Ph1a - Flipped Section

Problem Set 14

November 21, 2019

1. Elliptic Orbit

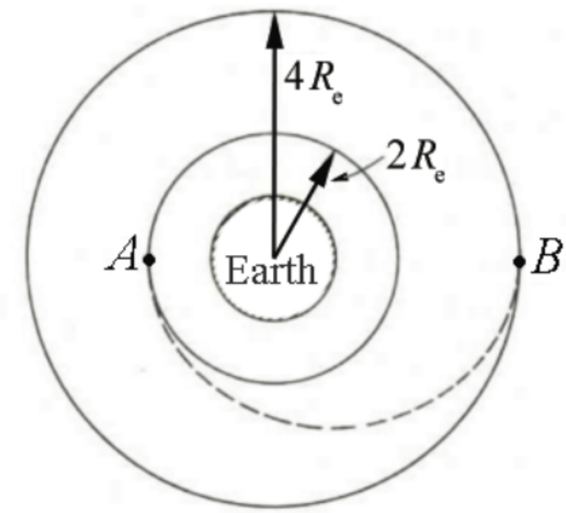
A satellite of mass m_s is in an elliptical orbit around a planet of mass $m_p >> m_s$. The planet is located at one focus of the ellipse. The satellite is at the distance r_a when it is furthest from the planet. The distance of closest approach is r_p. What is (i) the speed v_p of the satellite when it is closest to the planet and (ii) the speed v_a of the satellite when it is furthest from the planet?

2. Transfer Orbit

A space vehicle is in a circular orbit about the earth. The mass of the vehicle is m_s and the radius of the orbit is $2R_e$ where R_e is the radius of the earth. It is desired to transfer the vehicle to a circular orbit of radius $4R_e$. Suppose that the mass of the earth is denoted M_e.

a. What is the minimum energy expenditure required for the transfer?

b. An efficient way to accomplish the transfer is to use an elliptical orbit from point A on the inner circular orbit to a point B on the outer circular orbit (known as a Hohmann transfer orbit). What changes in speed are required at points A and B in order to accomplish this maneuver?



3. Central Force Proportional to Distance Cubed

A particle of mass m moves in a plane about a central point under an attractive central force of magnitude $F = br^3$. The magnitude of the angular momentum about the central point is equal to L.

In this problem, we will develop the method of "effective potential energy" that lends a very natural energy interpretation to orbits under central forces.

a. Find the potential energy associated with this central force, using the convention $U(r = 0) = 0$ and now write down an expression for the total mechanical energy E of the particle, explicitly decomposing the kinetic part into its radial and angular contributions.

b. We have learnt that angular momentum L is a constant of motion for central force motion. Use this fact to eliminate any angular dependence in the mechanical energy found above in terms of L and the radial coordinate r. Notice, your expression for energy now resembles the motion of a particle in one-dimension, namely the "radial direction", but with a modified potential energy than what we found in (a.)[see next part].

c. Find this "effective potential energy" U_{eff} and make a sketch of the effective potential energy as a function of r (Though we are working for a particular form of the central force $F = br^3$ in this problem, this effective potential construction is rather general). The contribution to the effective potential energy from the angular momentum term is called the "Centrifugal Potential Barrier" and it effectively tries to keep the particle from falling towards the force center.

However, the contribution to the actual central force is only from the central potential we found in part (a). Now let us add some "orbit understanding" to this picture. We know the total energy of the particle will be a constant of motion. For $E > min\,(U_{eff})$, the straight line plot of E will intersect the effective potential energy curve, which would be the radial bounds for the particle's orbit(think and make sure you are convinced of this). Since $E =$ radial KE + $U_{eff}(r)$, in the region where $E > min\,(U_{eff})$, what does this difference $E - U_{eff}$ correspond to?

d. Think further on these lines and figure out which point or points in this plot correspond to circular orbits. Hint: What is the radial KE for a circular orbit? Now, indicate on your sketch of the effective potential the total energy required for circular motion.

e. Next, suppose that the radius of the particle's orbit varies between r_0 and $2r_0$(so, this is obviously not a circular orbit). Find r_0.