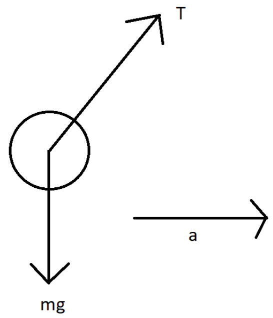
Ph1a - Flipped Section

Solutions

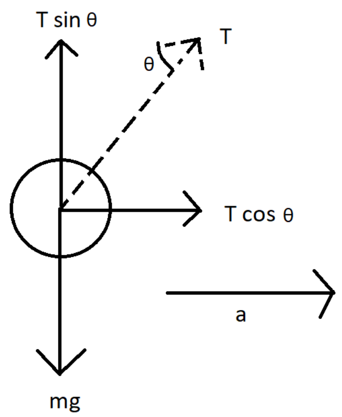
October 10, 2019

1. Accelerometer

a. The two forces acting on the ball are gravity mg and the string tension T. The force of gravity points straight down, while the string force points along the string, as shown on the free-body diagram. (a is the acceleration of the car and the arrow shows its direction.)



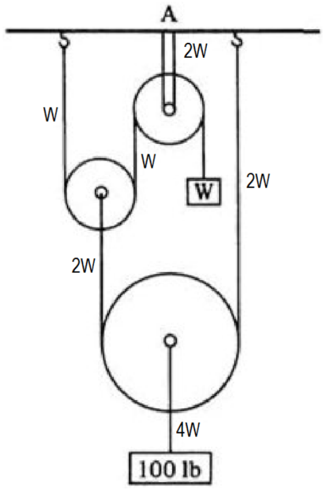
b. Since the car (and therefore the ball) is accelerating to the right with acceleration a, the net force cannot be zero. There must be a net force to the right. Since there is no vertical acceleration (the ball does not move up or down), the vertical forces must sum to zero.



Balancing the forces in the vertical direction, $T\sin\theta = mg \implies T = \frac{mg}{\sin\theta}$.

The ball has to move in the horizontal direction with the same acceleration as the car. Therefore, $T\cos\theta = ma \implies a = \frac{T\cos\theta}{m} = \frac{mg\cos\theta}{m\sin\theta} = \frac{g}{\tan\theta} = \frac{9.8}{\tan 83^\circ}$ m/s^2 = 1.2 m/s^2.

2. Pulley System



We can most easily solve this problem by first labeling all of the tensions on all of the rope segments. The rope supporting weight W must have tension = W, which propagates the length of the rope. This means the pulley under strut A has two tension forces pulling down on it for a total force of 2W. The other small pulley is being pulled up with a force 2W, and since it is in equilibrium, the second rope attached to the small pulley must have a downward force of 2W to oppose this. The entire second rope must be under tension 2W, meaning the large pulley has a total tension 4W pulling up on it, exactly counteracting the 100 lb force pulling down.

Thus 4W= 100 lbs and so W= 25 lbs. The downward pull of the support at A is thus 50 lbs.

3. Leaning Ladder

Gravity acts on the center of the ladder with force mg, downward; the ground pushes against the ladder at its contact point with the ladder with normal force N_1, upward; the wall pushes against the ladder at its contact point with the ladder with normal force N_2 to the right (if the wall is on the left of the ladder); friction from the ground acts on the ladder at its contact point with force f, to the left.

Balancing forces in the y-direction: $N_1 = mg$.

Balancing forces in the x-direction: $f = N_2$ where $f \leq \mu N_1$.

Consider torque about the point of contact between the ladder and the ground:the wall, with normal force N_2, contributes torque $N_2 d \sin\theta$, clockwise. Gravity acting on the center of the ladder contributes a torque $mg\frac{d}{2}\cos\theta$, counterclockwise.

Hence we require:

$$N_2 d \sin\theta = mg\frac{d}{2}\cos\theta,$$

from which we get

$$\tan\theta = \frac{mg}{2N_2}.$$

Since $N_2 = f \leq \mu mg$, we get $\tan\theta \geq \frac{1}{2\mu}$.

(Physically, the equation means that for a given value of θ, we need there to be a corresponding N_2; this N_2 in turn depends on the friction f, which will always be exactly the amount which prevents any motion, up to μmg, after which the ladder will slip.)

So we conclude that for the ladder to remain in place, θ can be as small as $\arctan(\frac{1}{2\mu})$, and as large as you want (with $\frac{\pi}{2}$ being the maximum that makes sense).

4. Inclined Block

First, note that when $M \to \infty$, this is just an inclined plane problem: the large block doesn't move, $A = 0$. When $M \to 0$, the large block doesn't oppose the motion of the small block in any way, and the small block just falls freely.

(Convention: a_x and A are positive toward the right; a_y is positive toward up.)

Consider the small block: it has normal force N from the large block, and also gravity acting on it. The equations of motion are: x: $N \sin\theta = ma_x$ and y: $N\cos\theta - mg = ma_y$.

For the large block, which has a normal force, equal and opposite to the N above, acting on it, the equation of motion in the x-direction gives

$$-N \sin\theta = mA.$$

We require that the small block remains on the slope of the large block (before it falls off, at least). With respect to the outside observer, the the position of the small block after time t is given by

$$(\frac{1}{2}a_x t^2, \frac{1}{2}a_y t^2)$$

if the position at $t = 0$ is $(0, 0)$.

The position of the large block after time t is given by

$$(\frac{1}{2}At^2, 0)$$

since it only moves in the x-direction.

This means that, in the frame of the large block, the position of the small block would be given by

$$(\frac{1}{2}(a_x - A)t^2, \frac{1}{2}a_y t^2).$$

(Note that $a_x - A > a_x$ since A should be negative.)

From the point of view of someone on the large block, all we see is that the small block slides down on the inclined plane, and it slides down with the x and y acceleration given above. From this point of view it is clear that the above vector, which describes the position of the small block with respect to the large inclined plane, should always satisfy the condition that $\tan\theta = -(\frac{1}{2}a_y t^2)/(\frac{1}{2}(a_x - A)t^2)$ (Note that $a_y t^2$ is negative.)

This gives:

$$-\frac{1}{2}a_y t^2 = \tan\theta(a_x - A)\frac{1}{2}t^2.$$

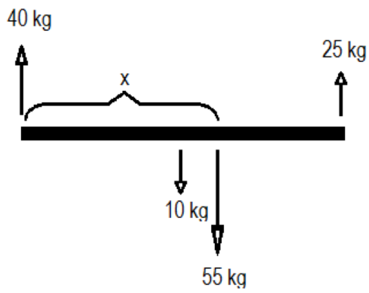
So we get $\tan\theta(a_x - A) = -a_y = -\frac{mg - N\cos\theta}{m}$,

$$-\tan\theta A = -\frac{mg - N\cos\theta}{m} - N\sin\theta/m$$

and $N = -mA/\sin\theta$ so we can solve for A.

5. Three Clowns

We know that the total weight supported by Orsene and Waldo is 65 kg. When Bobo is closer to Orsene, he supports most of the weight, and when Bobo is closer to Waldo, he supports most of the weight. We want to find x: how far away is Bobo from Orsene, when Orsene is supporting 40 kg?



Using the figure above and Orsene as our reference point, we write down the torque balance:

$$(1.5\text{m} \times 10\text{kg}) + (x\text{m} \times 55\text{kg}) - (3\text{m} \times 25\text{kg}) = 0 \rightarrow x = 60/55\text{m} \tag{1}$$

Thus, if Bobo is closer than 60/55 m away from Orsene, Orsene needs to support more than 40 kg. Assuming Bobo can reverse direction instantly when he reaches the end of the plank, we can find the speed he needs to ride:

$$(60/55\text{m})/(5\text{sec}/2) = \frac{24}{55}\text{m/s} \tag{2}$$

If Bobo is traveling this speed or faster, Orsene will be supporting 40 kg for less than 5 seconds.