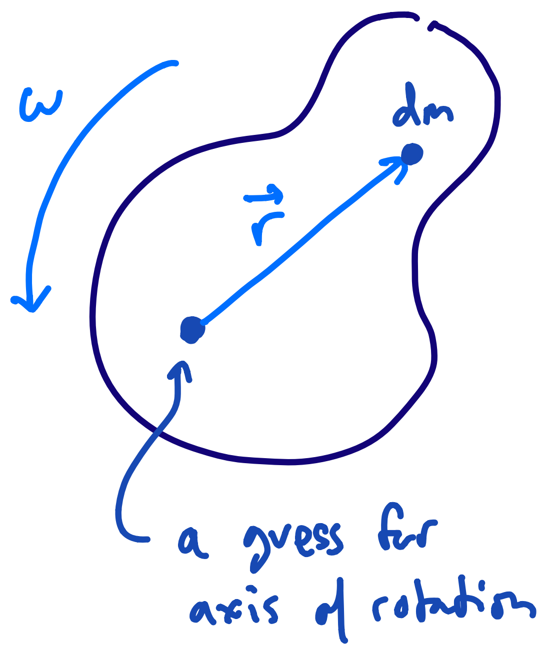
# Lecture 11: Rotational Dynamics II

## Free Rotation

Consider an object spinning off in empty space.



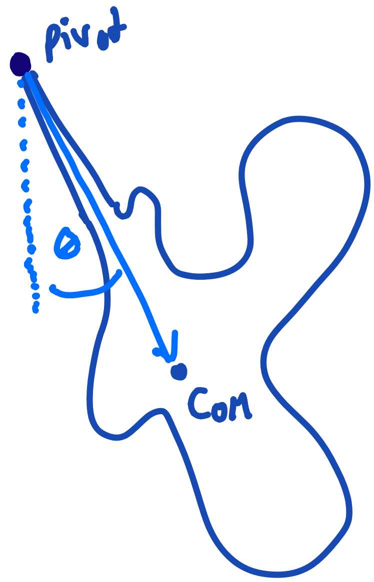

Centripetal force on dm $= d\vec{F} = -dm\,\omega^2\,\vec{r}$

no external force $\Longrightarrow 0 = \vec{F} = \int d\vec{F} = -\omega^2 \int dm\,\vec{r}$

$\int dm\,\vec{r} = 0$ implies that $\boxed{\text{Pivot axis} = \text{CoM axis}}$

bottom line: objects in empty space rotate about there center of mass

## Physical Pendulum




gravity acts to decrease $\theta$

$= \frac{d^2\theta}{dt^2}$

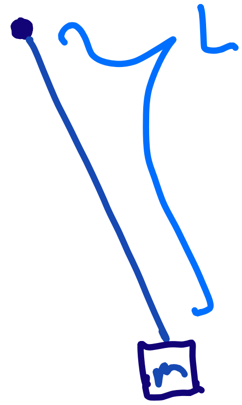
$$\tau = -mgr\sin\theta = I\ddot{\theta}$$

for small $\theta \ll 1$,

we will see later, this defines oscillatory motion w/ the period $T = 2\pi\sqrt{\frac{I}{mgr}}$

$$\ddot{\theta} = -\frac{mgr}{I}\theta$$

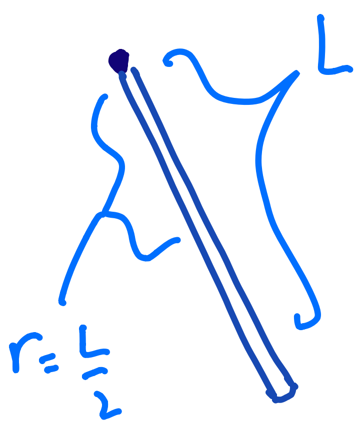
Compare a simple versus stick pendulum.




$$I_{simp} = mL^2 \quad \text{and} \quad r = L$$

$$\Downarrow$$

$$T_{simp} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$



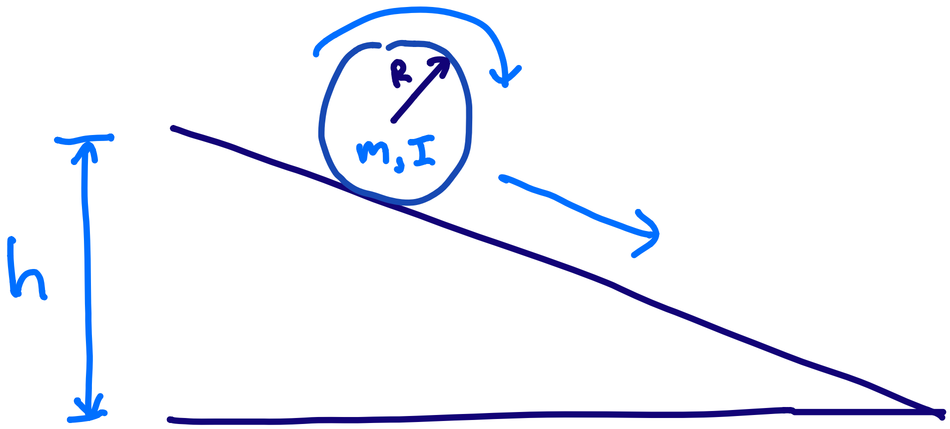

$$I_{stick} = \int dm\, l^2 = \int_0^L dl\, \overbrace{\frac{m}{L}}^{\text{density}}\, l^2$$

$$= \frac{m}{L} \cdot \frac{L^3}{3} = \frac{1}{3} mL^2$$

$$T_{stick} = 2\pi\sqrt{\frac{\frac{1}{3} mL^2}{mgL/2}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\Longrightarrow \quad T_{simp} > T_{stick}$$

# Rolling without Slipping




Energy Conservation:

final velocity and rotation

$$mgh = \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

initial state only has potential energy

final state only has kinetic energy

define $I = kmR^2$

$k$ = numerical constant

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}km\underbrace{R^2\omega^2}_{=v^2}$$

$$2gh = v^2(1+k)$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2gh}{1+k}}}$$

| type | $k$ | $v/\sqrt{2gh}$ |
| --- | --- | --- |
| point | 0 | 1 |
| ring | 1 | $\sqrt{1/2} \sim 0.71$ |
| disc | $\frac{1}{2}$ | $\sqrt{2/3} \sim 0.82$ |
| shell | $\frac{2}{3}$ | $\sqrt{3/5} \sim 0.77$ |
| ball | $\frac{2}{5}$ | $\sqrt{5/7} \sim 0.85$ |