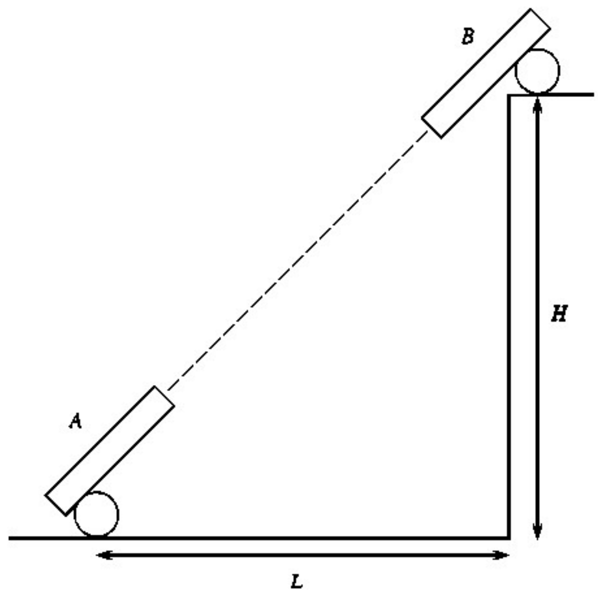
FP1

Cannon A is located on a plain a distance L from a wall of height H. On top of thise wall is an identical cannon (cannon B). Ignore air resistance throughout this problem. Also ignore the size of the cannons relative to L and H.



a) (3 points) The two groups of gunners aim the cannons directly at each other. They fire at each other simultaneously, with equal muzzle speeds v_0. What is the value v_{min} of v_0 for which the two cannon balls collide just as they hit the ground?

b) (3 points) Describe what happens for muzzle velocities greater than v_{min} and less than v_{min}?

c) (2 points) Cannon B breaks, and the gunners don't know how to fix it, so they decide to use a large sling, which hurls rocks. The sling has a radius of 5 m, rotates at 10 revolutions/minute, and hurls objects out in the direction of cannon A with a purely horizontal velocity. If $H = 1$km, where will the projectiles from this sling land?

d) (2 points) The rocks from the sling fall short of the location of cannon A, hitting the plain at a distance only 1/4 of the way from the base of the wall to cannon A. Assuming that the sling can only hurl rocks with a horizontal velocity, but has an angular velocity adjustable smaller or larger by a factor of two and a radius adjustable smaller or larger by the same factor, how can the sling be adjusted so that the rocks hit cannon A, or is this impossible to do with the sling?

FP2

Two particles of masses $m_1 = m$ and $m_2 = am$ collide after traveling in the $x - y$ plane with inital velocities $\mathbf{v}_1 = v\hat{x}$ and $\mathbf{v}_2 = bv(cos\theta_{\hat{x}} + sin\theta_{\hat{y}})$ where a and b are postive constants. Gravity is not present in this problem.

a) (2 points) What is the total energy and linear momentum of the two-mass system prior to the collision?

b) (3 points) If the collision is totally inelastic (ie., the masses stick together), how much kinetic energy is lost in the collision?

FP3

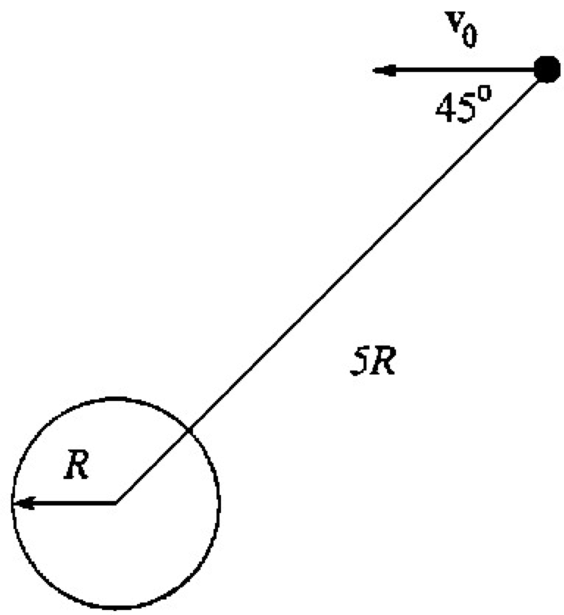
An ideal fluid of density 6000 kg/m^3 flows steadily in a hose with a velocity of 5 m/s at an absolute pressure of 0.5 atm. The hose has a uniform radius of 5 cm. It runs out a window and the lower end lies on the ground below the window. The lower end is open to the air, but someone blocks off half of its open area.

a) (2 points) What is the velocity of this fluid immediately outside the end of the hose?

b) (3 points) How far above the ground is the window located?

FP4

From a distance of $5R$ from the center of a planet of mass M and radius R, a satellite of mass m is launched with a speed $v_0 = \sqrt{\frac{GM}{5R}}$ at time $t = 0$ in the direction shown in the figure.



a) (2 points) Calculate the energy E of the satellite in terms of G, m, M and R. What type of orbit (i.e. what type of curve) does the satellite follow?

b) (2 points) Calculate the magnitude of the angular momentum in terms of G, m, M, and R.

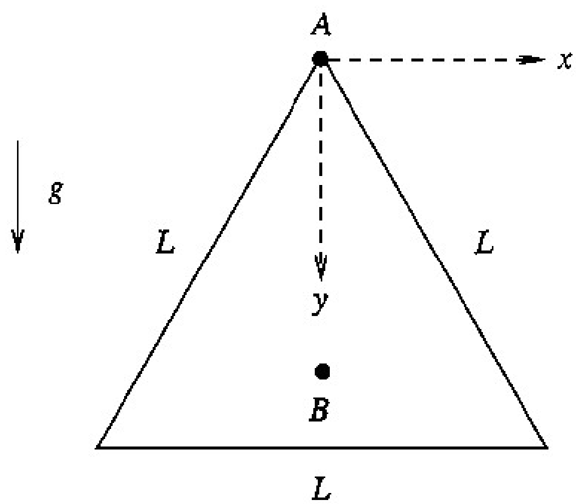
c) (4 points) Using conservation of energy and angular momentum, calculate the speed of the satellite at its perigee (the point of closest approach) in terms of G, m, M and R. *Hint:* At perigee, the velocity v is perpendicular to the radius vector $\mathbf{r}$.

   Suppose that a disturbance at the core of the planet suddenly reduces the planet's mass to $M/2$. The remainder of the planet's mass is vaporized and is quickly ejected to distances well outside the orbit of the satellite. The center of mass of the planet is not accelerated by the disturbance (the vaporized matter is ejected symmetrically) and the satellite is not struck by the flying debris.

d) (3 points) If the disturbance occurs at time t = 0 (the time depicted in the figure), what type of orbit does the satellite follow?

FP5

A thin uniform plate (mass M), in the shape of an equilateral triangle (side L), is suspended from one vertex (at A in the figure), forming a physical pendulum. The triangle swings about an axis perpendicular to the plate through point A. Take $x-y$ coordinates as shown, so that $w(y) = \frac{2}{\sqrt{3}}y$ is the width of the triangle a vertical distance y from A. Our goal is to calculate the period for small oscillations about A.



a) (3 points) Find the coordinated of the center of mass (x_{cm},y_{cm}). *Hint:* One method involves breaking the triangle into horizontal strips of mass dm and then integrating. There is also a symmetry argument.

b) (4 points) Calculate the moment of inertia I_A about the axis through A. *Hint:* Apply the parallel axis theorem to each horizontal strip and then integrate.

c) (2 points) What is the period of small oscillations about A? Leave your answer in terms of I_A if you were unable to solve part (b).

d) **Extra Credit** (2 points) We now move the suspension to a second point B on the y axis such that, when the system is inverted, small oscillations have the same period as about A. Find the coordinates y_B of this point relative to the coordinate system centered on point A.

FP6

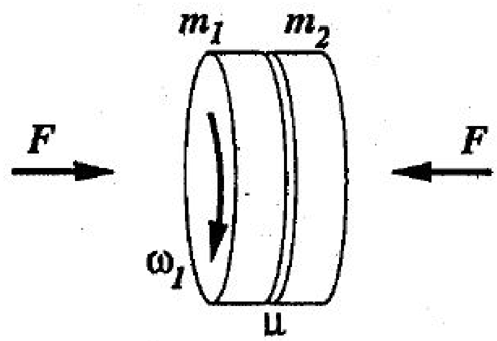
For the following problem use on physics from this term and imagine that light is composed of particles called photons. A black hole is an object whose gravity is so strong that even light cannot escape. The radius, or "event horizon", of a black hole can be defined by the innermost distance from which light can escape. Recall that light propagates with velocity $c = 3\,\mathrm{x}10^8$ m/s. Hint: note that the escape velocity of a particle, including a photon is independent of its mass.

a) (4 points) Find an expression for the radius of a black hole of mass M. What is this radius for a black hole with the mass of our sun $M_\odot = 2.0\,\mathrm{x}10^{30}$kg.

b) (3 points) If you find yourself standing at the event horizon of a black hole, the force of gravity at your feet is stronger than at your head. Assuming that your height $h = 2m$, what is the difference in the acceleration due to gravity between your head and your feet for a solar mass black hole? (You may assume that $h << R$ if it is convenient.)

c) (3 points) Considering your answer to part (b), would you be better off standing at the event horizon of a solar mass black hole or a much larger supermassive black hole ($M_{SM} = 10^7 M_\odot$) like those found in the center of most galaxies? Explain.

FP7



A rudimentary transmission can be made by forcing two uniform cylindrical wheels with frictional coefficient μ together. The wheels have masses m_1 and m_2, and both have radius R. Initially, wheel 1 is rotating with angular velocity ω_1 and wheel 2 is at rest. The wheels are being forced together with a constant force F, uniformly distributed across each wheel's face.

a) (3 points) When the wheels are first brought together, what is the magnitude of the torque that wheel 1 applies to wheel 2 via friction? Hint: split wheel 1 up into infinitesimal concentric rings of radius r and width dr, and calculate the torque exerted by them.

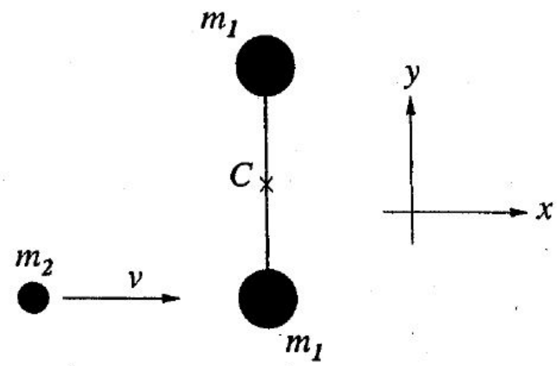
   If you could not answer part (a) use $r = \mu FR$ in what follows.

b) (2 points) What is the final angular velocity, ω_f, of the two wheels (in terms of constants and ω_1)?

c) (3 points) How long does it take to reach ω_f?

d) (2 points) What was the change in energy of the system as a fraction of the initial energy? Where did this energy go?

FP8



A massless rod of length $2R$ has masses m_1 attached to each end. The rod is free to rotate on the surface of a frictionless table about a pin attached to its center C. A mass m_2 is moving across the table at speed v perpendicular to the rod and is aimed directly at one of the masses m_1 as shown in the diagram. At time $t = 0$, m_2 collides with m_1 and sticks instantaneously, setting the rod into rotation about C.

a) (2 points) Considering only the rod and masses, which, if any, of the following quantities are conserved in this collision:

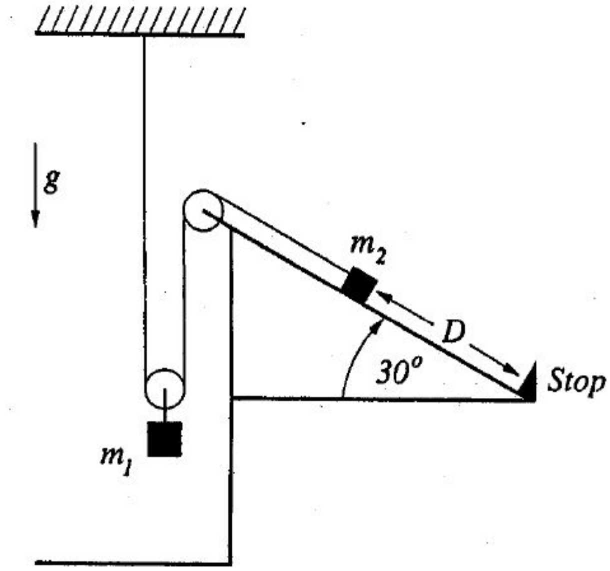
   1) Linear Momentum?
   2) Anguluar Momentum about C?
   3) Kinetic Energy?

b) (3 points) What is the angular velocity ω following the collision, and is it constant?

c) (3 points) Determine the total linear momentum P of the rod and mass system following the collision. Give both the x and y components and specify their time dependence, if any.

d) (2 points) Determine the force F exerted by the pin at C. Again, give both the x and y components and specify their time dependences, if any. Also give the magnitude of the force.

FP9



Two masses are connected by a string as shown. m_2 slides without friction on a fixed incline at an angle of 30° with respect to the horizontal. Neglect the mass and friction of the pulleys, and the mass of the string.

a) (2 points) Find the ratio of the masses m_2/m_1 such that the masses will remain stationary, if they are initially at rest.

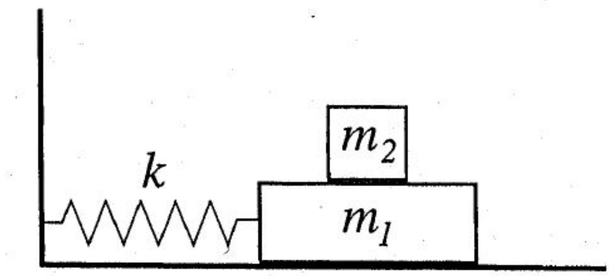
b) (1 points) If the mass m_2 moves a small distance ΔD_2 along the incline, find the distance ΔD_1 that the mass m_1 moves.

c) (3 points) If $m_2 = 2m_1$, adn the masses are initially at rest as shown, find the acceleration of m_2.

d) (3 points) If m_2 slides a distance, D down the incline before encountering the stop at the bottom, what are the speeds of m_2 and m_1 just before encountering the stop?

e) (1 point) WHen the moments of interia of the pulleys are taken into account, do the speeds of the masses in part (d) increase, decrease, or remain the same?

FP10



A large block of mass m_1 executes horizontal simple harmonic motion as it slides across a frictionless surface under the action of a spring constant k. A block of mass m_2 rests upon m_1. The coefficient of friction between the two blocks is μ. Assume that m_2 does not slip relative to m_1.

a) (3 points) Draw the free body diagrams for m_1 and m_2 at a time when the spring is stretched a distance x beyond equilibrium to the right.

b) (2 points) Write down the horizontal and vertical equations of motion for blocks m_1 and m_2.

c) (2 points) What is the angular frequncy of oscillation, ω, of the system?

d) (3 points) What is the maximum amplitude of oscillation, A, that the system can have if m_2 is not to slip relative to m_1? (Write your answer in terms of ω.)

FP11

An ice cube of side a floats in a much larger glass of water.

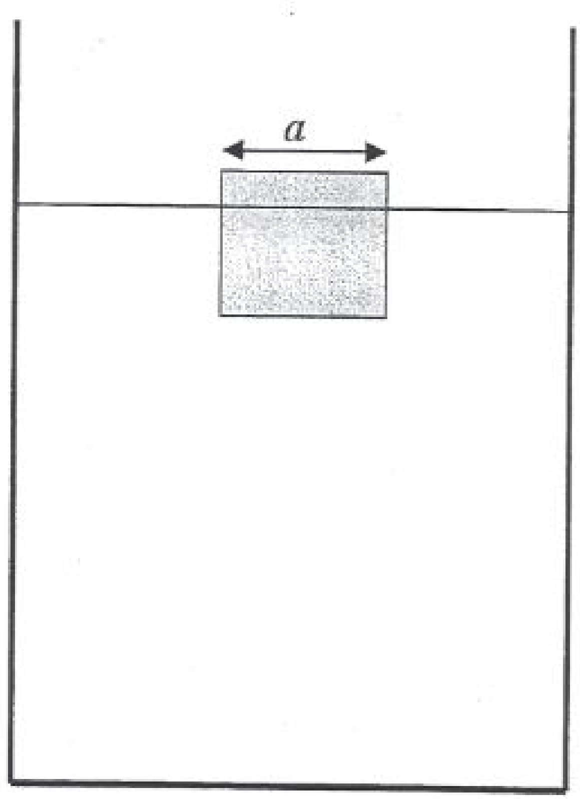
a) (2 points) In terms of the densities of water (ρ_W) and ice (ρ_I), how far below the surface of the water is the bottom face of the cube?

   The cube is now lifted distance y_0 above its equilibrium position. At time $t = 0$ the cube is released.

b) (1 point) Draw a free body diagram of the ice cube, valid at $t = 0$.

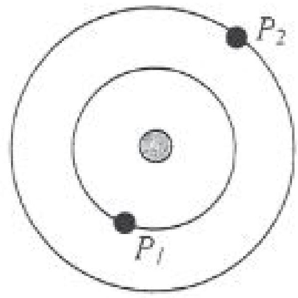
c) (4 points) Show that the vertical motion for $t > 0$ is simple harmonic. Find the frequency of the oscillations, and give a precise expression for the displacement of the cube from its equilibrium position as a function of time t. Assume that there is no damping of the oscillations.

d) (3 points) Eventually, the ice cube melts. Relative to the water level before the ice melts, where is the level after melting? Neglect any temperature variations of the density of ice and water. Justify your answer.



FP12

Two planets each of mass m are in counter-clockwise circular orbits around a star of mass M (see the figure below). The inner planet P_1 is in an orbit of radius R, while the outer planet P_2 is in an orbit of radius $2R$. Express your answer to all parts of this problem in terms of R, M, m and G, Newton's gravitational constant. Assume $m << M$.

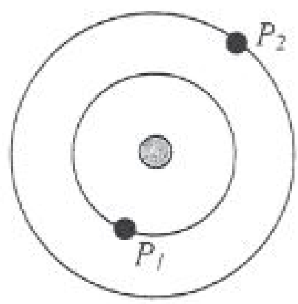


a) (2 points) What is the ratio T_2/T_1 of the orbital period of planet P_2 to that of P_1? What is the value of T_1?

b) (2 points) What is the ratio of E_2/E_1 of the total energies of the orbits? What is the value of E_1?

c) (2 points) What is the ratio L_2/L_1 of the orbital angular momenta of the orbits? What is the value of L_1?

Now suppose that we send a spacecraft of mass m_s from P_1 to P_2 along the elliptical transfer orbit shown below. The orbit has a "periastron" (the distance of closest approach to the star about which the planets are orbiting) of R and an "apastron" (the greatest distance to the star) of $2R$.



d) (4 points) What are the semi-major axis a, the energy E, the angular momentum L and the eccentricity e of the transfer orbit?

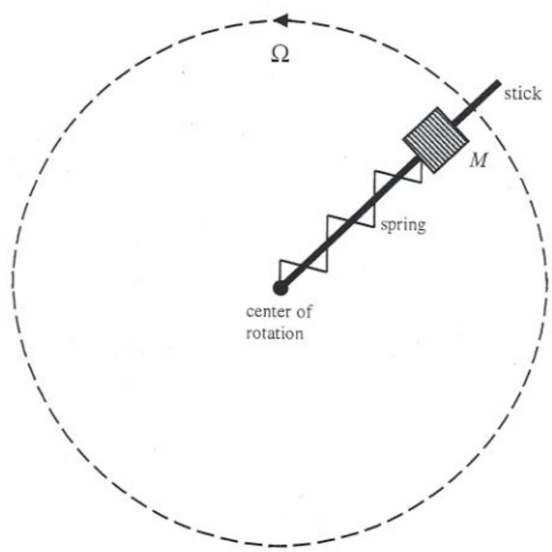
FP13

A long massless stick is rotating about one of its ends at angular frequency Ω. This motion is enforced by a motor which can maintain the rotation frequency constant no matter what. A massless spring of force constant k is wrapped around the stick with one end attached to the motionless end of the stick. The spring's equilibrium length is R and it is free to expand and contract without friction along the stick. A mass M is attached to the free end of the spring and can slide along the stick without friction. There is no gravity in this problem. *Hint:* You may find it easier to think about this problem in the rest frame of the rotating stick.

a) (5 points) Assuming there are no oscillations, find the equilibrium position r_0 of the mass M, as a function of Ω, k, R and M. Determine the rotation frequency Ω above which the spring will be stretched indefinitely.

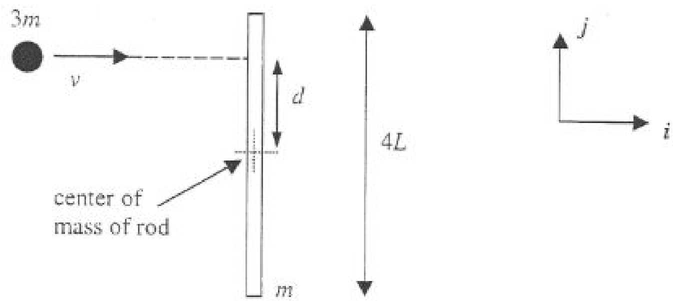
b) (5 points) Find the frequency ω of small oscillations of the mass M about the equillibrium position, assuming the rotation frequency Ω is not too large.

c) (5 points) Assuming the mass is executing simple harmonic motion $r = r_0 + Acos(\omega t)$ with t the time, find an expression for the torque τ needed to maintain the stick's constant rotation rate.



FP14

As shown below, a uniform rod of mass m and length $4L$ resting on a frictionless surface is struck perpendicularly at a postion d above its center of mass by a small piece of clay of mass $3m$ having velocity v= vi. Upon impact, the clay sticks to the rod, and then combined clay-rod system moves off together.



a) (2 points) What is the location of the center of mass of the clay-rod system at the moment of impact?

b) (3 points) What is the moment of inertia, I_{new}, of the combined clay-rod system about its center of mass?

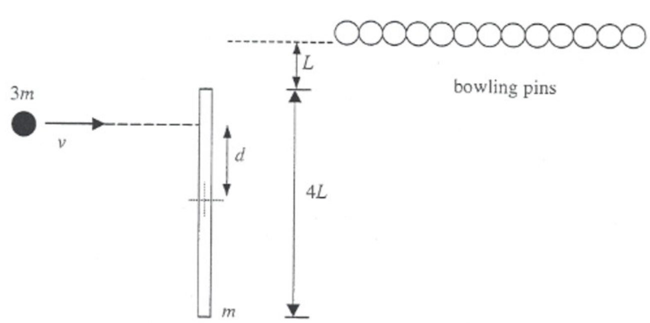
c) (3 points) What value of d maximizes the kinetic energy of the clay-rod system as it moves off following the collision?

   As the system moves off following the collision, it rotates.

d) (3 points) Find the linear velocity v_{linear} and the angular velocity ω of the clay-rod system following the collision. Express your answer in terms of v, m, L, d and I_{new}.

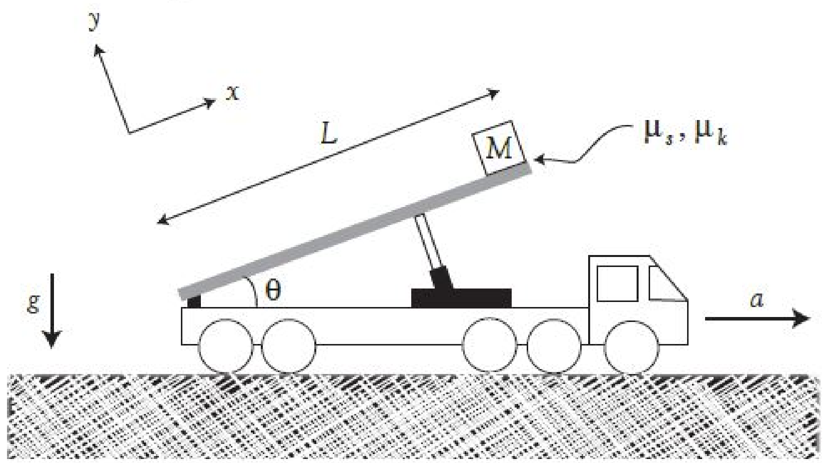
e) (2 points) How far has the system travelled horizontally (in the i direction) when it returns to a vertical orientation (with the long axis of the rod along the j direction) for the first time following the collision?

f) (2 points) Now suppose that a row of bowling pins is placed a distance L above the top of the rod, as shown below. What is the maximum value that d may have without the rod-clay system knocking down at least one pin as a result of its motion following the collision?



FP15

A crate of mass M, which contains an expensive piece of scientific equipment, is being delivered to Caltech. The delivery truck has a freight bed of lenth L (see the figure), with a coefficient of static friction μ_s and a coefficient of kinetic friction μ_k. Rather than move the heavy crate himself, the driver tilts the truck bed by an angle θ and then drives the truck forward with increasing acceleration a, until the crate begins to slide.



For this problem, use the x, y coordinate system shown in the figure. These coordinates are fixed with respect to the truck's bed, **not** to the ground.

a) (2 points) Draw a free-body force diagram for the crate in the truck's frame of reference.

b) (3 points) Write down Newton's second law for the motion of the crate in the $x-$ and $y-$ directions, just before it begins to slip.

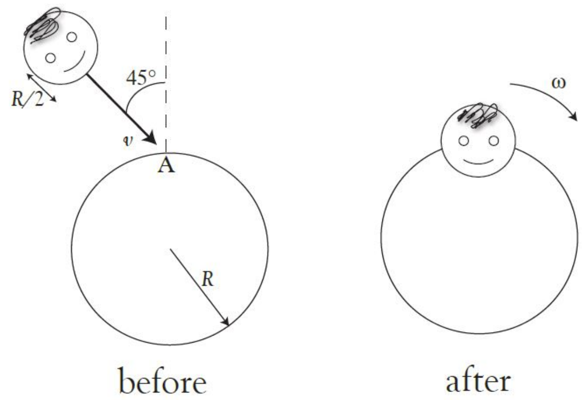
c) (2 points) Determine the minimum acceleration a_{min} for which the crate will begin to slip. Express your answer in terms of the constants shown in the figure.

When the truck reaches a_{min} and the driver notices the crate beginning to slide, he continues at that constant acceleration.

d) (3 points) Find the speed of the crate along the $x-$ direction when the crate leaves the truck bed. Neglect the size of the crate. You may leave your answer in terms of a_{min}.

FP16

A playground merry-go-round of uniformly distributed mass M and radius R is initially at rest. A kid with uniformly distributed mass $2M$ (whose shape is remarkably close to that of a disk of radius $R/2$) runs at a constant velocity $\vec{v}$ towards point A on the edge of the merry-go-round. The angle between $\vec{v}$ and the line that runs from A to the center of the merry-go-round is 45°, as shown in the figure.



At the instant when she jumps aboard, the child grabs hold of the edge so that she is fixed with respect to the merry-go-round, with her center of mass at point A. The merry-go-round then begins turning with angular velocity ω.

a) (2 points) Find the total moment of interia I_{tot} of the system about the axis of the merry-go-round, after the child has jumped on.

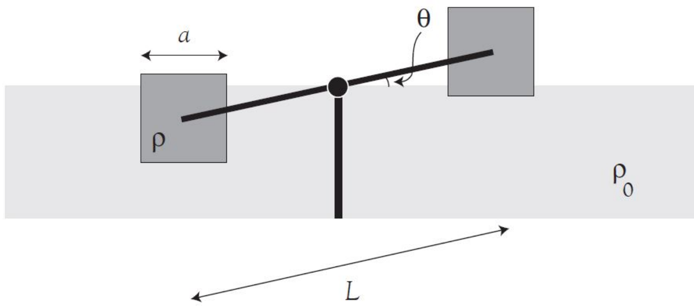
b) (3 points) Find the angular velocity ω of the system after the jump. Express your answer in terms of v, M, and R.

c) (2 points) Find the fraction of the initial kinetic energy that is lost in the "collision" between the kid and the merry-go-round.

d) (3 points) After a few revolutions at a constant angular velocity ω, the kid decides to move to the center of the merry-go-round with a constant radial speed $v_r = -dr/dt$. Find the angular acceleration α of the merry-go-round as a function of r. Express your answer in terms of M, R, v_r and the initial angular velocity ω.
Hint: You might want to save yourself some computation by first finding an expression for ω as a function of r, and then using the chain rule $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{dr}\frac{dr}{dt}$.

FP17

Two solid cubed of edge length a and uniform density ρ are partly immersed in an ideal fluid of density ρ_0. The cubes are connected at their centers by a rigid, massless rod of length L, with a fixed pivot point that lies on the surface of the liquid, as shown in the figure. The system is built so that the cubes remain upright even as the rod turns on its pivot.



Suppose that the system is tilted until the rod makes an angle θ with the surface of the fluid. Assume that for this value of θ both cubes remain in contact with the fluid.

a) (2 points) Calculate the net force on the rod's pivot. For what value of ρ/ρ_0 is this force zero? Hint: The value is independent of θ.

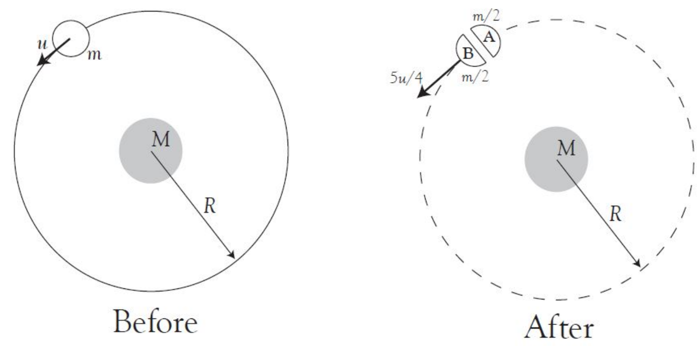
b) (2 points) Calculate the torque τ about the pivot as a function of θ and the other parameters specified in the problem.

c) (3 points) If the system is tilted by an angle θ_0 and then released, it will oscillate. Assume that $a << L$, so that the cubes may be treated as point masses for the purposes of computing the moment of interia. Find an expression for the angular acceleration $\ddot{\theta}$ of the rod.

d) (3 points) Now assume that θ_0 is very small. Express the period T of the oscillation in terms of the parameters in the problem.

FP18

A small spherical satellite of mass m orbits a planet of much larger mass M at a speed u on a circular orbit of radius R. At a time $t = 0$, an internal explosion breaks the satellite into two equal hemispheres A and B, each of mass $m/2$. Immediately after the explosion, the speed of piece B is $5u/4$, and it is moving along the same direction as it was before the explosion. The figures show the system immediately before and immediate after the satellite's explosion.



a) (3 points) What is the speed u, angular momentum L, and total energy E (kinetic plus graviatational potential) of the satellite just before the explosion? Choose your energy scale so that a stationary object infinitely far away from the planet will have zero energy. Express your answers in terms of G, m, M, and R.

b) (2 points) What are the angular momenta L_A and L_B and the total energies E_A and E_B for parts A and B for times $t > 0$? You may leave your answer in terms of u if you did not complete part (a).

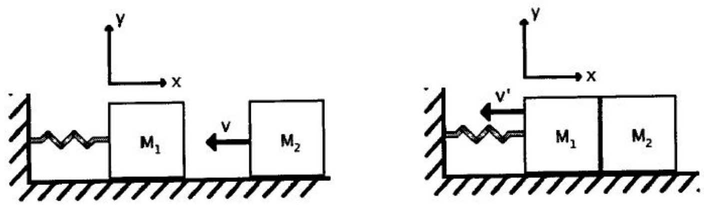
c) (2 points) How much mechanical work was done by the explosion that broke the satellite apart?

d) (1 point) Find the length of the semimajor axes a_A and a_B for the orbits of the pieces A and B for $t > 0$.

e) (2 points) Sketch the orbits of pieces A and B for $t > 0$. Clearly label the point at which the explosion occurred.

FP19

A mass m_1 sits on a frictionless surface and is attached to one end of a spring with spring constant k. The other end of the spring is attached to the wall. The mass and the spring are initially at rest.



A second mass m_2 comes sliding in which velocity $-v\hat{x}$, hits the first mass m_1 at time $t = 0$, and sticks to it. This includes oscillations in the spring, which can then be measured. This in turn can be used to determine the mass m_2 of the impinging object.

a) (3 points) What is the velocity $\vec{v}\,'$ of the two masses immediately after the collision? Express your answer in terms of v, m_1, and m_2.

b) (3 points) Find an expression from m_2 in terms of m_1, k and the angular frequency ω_0 of the observed oscillations.

A function which describes the position for the two masses for all time following the collision is $x = Asin(\omega_0 t) + Bcos(\omega_0 t)$ where A and B are unknown constants, $t = 0$ is the time of the collision, and $x = 0$ is the equilibrium position of the spring.

c) (4 points) What are the values of A and B? Express your answer in terms of ω_0, m_1, m_2 and v.

FP20

A meteor of mass m is in a circular orbit about an airless planet of radius R and mass M at an altitude of $h = 3R$ about the planet's surface. The meteor suddenly undergoes a head-on collision with a small piece of space debris. As a result of the collision, the meteor loses half its kinetic energy without changing its direction of motion or its total mass. Answer the following questions about the meteor and its orbit *after the collision.*

You may find it useful to recall that the total energy of an elliptical orbit is $E = -GMm/2a$ where a is the semi-major axis. You may also wish to recall Kepler's third law: $T^2 = (4\pi^2/GM)a^3$.

a) (2 points) Find the kinetic energy K, the potential energy U, and the angular momentum L of the meteor immediately after the collision.

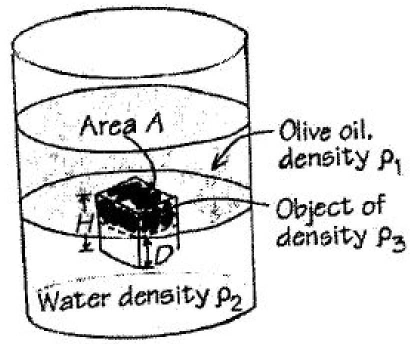
b) (2 points) What is the shape of the meteor's orbit? Use the results from part (a) to justify your answer. Make a sketch of the orbit and indicate the meteor's initial position (i.e., the position of the meteor at the time of the collision).

c) (2 points) Find the minimum distance h_{min} of the meteor from the surface of the planet, and its maximum speed v_{max}. Hint: use conservation of energy and angular momentum.

d) (2 points) Find the time it takes for the meteor to travel from its position in part (c) back to the position when it had the collision.

FP21

Olive oil floats on water. Take ρ_1 to be the density of the oil and ρ_2 to be the density of the water. Consider an oil-water interface across which a bouillon cube of density ρ_3 floats, as shown in the figure below.



a) (3 points) What is the condition on ρ_3 in terms of ρ_1 and ρ_2 such that the cube floats?

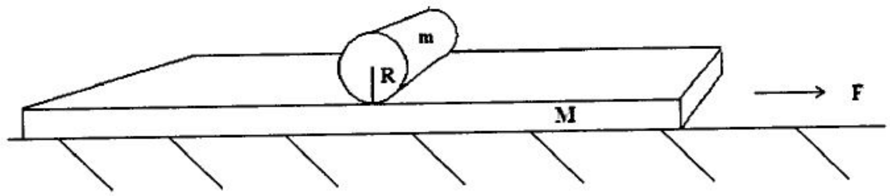
b) (4 points) If the height of the cube is H and the depth of the bottom of the cube below the oil-water interface is D, find D/H in terms of ρ_1, ρ_2 and ρ_3.

FP22

Students would like to send a water stream 24 m across their neighbor's yard to hit a target lying on the ground on his patio. They have a hose of inside diameter 1.5 cm, but when they turn on the water they find they can spray a distance of only 1.5 m. So they attach a nozzle to the house and now find they can just hit their target. During the students' experimentation, they held the end of the hose at the same height as the target.

(5 points) What is the inner diameter of the nozzle?

FP23



As depicted in the diagram above, a large flat board, sitting on the ground, of mass M is pulled with a force F, while a cylinder of mass m, radius R and moment of interia $I = (1/2)mR^2$ rolls withough slipping on the board. Assume there is friction between the board and the cylinder to ensure rolling without slipping, but for simplicity, assume there is no friction between the board and the ground. In addition, assume that the board is sufficiently long that the cylinder remains on the board for the problem. Finally, for calculations in this problem, define a to be the acceleration of the board with respect to the ground a_1 to be the acceleration of the center of mass of the cylinder with respect to the accelerating board and a_2 to be the acceleration of the cylinder with respect to the ground.

a) (2 points) Draw free body diagrams for the cylinder and the board.

b) (3 points) Write down Newton's equations (listing all relevant forces) for the linear motion of the board and cylinder. In addition, write down an expression describing the rotational motion of the cylinder. Finally, find a constraint associated with rolling without slipping. For each of the equations, list which frame of reference it is in.

c) (1 point) Does the cylinder roll clockwise or counter-clockwise?

d) (2 points) Solve the equations you found for a, in terms of F, M and m.

e) (2 points) Solve the equations for a_1 and a_2, again in terms of F, M and m.