Ph1a - Flipped Section

Problem Set 8

October 31, 2019

1. Rockets

a. Consider a rocket of mass m flying at velocity $\mathbf{v}$ which is ejecting mass at a rate dm/dt and velocity $\mathbf{u}$ with respect to the rocket. The sum of external forces on the rocket is $\mathbf{F}$. What is the formula for the change in momentum of the rocket, $d\mathbf{p}/dt$?

b. A rocket (of total mass m_0 initially) is fired from rest from the surface of the Earth, ejecting mass at a rate $dm/dt = -k$ with $k > 0$, and the ejected mass has speed constant speed u relative to the rocket. Find the rocket's velocity as a function of time.

2. Bouncing Balls

A tennis ball with a small mass m_2 sits on top of a basketball with a large mass $m_1 \gg m_2$. The bottom of the basketball is a height h above the ground, and the bottom of the tennis ball is a height $h + d$ above the ground. The balls are dropped. To what height does the tennis ball bounce? (Assume that the bounce is elastic.)

3. Angular momentum when total linear momentum vanishes

Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same about any origin.

4. Different collisions on each side

Consider two point masses of mass M each, connected by a massless rod of length L, lying on a frictionless plane. Two point-like masses, of mass m each, come from opposite sides with speed v to hit the M masses. The one on the top collides elastically, while the one on the bottom sticks together with the bottom M.

a. Find the angular momentum of the system.

b. If the rod rotates with some angular frequency ω, find the point which does not rotate, and hence is the axis about which the rod rotates. This point can be regarded as the center of mass of the rod.

c. Find the linear motion of the center of mass of the rod.

d. With what angular frequency ω does the rod rotate?

Some formulae:

$$\vec{L} = \vec{r} \times \vec{p}, \qquad (L = m\ v\ d)$$

$$K = \frac{1}{2}mv^2, \qquad v = r\omega$$