Ph1a - Flipped Section

Problem Set 13

November 18, 2019

1. Acceleration in polar coordinates

(Based on Frautschi 17.3 and 17.4)

a. What is the formula for velocity in polar coordinates?

b. By differentiating the formula for velocity in polar coordinates find the formula for the acceleration vector $\mathbf{a} = d\mathbf{v}/dt$. The factors multiplying $\hat{\mathbf{r}}$ and $\hat{\theta}$ are called, respectively, the radial and transverse components of acceleration.

c. Show that the transverse component of the acceleration is equal to

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right).$$

Deduce that the acceleration is radial (parallel to $\mathbf{r}$) if and only if the position vector $\mathbf{r}$ sweeps out area at a constant rate.

2. Bead on a spoke

A bead moves along the spoke of a wheel at constant speed u. The wheel rotates with uniform angular velocity $\dot{\theta} = \omega$ about an axis fixed in space. At $t = 0$ the spoke is along the x axis, and the bead is at the origin.

a. Find the velocity at time t in polar coordinates and in cartesian coordinates.

b. Find the acceleration in polar coordinates.

3. Radial motion without radial acceleration

A particle moves with $\dot{\theta} = \omega = \text{constant}$ and $r = r_0 e^{\beta t}$, where r_0 and β are constants.

a. At what value of β does the radial component of the acceleration vanish?

b. Explain how zero radial acceleration still allows for $r = r_0 e^{\beta t}$.