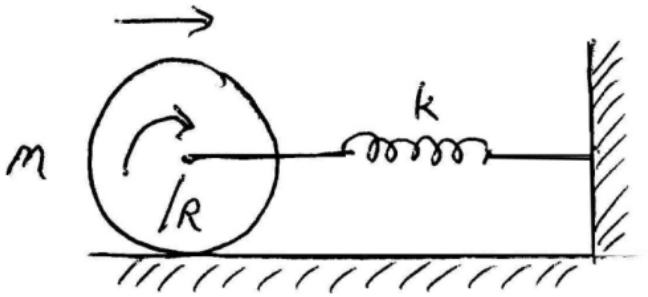
Ph1a - Flipped Section

Problem Set 12

November 14, 2019

1. Oscillating and Rolling



Attach a solid cylinder of mass M and radius R to a horizontal massless spring with spring constant k so that it can roll without slipping along a horizontal surface. Let us try to find the period of the simple harmonic motion for the center of mass of the cylinder.

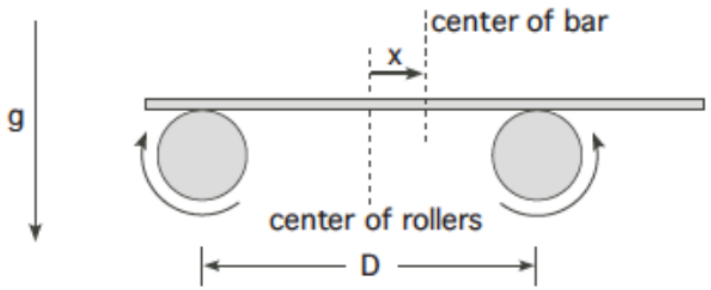
a. Write down the energy of the system.

b. Conservation of energy tells us that $dE/dt = 0$. Use this to write down the equation of motion for the position of the cylinder, and find the period.

c. We can also solve the problem using forces. Write down the force equation and the torque equation for the center of mass of the cylinder.

d. The condition of rolling without slipping tells us that $a = R\alpha$. Use this to obtain the equation of motion for the position of the cylinder, and find the period.

2. Oscillating Bar



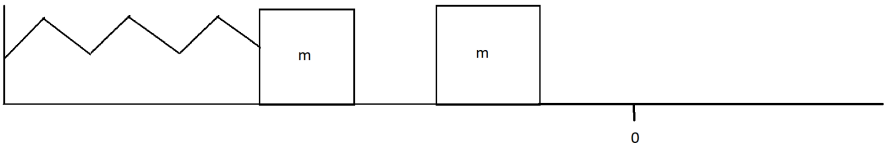
A bar of mass m and negligible height is lying horizontally across and perpendicular to a pair of counter rotating rollers as shown in the figure. The rollers are separated by a distance D. There is a coefficient of kinetic friction μ between each roller and the bar. Assume that the bar remains horizontal and never comes off the rollers.

a. The bar is released from rest at $x = x_0$ at $t = 0$. Find the subsequent location of the center of the bar $x(t)$. (Hint: the net torque on the bar is zero.)

b. Now suppose the rollers roll in the opposite direction. Find $x(t)$ with the same initial conditions as part (a).

3. Mass on a spring hitting another mass

Consider a mass m attached to a spring of spring constant k oscillating with amplitude d. Let $x = 0$ be the position of the mass when the spring is at equilibrium. As the spring is stretching out, at $x = -d/2$ it hits another mass (also with m) and afterward the two masses stick and oscillate together.



a. What is the velocity v_i of the moving mass right before the collision?

b. What is the velocity v_f of the combined mass right after the collision?

c. Find the motion of the combined mass after the collision, in the form $x(t) = A\cos\omega t + \delta$, i.e. find the amplitude, angular frequency, and the phase after the collision, where we take $t = 0$ to be the moment of collision.