Ph1a - Flipped Section

Problem Set 7

October 28, 2019

1. Mid-air explosions

A cannon shoots a grenade horizontally into a vacuum with no gravitational force acting on it. The grenade has a mass of 50 kg and travels to the right at 100 m/s.

a. What is the initial momentum of the grenade?

b. The grenade explodes in mid-air and breaks into three pieces. You manage to measure the mass of all fragments, but the velocity of only two of them:

the left-most piece is 20 kg with a speed of 30 m/s in the leftward direction, the middle piece has a mass of 15 kg and travels at 20 m/s to the right, and the right-most piece is also 15 kg with an unknown velocity.

Find the velocity of the third fragment.

c. You now shoot off a new grenade, with identical mass and velocity to the one in part (a). Again, it explodes in mid-air. This time, however, it forms just two fragments each of which have a mass of 25 kg. One flies straight up in the vertical direction at 10 m/s. Find the velocity of the second fragment.

2. Spring railway gun

A compressed massless spring is mounted on a toy train, which is on a horizontal frictionless track. Initially everything is at rest. When the spring is released it fires a marble of mass m horizontally, and the train of mass M recoils in the opposite direction. If all potential energy $\frac{1}{2}kx^2$ of the spring is converted into kinetic energy, calculate

a. the speed of the wagon and of the marble,

b. the kinetic energy of each of those bodies.

The marble hits a wall, bounces back elastically, and comes back to the train, hitting the spring again.

c. How much will the spring compress now? (Hint: when the spring is compressed as much as possible, the marble comes to rest with respect to the train.)

3. Maximizing transverse velocity

A particle of mass m_1 and initial velocity $v_0 = v_{0i}$ scatters elastically off a particle of mass m_2 that is initially at rest. The particles emerge at angles θ_1 and θ_2. At what θ_1 does the target particle have maximum transverse velocity v_{2y}?

(Hint: use the center of mass frame)

4. Rockets

a. Consider a rocket of mass m flying at velocity $\mathbf{v}$ which is ejecting mass at a rate dm/dt and velocity $\mathbf{u}$ with respect to the rocket. The sum of external forces on the rocket is $\mathbf{F}$. What is the formula for the change in momentum of the rocket, $d\mathbf{p}/dt$?

b. A rocket (of total mass m_0 initially) is fired from rest from the surface of the Earth, ejecting mass at a rate $dm/dt = -k$ with $k > 0$, and the ejected mass has speed constant speed u relative to the rocket. Find the rocket's velocity as a function of time.

5. Bouncing Balls

A tennis ball with a small mass m_2 sits on top of a basketball with a large mass $m_1 \gg m_2$. The bottom of the basketball is a height h above the ground, and the bottom of the tennis ball is a height $h + d$ above the ground. The balls are dropped. To what height does the tennis ball bounce? (Assume that the bounce is elastic.)