Summary of Main Equations

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{r}.$$

$$K = \frac{1}{2} m v^2.$$

$$U_{spr} = \frac{1}{2} k x^2.$$

$$U_{grav} = mgz.$$

$$U(r) = \frac{-GmM_e}{r}$$

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Strategy

$$U_A + K_A = U_B + K_B.$$

(i) Define your system.
(ii) Pick one reference position for $U = 0$ and use it consistently.
(iii) Write down the total energy of the system at the point, say A, where you want to determine some unknown quantity (like speed or height); $E_A = U_A + K_A$.
(iv) Find another point, say point B, where you know everything about the object's motion and write down the total energy at that point; $E_B = U_B + K_B$.
(v) Conservation of energy implies that $E_A = E_B$; equate the two energies and solve for the unknown quantity.

(vi) If there's any friction, add energy loss (notice $W_{friction}<0$):

$E_A = E_B - W_{friction}$ from A to B

Let’s practice

Example 7

Estimate the escape velocity from the earth.

Reminder: The escape velocity is the instantaneous speed at the Earth’s surface that would allow a rocket to get to spatial infinity with zero speed.

Let's practice

Example 7

Estimate the escape velocity from the earth.

We know from Chapter 7 that

$$g = GM_e/R_e^2,$$

so we can write the escape velocity from the earth in the convenient form

$$v = \sqrt{2gR_e}.$$

Knowing that g is about 10 m/s^2 and the radius of the earth is approximately 6.4×10^3 km, we easily find that the escape velocity is about 11 km/s, or 7 mi/s (= 25,000 mph), a large but not impossible speed to attain.

A real case from 2018

Falcon Heavy / **Max speed**

11 kilometers per second

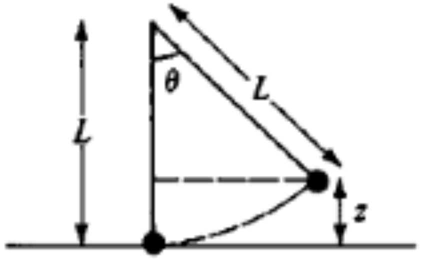
The Falcon Heavy will dispatch the Roadster — weighing around 2,760 pounds (1,250 kilograms) on the street — with enough velocity to escape Earth's gravitational bonds, reaching a maximum speed of around **7 miles per second** (**11 kilometers per second**; **24,600 mph**). Feb 5, 2018

Energy and dynamics

Consider an arbitrary initial θ and L.

Example 3

A pendulum bob is pulled aside from the vertical through an angle θ and released. Find the speed of the bob and the tension in the string at the lowest point of the swing, assuming $L = 0.3$ m, $\theta = 30°$, and $m = 0.5$ kg.



Energy and dynamics

$$E_A = mgz,$$

where z is the vertical height above B. Using geometry, we see that $z = L - L\cos\theta = L(1 - \cos\theta)$, so the initial energy is

$$E_A = mgL(1 - \cos\theta).$$

The total energy at B is purely kinetic energy because we have chosen $z = 0$ there, so $E_B = \frac{1}{2}mv_B^2$. Therefore the law of conservation of energy, $E_A = E_B$, implies

$$mgL(1 - \cos\theta) = \frac{1}{2}mv_B^2.$$

Solving for the speed v_B we find

$$v_B = \sqrt{2gL(1 - \cos\theta)}\ .$$

Substituting numbers, this turns out to be 0.9 m/s in our case.

Energy and dynamics

$$T_B - mg = mv_B^2/L,$$

which implies

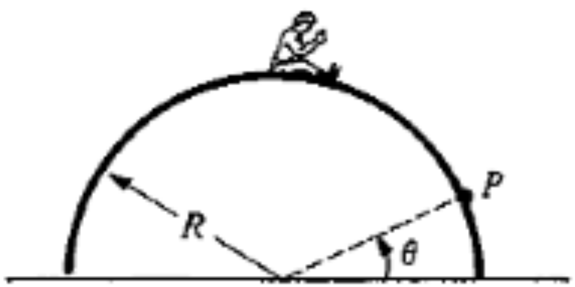
$$T_B = mg + mv_B^2/L.$$

Substituting for v_B from our law of conservation of energy we get for the tension in the string when the bob is at point B

$$T_B = mg + 2mg(1 - \cos\theta) = mg(3 - 2\cos\theta).$$

Energy, dynamics and circular motion

21. A child sits on a hemispherical mound of ice. Assuming the mound to be frictionless, if the child is given a slight nudge and slides off, find the angle from the horizontal at which the child leaves the mound.



Energy, dynamics and circular motion

***Main idea**: When **N = 0**, the boy and the mound are not in contact anymore.*

From the non-inertial view of the boy, the forces at that moment are:

1) The portion of the weight pointing toward the center of the mound, and
2) The centrifugal force felt by the boy.

When $\boldsymbol{N} = \mathbf{0}$, we can write:

$$m\, v^2_{off}/R = m\, g \sin\theta \;, \qquad \text{or} \qquad v^2_{off} = g\, R \sin\theta \;.$$

Now, let's use conservation of energy between the top of the mound and the instant when the boy will lose contact with the mound:

$$mgR + 0 = mgR \sin\theta + \tfrac{1}{2} mv^2_{off}, \qquad \text{or} \qquad mgR = 3mgR \sin\theta/2.$$

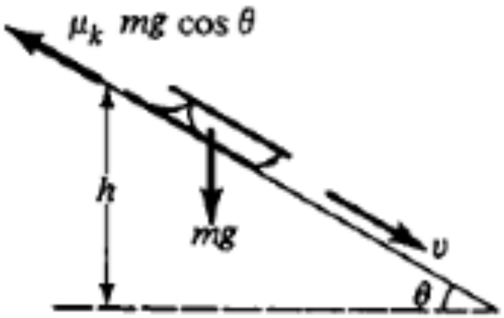
Finally, $\sin\theta = 2/3$ and $\theta =$ 41.81 degrees.

Energy and friction

Do it without numerical values

Example 10

Starting at a height of 25 m, a sled of mass 20 kg slides down a hill with a 30° slope. If the sled starts from rest and has a speed of 15 m/s at the bottom of the hill, calculate the energy dissipated by friction along its path and calculate the coefficient of friction.



Energy and friction

In sliding down the hill, the sled converts mgh of potential energy into $\frac{1}{2}mv^2$ of kinetic energy and an amount of work W done against friction. Therefore W is

$$W = mgh - \tfrac{1}{2}mv^2 = (20\text{ kg})(10\text{ m/s}^2)(25\text{ m}) - \tfrac{1}{2}(20\text{ kg})(15\text{ m/s})^2$$

$$= 5000\text{ J} - 2250\text{ J}$$

$$= 2750\text{ J}.$$

The sled slides a distance $h/\sin\theta$ down the hill, with friction exerting a backward force $\mu_k N = \mu_k mg\cos\theta$ during this time. So the work done against friction is

$$W = (\mu_k mg\cos\theta)(h/\sin\theta).$$

Distance along the force (scalar product in the W formula!)

Combining the two equations for W we find

$$\mu_k mgh\cot\theta = mgh - \tfrac{1}{2}mv^2$$

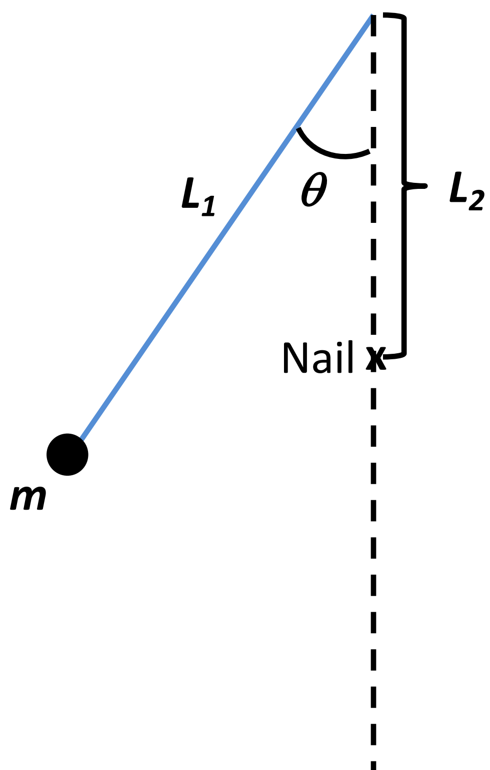
or

$$\mu_k = (\tan\theta)(1 - v^2/2gh).$$

Using the given values for θ, v, and h, we find $\mu_k = 0.32$. As a check, note that for a frictionless incline ($\mu_k = 0$, $\tan\theta \neq 0$), our equation yields $\frac{1}{2}mv^2 = mgh$, the well-known result for conservative motion in a uniform gravitational field.

Energy and rotation

Consider a pendulum made out of a rope and a bead. The bead is raised to some height with the rope taut and then set free from rest. Where should we hammer a nail along the mid vertical line so that the bead makes a full turn? The rope is massless and cannot be stretched. Its length is L_1. The bead has mass m. Consider the acceleration of gravity to be constant and neglect any friction with air.



We'll work out the solution in the next session: Monday 10/28/19