Ph1a - Flipped Section

Problem Set 5

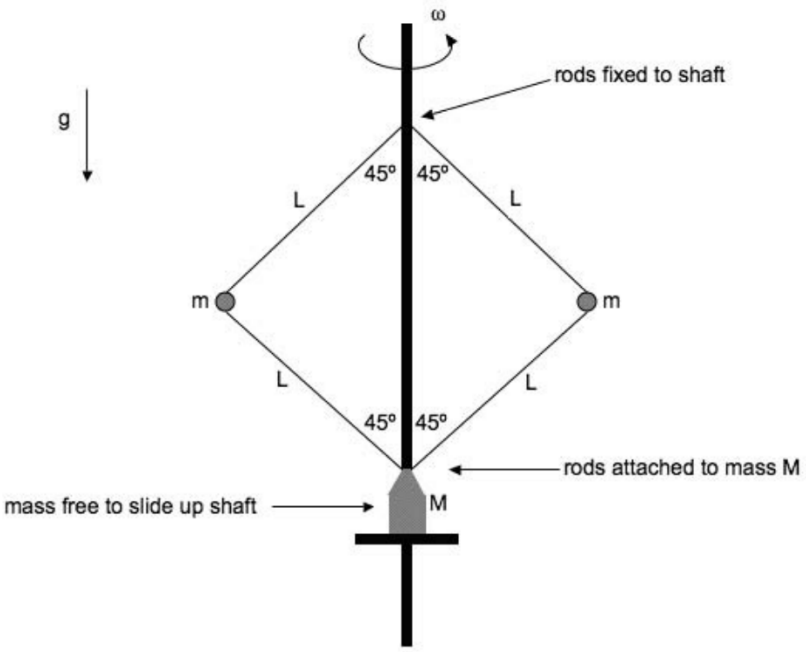
October 21, 2019

1. Some more astrophysics

A spherical object rotates with angular velocity ω. If the only force preventing centrifugal disintegration is gravity, what minimum density should the object have? Use this to estimate the minimum density of the Crab Pulsar which rotates 30 times per sec.

2. Spinning masses

Two equal masses m are attached by hinges and massless rods of length L to a rotating shaft and to a mass M which rotates with the shaft but is free to slide up it without friction. The upper rods are hinged to a fixed point on the shaft. At low angular velocity ω, the mass M sits on a seat attached to the shaft and the rods make a 45° angle with the shaft, as shown. However, if ω exceeds a particular value ω_0, then the mass M is no longer in contact with its seat but instead begins to slide up the shaft (thus increasing the angle between the rods and the shaft). Gravity acts downwards.



a. Draw force diagrams for the three masses.

b. Calculate the critical angular velocity ω_0 at which the mass M just begins to rise off the seat and up the shaft.

Hint: The tension in the upper rods is not equal to the tension in the lower rods.

3. Where can I weigh the least!

(Following Frautschi 9.6)

If the Earth were exactly spherical and did not rotate, the weight of an object would point directly toward the center of the earth and the acceleration due to gravity $\vec{g_0}$ will be radially inward everywhere on Earth. But this is not exactly true when the earth's rotation is taken into account. Let us try and determine the latitude dependence of the weight of a mass m on the surface of the Earth.

Consider the Earth as a rotating sphere of radius R, rotating with an angular velocity ω about the North-South axis.

a. What (contact) force will be identified as the weight of the object? Use this to define an apparent acceleration due to gravity $\vec{g}$.

b. Now try and work in the rotating Earth's frame and figure out all the forces acting on the body when placed at a latitude θ and use equilibrium conditions to determine $\vec{g}$ in terms of $\vec{g_0}$. Making reasonable approximations, work out the magnitude of the vector $\vec{g}$ in order to determine the latitude dependence of the weight of the object.

4. Roller coasters

Suppose you are riding a roller coaster of mass m that goes through a vertical loop of radius R, maintaining a constant speed u. (In order to achieve this, the roller coaster track exerts a force on the car tangential to the loop.) What is the magnitude of the total force (including the normal force) exerted by the track on the roller coaster car as a function of its position θ on the loop?