Ph1a - Flipped Section

Problem Set 10

November 7, 2019

1. Moment of inertia of a filled square

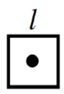


Figure 1

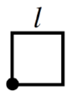


Figure 2

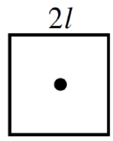


Figure 3

The figure shows three thin, planar squares which are rotated around the axes denoted by the black dots. All rotation takes place in the plane of the page.

a. Set up and evaluate the integral for the moment of inertia for the thin square rotating about an axis through its corner (above, middle). *Hint: It might be useful to split the square into infinitesimal elements of width dx and height dy and integrate over the entire width and height of the square. If you are uncomfortable with evaluating double integrals, call over a TA, or skip this part.*

While finding moments of inertia of spheres and cylinders can be done relatively easily, finding moments of inertia of geometric objects often leads to multidimensional integrals (which aren't always pretty). Now we will derive the same result, but in a much neater fashion... without any integrals!

b. Relate the moments of inertia of the left and middle squares using the parallel axis theorem, i.e find $I_1 + ? = I_2$.

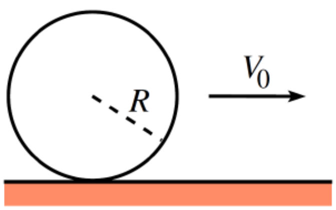
c. Relate the moments of inertia of the middle and right squares, i.e. find α such that $I_3 = \alpha I_2$.

d. Recall that moment of inertia scales quadratically with length and linearly with mass. How are the moments of inertia of the left and right squares related (assume they have the same density)? (Find β so that $I_3 = \beta I_1$).

e. The three previous parts should have given you a system of 3 equations in 3 unknowns. Solve them for the moment of inertia of the left-most square and verify that it matches your answer in part (a).

2. Sliding to a roll

(Morin, Introductory Mechanics, Problem 7.17)



A ball initially slides, without rotating, on a horizontal surface with friction. The initial speed of the ball is V_0, and the moment of inertia about its center is $\eta m R^2$.

a. Without knowing anything about how the friction force depends on position, find the speed of the ball when it begins to roll without slipping. Also, find the kinetic energy lost while sliding.

Hint: Friction acts as a force to slow down the ball, and also as a torque to speed up the rotation. Write down the two equations describing these processes, eliminate the Friction force, and integrate over time.

b. Now consider the special case where the coefficient of sliding friction is independent of position. At what time, and at what distance, does the ball begin to roll without slipping?

Verify that the work done by friction equals the loss in energy calculated in part a) (Hint: Note that if something is rolling without slipping, friction doesn't do any work at all. It only does work when something is sliding.).

3. Rolling uphill and downhill

Consider a wheel of radius R with mass m and moment of inertia $\eta m R^2$ about its axis of rotation, which is rolling without slipping with speed v_0 on a flat surface with coefficient of friction μ. It approaches an incline with angle θ with respect to the horizontal, whose surface is friction-less.

a. What height h does the wheel reach? In what direction is it spinning when it is coming down the incline?

b. After coming down the incline, it again rolls on the frictionful flat surface. What is the final speed of the wheel? How much time does it take to reach that speed?