

FP13

A long massless stick is rotating about one of its ends at angular frequency Ω . This motion is enforced by a motor which can maintain the rotation frequency constant no matter what. A massless spring of force constant k is wrapped around the stick with one end attached to the motionless end of the stick. The spring's equilibrium length is R and it is free to expand and contract without friction along the stick. A mass M is attached to the free end of the spring and can slide along the stick without friction. There is no gravity in this problem. *Hint:* You may find it easier to think about this problem in the rest frame of the rotating stick.

- (5 points) Assuming there are no oscillations, find the equilibrium position r_0 of the mass M , as a function of Ω , k , R and M . Determine the rotation frequency Ω above which the spring will be stretched indefinitely.
- (5 points) Find the frequency ω of small oscillations of the mass M about the equilibrium position, assuming the rotation frequency Ω is not too large.
- (5 points) Assuming the mass is executing simple harmonic motion $r = r_0 + A\cos(\omega t)$ with t the time, find an expression for the torque τ needed to maintain the stick's constant rotation rate.

