vec{L}}{dt}$ in analogy with $\vec{v} \perp \vec{a}$ for circular motion.

For example, recall that for circular orbits,



so $\vec{p} \perp \frac{d\vec{p}}{dt}$



$$dL = L\, d\phi$$

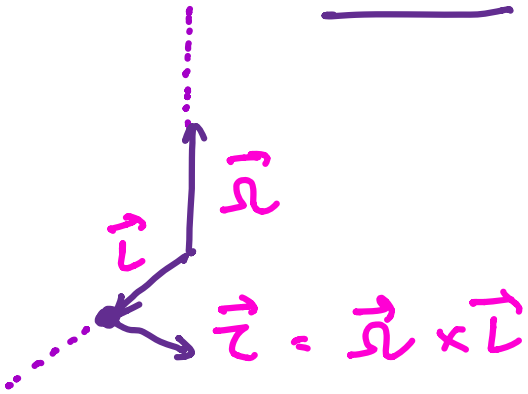
$$\Downarrow$$

$$\frac{dL}{dt} = L\frac{d\phi}{dt} = L\Omega$$

frequency of "precession"

In terms of vectors,

Side view

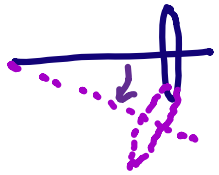


Conservation of Energy + Angular Momentum

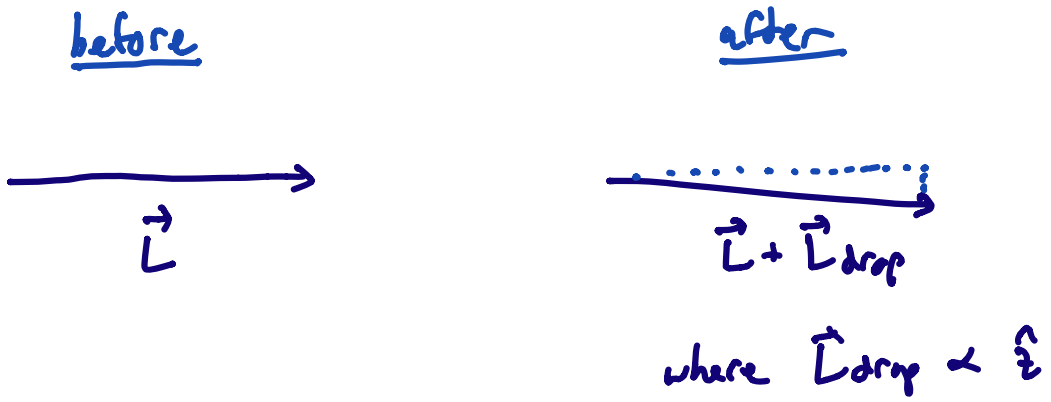
kinetic energy of precession $= \frac{1}{2} I_p \Omega^2$

I_p: moment of inertia about precession axis

This energy comes from potential energy released:



However, this slight angle implies a slight $\hat{z}$ torque.



where $\vec{L}_{drop} \propto \hat{z}$

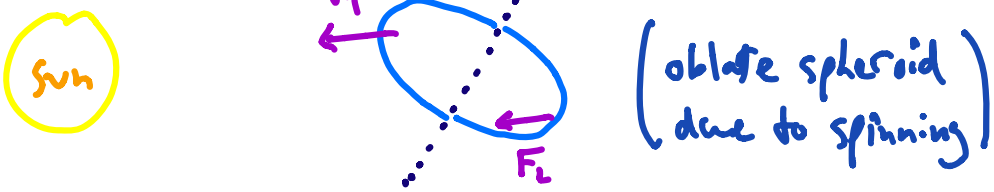
But there is no torque in $\hat{z}$ direction!

This is consistent because $\vec{L}_{drop} + I_p \vec{\Omega} = 0$

balanced

Examples of Precession:

i) Earth



Since $|\vec{F}_1| > |\vec{F}_2|$, there is a torque.

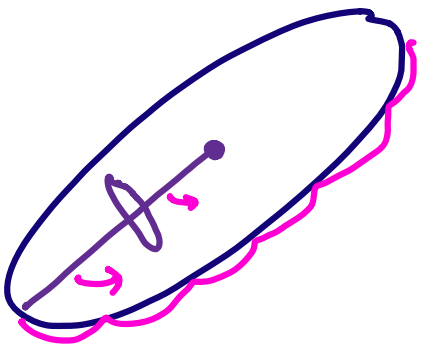
With torque there is precession ($\Omega \sim {}^{2\pi}/_{25{,}000\ yr}$)

ii) gyroscope

demo: "nutation"

Nutation

In reality, if you "drop" the bicycle wheel:



nutation = oscillation about equilibrium