Physics 1A Quiz 1 Solutions

1 Problem 1: Drag Race

1a) 3 pt

Two cars, **A** and **B**, start a race at rest at $t = 0$. After traveling a distance L they both reach the finish line at the exact same time t_L. During the race, their position is described by the equations

$$x_A(t) = \alpha t^2$$
$$x_B(t) = \beta t^3$$

Solve for α and β given the information provided.

Solution: The key is to use **all** of the information provided to you.

$$x_A(t_L) = L = \alpha t_L^2 \qquad x_B(t_L) = L = \beta t_L^3$$

$$\boxed{\alpha = \frac{L}{t_L^2} \qquad \beta = \frac{L}{t_L^3}}$$

Because you are given enough information to solve for α and β individually, leaving an answer in the form of $\alpha = \beta t_L$ or $\beta = \frac{\alpha}{t_L}$is not sufficient

1b) 1 pt

What are the SI units of α and β?

Solution: Meters(m) are the SI unit for length and seconds(s) are the SI unit for time. So, from 1a) it is clear that α has units of $\frac{m}{s^2}$ and β has units of $\frac{m}{s^3}$.

1c) 2 pt

Make a sketch that illustrates the positions of the two cars as a function of time. Which car (if any) is ahead at $t = t_L/2$?

Solution:

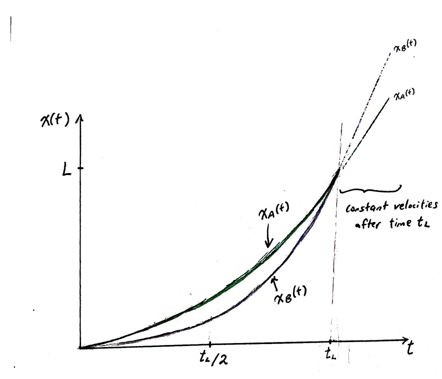

Figure 1: Position vs Time

From the graph it is clear that **Car A wins**. But this can also be proven analytically.

$$x_A(t_L/2) = \alpha(t_L/2)^2 = \frac{L}{t_l^2} \cdot \frac{t_l^2}{4} = \frac{L}{4}$$
$$x_B(t_L/2) = \beta(t_L/2)^3 = \frac{L}{t_l^3} \cdot \frac{t_l^3}{8} = \frac{L}{8}$$

since $\frac{L}{4} > \frac{L}{8}$ Car A is ahead at $t = t_L/2$.

1d) 2 pts

Calculate the accelerations $a_A(t)$ and $a_B(t)$ and make a sketch that illustrates these as a function of time. Use the same vertical scale for both accelerations.

Solution:

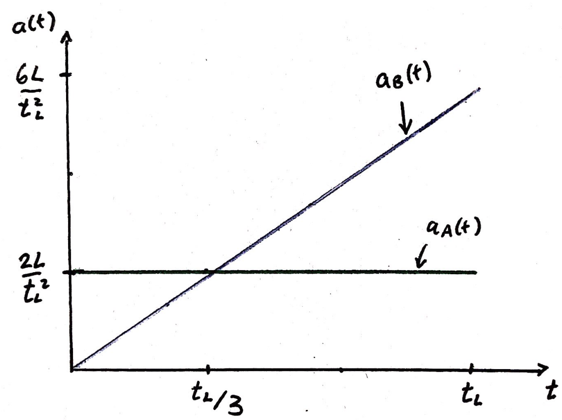

Figure 2: Acceleration vs Time

$$a_A(t) = \ddot{x}_A(t) = \frac{d}{dt}\dot{x}_A(t) = \frac{d}{dt}2\alpha t = 2\alpha = \frac{2L}{t_L^2}$$
$$a_B(t) = \ddot{x}_B(t) = \frac{d}{dt}\dot{x}_B(t) = \frac{d}{dt}3\beta t^2 = 6\beta t = \frac{6L}{t_L^3}t$$

$a_A(t)$ is a constant function at $\frac{2L}{t_L^2}$ while $a_B(t)$ is a linear function of t that starts at 0 and reaches $\frac{6L}{t_L^2}$ at $t = t_L$.
In order to receive credit for the vertical scaling of the graph, you must show either that $a_B(t)$ was 3 times larger than $a_B(t)$ at $t = t_L$. or that $a_B(t) = a_A(t)$ at $t = t_L/3$. This was possible even if you were not able to solve for α and β in terms of L in part 1a).

1e) 2 pts

Suppose the cars continue "coasting" at their final velocities after they pass the finish line. Modify the sketch in part (c) (or make a new one) to show what happens. Do the cars stay even? If not, which one pulls ahead?

Solution: Since the cars are "coasting" they stop accelerating and maintain their velocity after $t = t_L$. This is represented on the graph by drawing a line tangent to the curves so show a constant velocity (see Figure 1 in part 1c) . Clearly Car B has a higher velocity after $t = t_L$ so **Car B pulls ahead**.
Similar to 1b), you could analytically find that the velocity of Car B was higher.

$$v_A(t) = \dot{x}_A(t) = 2\alpha t = \frac{2L}{t_L^2}t \qquad v_A(t_L) = \frac{2L}{t_L}$$

$$v_B(t) = \dot{x}_B(t) = 3\beta t^2 = \frac{3L}{t_L^3}t^2 \qquad v_B(t_L) = \frac{3L}{t_L}$$

$$v_B(t_L) > v_A(t_L)$$

2 Problem 2: Department Store Escalator

Kids of all ages enjoy running up and down escalators in department stores, especially in the wrong direction. Consider an escalator going down at an incline at a speed v_e, with steps that are a distance ΔL apart. The full length of the escalator is L.

2a) 3pt

Suppose I can run down the length of the escalator in 1/3 of time that it takes to run up the escalator, but that my running speed v_{me} *relative to the escalator steps* is the same in each case. What is my running speed?

Solution: First, it is important to determine what your net speed is going down and up the escalator is. Your total speed going down is the sum of your running speed and the escalators speed, $v_{me} + v_e$. Going up, your running speed is countered by the escalators speed, hence your net speed is $v_{me} - v_e$

$$v_{up} = v_{me} + v_e \qquad v_{down} = v_{me} - v_e$$

Let t be the time it takes for you to go down the escalator. Using the simple relation of distance = rate x time we can see that $t = \frac{L}{v_{me}+v_e}$. Knowing that the time going up is 3 times the time going down, we can see that $3t = \frac{L}{v_{me}-v_e}$. Therefore:

$$\frac{L}{3(v_{me} - v_e)} = \frac{L}{v_{me} + v_e}$$

$$3(v_{me} - v_e) = v_{me} + v_e$$

$$2v_{me} = 4v_e$$

$$\boxed{v_{me} = 2v_e}$$

Your running speed is **twice** the speed of the escalator.

2b) 3pt

How many steps do I pass on the way up the escalator? How many on the way down?

Solution: There are a multitude of ways to approach this problem. First an analytic approach:
Running down the escalator at a speed of v_{me} I will run a distance of $v_{me} \cdot t$ where t is defined in 2a). This must equal the total number of steps I pass, n, times ΔL. Substituting $t = \frac{L}{v_{me}+v_e}$:

$$n_{down}\Delta L = v_{me} \cdot \frac{L}{v_{me} + v_e}$$

Similarly, going up:

$$n_{up}\Delta L = v_{me} \cdot \frac{L}{v_{me} - v_e}$$

taking our solution from 2a), $v_{me} = 2v_e$:

$$\boxed{n_{down} = \frac{2Lv_e}{3\Delta L v_e} = \frac{2L}{3\Delta L}}$$

$$\boxed{n_{up} = \frac{2Lv_e}{\Delta L v_e} = \frac{2L}{\Delta L}}$$

An alternate more intuitive approach would be to consider how an escalator works. If you are moving down the escalator twice as fast as the descending steps, 1 step descends every 2 steps that you descend. You cannot pass by steps that have already descended below the floor, hence you only travel 2/3 the length of the escalator. If you are traveling up a descending escalator, the escalator is producing a new step for every 2 steps you take. Essentially, you only move the distance of 1 step for every 2 steps taken, thus you must travel double the normal amount of steps to reach the top.

2c) 4 pts

A small child starts running up the escalator with a running speed $v_{child} = 1.0$ m/s relative to the escalator steps, but when she gets half way to the top (in distance, relative to the fixed building. she starts decelerating at a rate of $a_{child} = 0.02$ m/s^2 as she gets tired. Let us also assume $v_e = 0.4$ m/s, $\Delta L = 0.3$ m, and $L = 16$ m. Does she make it to the top of the escalator? If so, what is her speed *relative to the fixed building* at the top? If not, how close does she get to the top at closest approach?

Solution: One particularly elegant approach to this problem is to use the equation $v_f^2 = v_o^2 + 2a\Delta x$ to describe the motion of the child in the top half of the escalator. If we use Δx = L/2 = 8 m, and a = -0.02 m/s^2 and $v_o = v_{child} - v_e = 1.0 - 0.4 = 0.6$ m/s (relative to the building), then if v_f is a real number, the child makes it up the escalator and her speed is v_f at the top (already relative to the fixed building).
Indeed, when we plug in the numbers:

$$v_f^2 = v_o^2 + 2a\Delta x = (0.6)^2 + 2(-0.02)(8) = 0.36 - 0.32 = 0.04$$

$$\boxed{v_f = 0.2\, m/s}$$

The child **makes it up the escalator**! Her speed at the top relative to the fixed building is 0.2 m/s.

Another approach is to use the equation $x_f = x_o + v_o t + \frac{1}{2}at^2$ to solve for the time it takes for the child to reach the top. Plugging in the same values as before you find that:

$$16 = 8 + 0.6t + \frac{1}{2}(-0.02)t^2 \rightarrow t = \frac{-0.6 \pm \sqrt{(0.6)^2 - 4(-0.01)(-8)}}{-0.02} = (30 \pm 10)\text{s}$$

The correct time to use is 20 seconds, because 40 seconds is the time that the child theoretically reverses back to the escalator.

The child's final speed v_f can be found using the equation:

$$\begin{aligned} v_f &= v_o + at \\ &= 0.6 - 0.02(20) \\ &= 0.2\,\mathrm{m/s} \end{aligned}$$

We get the same solution as before! She makes it to the top with a speed of 0.2 m/s relative to the escalator.

Note: it is crucial to use the correct relative speeds for calculations in this question. The values used must be consistent, all values must either be in the reference frame of the escalator or in the reference frame of the fixed building.